EE480

Review for Exam 2

Friday, Oct. 21st. 2021

- Impulse Response

$\delta(n) * \text{anything} = \text{anything}!$

$y(n) = x(n) * h(n) =$

↑ "Build"

↑ Delta Function

(Kronecker)

Assuming No memory: Zero-State Response

AKA: Kick the Box!

- Convolution!

- Math way

Keep! Flip & Shift

$$\sum_{k=-\infty}^{k=\infty} h(k)x(n-k)$$

$x(n) = \alpha^n u(n)$

↑ Amplitude

Can be complex!

- Drawing way

(flip shift method)

$y(n) = x(n) * h(n)$

$h(n) = \alpha^{n} u(n)$

General Def!

$x(n) = \beta^{n} u(n)$

Can be

Const

exponent

(sins, cosins)

finite & stable

Shift!

$$y(n) = \sum_{k=-\infty}^{k=\infty} x(k)\, h(n-k)$$

Flip!

Goes to ∞

Delay!

$$y(n) = \sum_{k=-\infty}^{k=\infty} \beta^{k} u(k)\, \alpha^{n-k}\, u(n-k)$$

Unit step!

$$y(n) = \sum_{k=0}^{k=n} \beta^{k} \alpha^{n-k}$$

$u(0) = 1$ $n \geq 0$

else 0 $n < 0$

$$= \alpha^{n} \sum_{0}^{n} \left(\frac{\beta}{\alpha}\right)^{k} = \boxed{\alpha^{n} \left(\frac{1-\left(\frac{\beta}{\alpha}\right)^{n+1}}{1-\left(\frac{\beta}{\alpha}\right)}\right) = y(n)}$$

∞ $|\alpha| < 1$

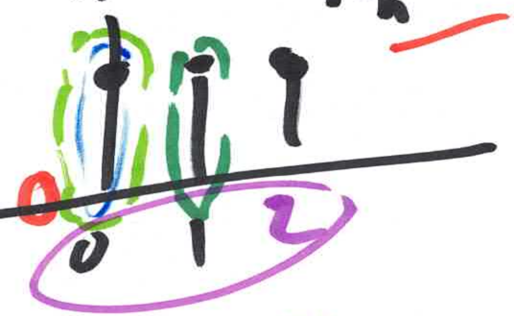
Geometric Series

$$\text{Rule: } \sum_{k=0}^{k=N} \alpha^{k} = \frac{1-a^{N+1}}{1-a}$$

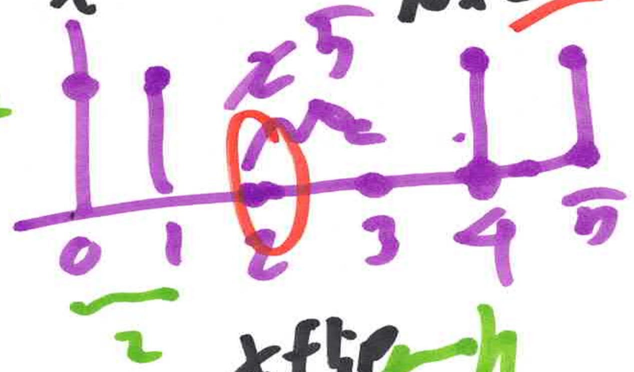
2)

Convolution: Drawing

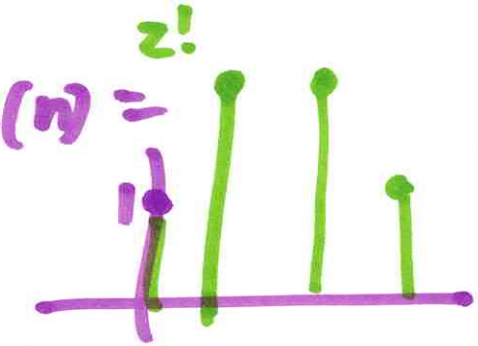
h $N_h = 3$



x Area! = 2 $N_x = 2$

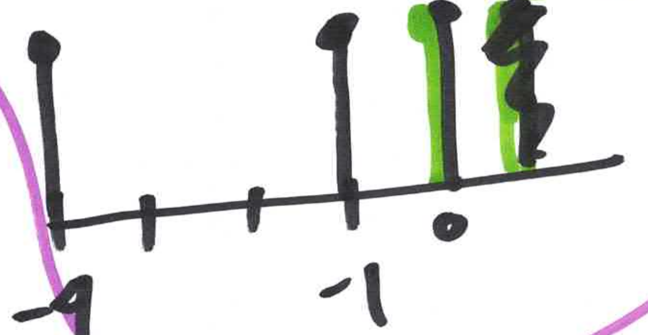


y(n) =

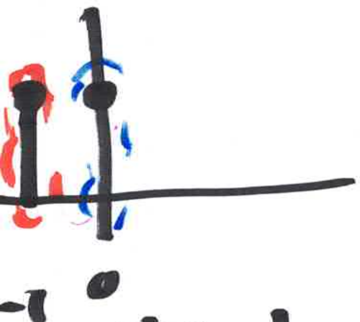


$N_y = N_h + N_x - 1$
$= \boxed{4}$

xflip ← h



xflip



xflip − 1

0,1,2

$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k)$

$u(n) = u(n-$

Raw def: of Convolution

back words!

$y(0) = h(0)x(0) + h(1)x(-1) + h(2)x(-2)$

forward!

$y(1) = h(0)x(1) + h(1)x(0) + h(2)x(-1)$

$y(2) = h(0)x(2) + h(1)x(1) + h(2)x(0)$

$y(3) = h(0)x(3) + h(1)\cancel{x(2)} + h(2)x(1)$

Fixed!

Doesn't exist

h x x ← Same & Shift

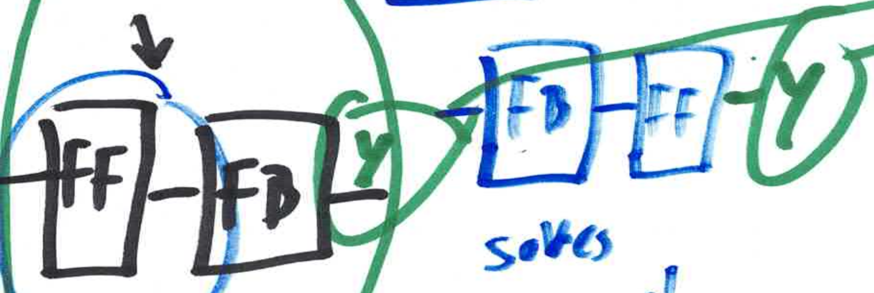
$[1 \times N_H]$ $[N_H \times 1]$

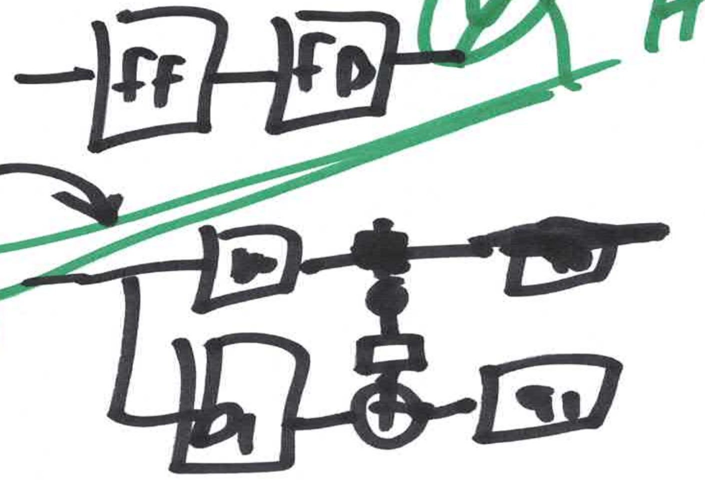
Matrices in MATLAB!

Impulse Response! - Building!

Block Diagram

DFI DFII TDFII





All give same output!

∴ Linear!

Direct!

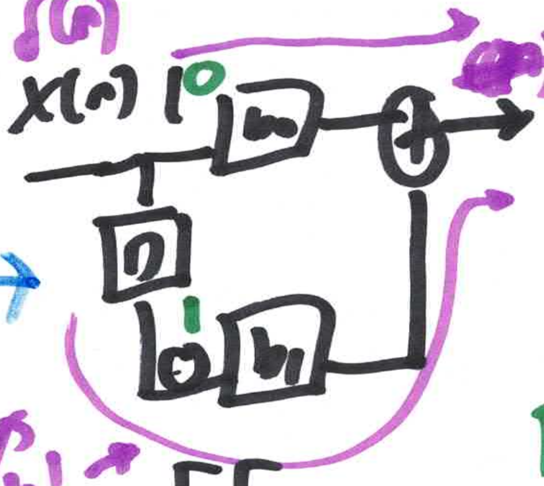
solves memory! but harder to stabilize

Dont worry!

Build me FF circuit!

$$y(n) + \alpha_1 y(n-1) = b_0 x(n) + b_1 x(n-1)$$

$$y(n) = b_0 x(n) + b_1 x(n-1) - a_1 y(n-1)$$



$$= h_{zeros}(n) = b_0 \delta(n) + b_1 \delta(n-1)$$

Built ~~First~~ first part!

Always stable!

FF

FIR → Finite Impulse Response

$$C(n) = h_{zeros}(n) * x(n)$$

δ(n)

$$= h_{zeros}(n)!$$

Take part!

u(n)

d(n)

built!

?

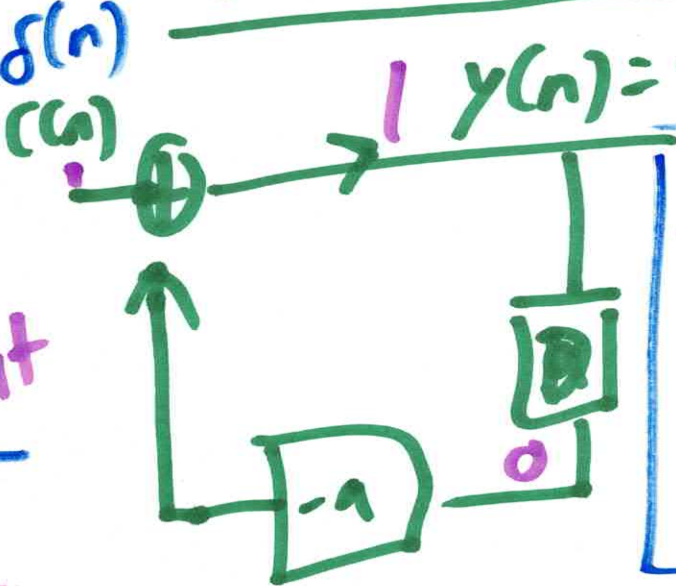
Poles FB

Ignore! (for now!)

$$y(n) + \underline{a_1}\, y(n-1) = b_0 x(n) + b_1 x(n-1)$$

$$y(n) = -a_1 y(n) + C(n)$$

Build me FB Circuit



$y(0) = 1$
$y(-1) = 0$
⋮
$y(-N) = 0$

Conditions for FB circuit Impulse Resp (ZSR)

ZSR → no memory!

$$y(n) + a_1 y(n-1) = \cancel{b_0 x(n) + b_1 x(n-1)}$$

Ignore

$$\lambda\left(\lambda^{-0} + a_1 \lambda^{-1} = 0\right)$$

$$y(n-N) \to \underset{\uparrow}{\lambda^{-N}}$$

Characteristic!

$$\lambda + a_1 = 0$$

$$\boxed{\lambda = -a_1}$$

Guess:

$$y_{puls}(n) = c_1 \lambda_1^n$$

$$= c_1 (-a_1)^n$$

$$y_{puls}(n) = \underset{?}{c_1}$$

from Conditions!

$$y_p(0) = 1 = c_1(-a_1)^0$$

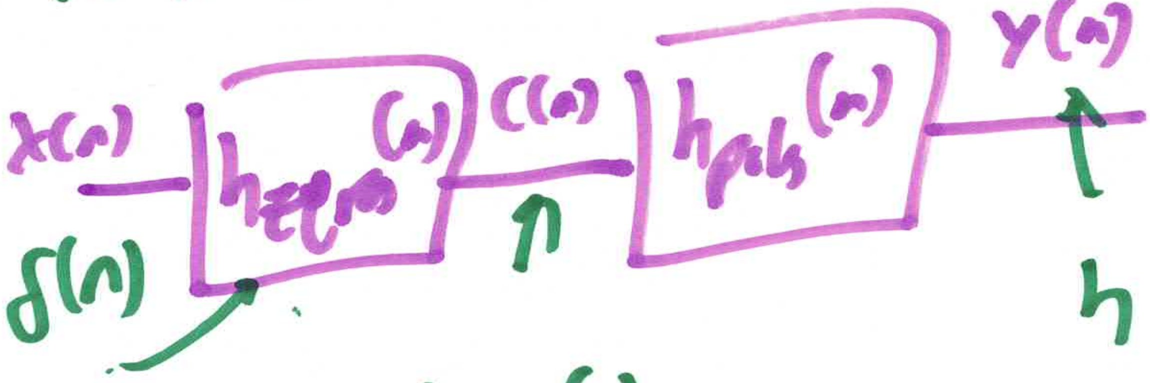
$$\therefore c_1 = 1$$

$$y_{puls}(n) = \boxed{h_{puls}(n) = (-a_1)^n u(n)}$$

Built FD Impulse Response!

Don't forget!

Put it together!



$y(n) = h_{zeros}(n) * h_{poles}(n)$

h(n) !!!

$h_{zeros}(n)$

$\rightarrow y(n) = x(n) * h(n)$

Zero-State Response (Impulse Response of a System)

$y(n) + a_1 y(n-1) = \cancel{b_0 x(n)} + \cancel{b_1 x(n-1)}$

ZIR

(Zero Input)

$x(n) = 0$

$y(n) + a_1 y(n-1) = 0$

Aux eqns!

Dr. Stubberud's ex:

Conditions ↗

memory!!

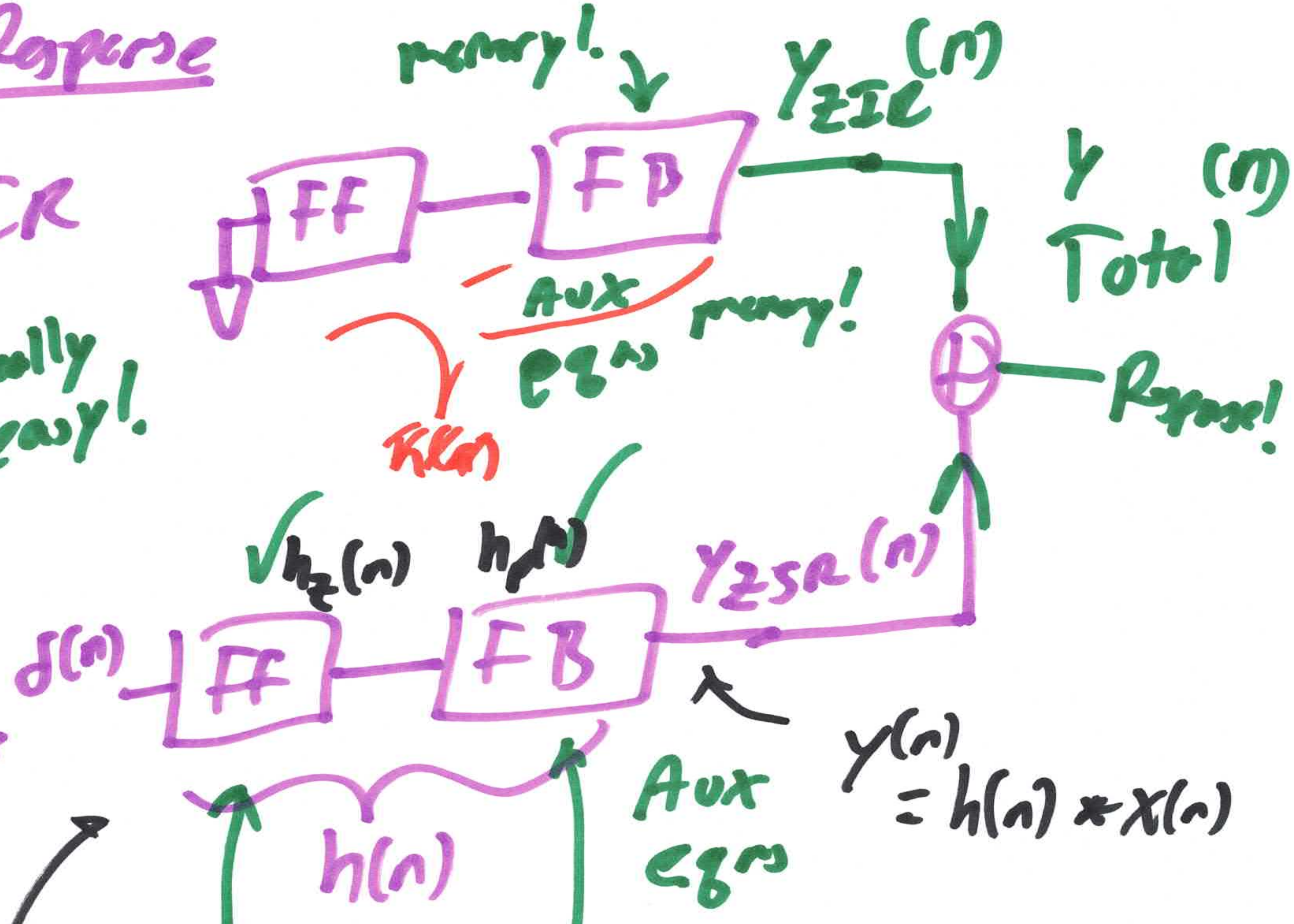
ex: $y(0) = 5,\ y(-1) = -2$

Total Response

~~ZSR~~ 1. ZIR

Solve First

Really easy!



2. ZSR

Impulse Response

Build it!

2nd order

Conditions

Easy numbers!

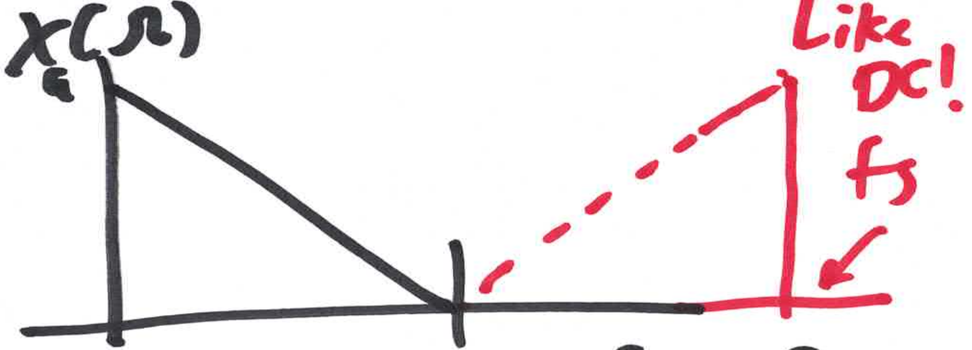
$$y(n) + a_1 y(n-1) + a_2 y(n-2)$$

$$\rightarrow \lambda^2 + \lambda a_1 + a_2 = 0$$

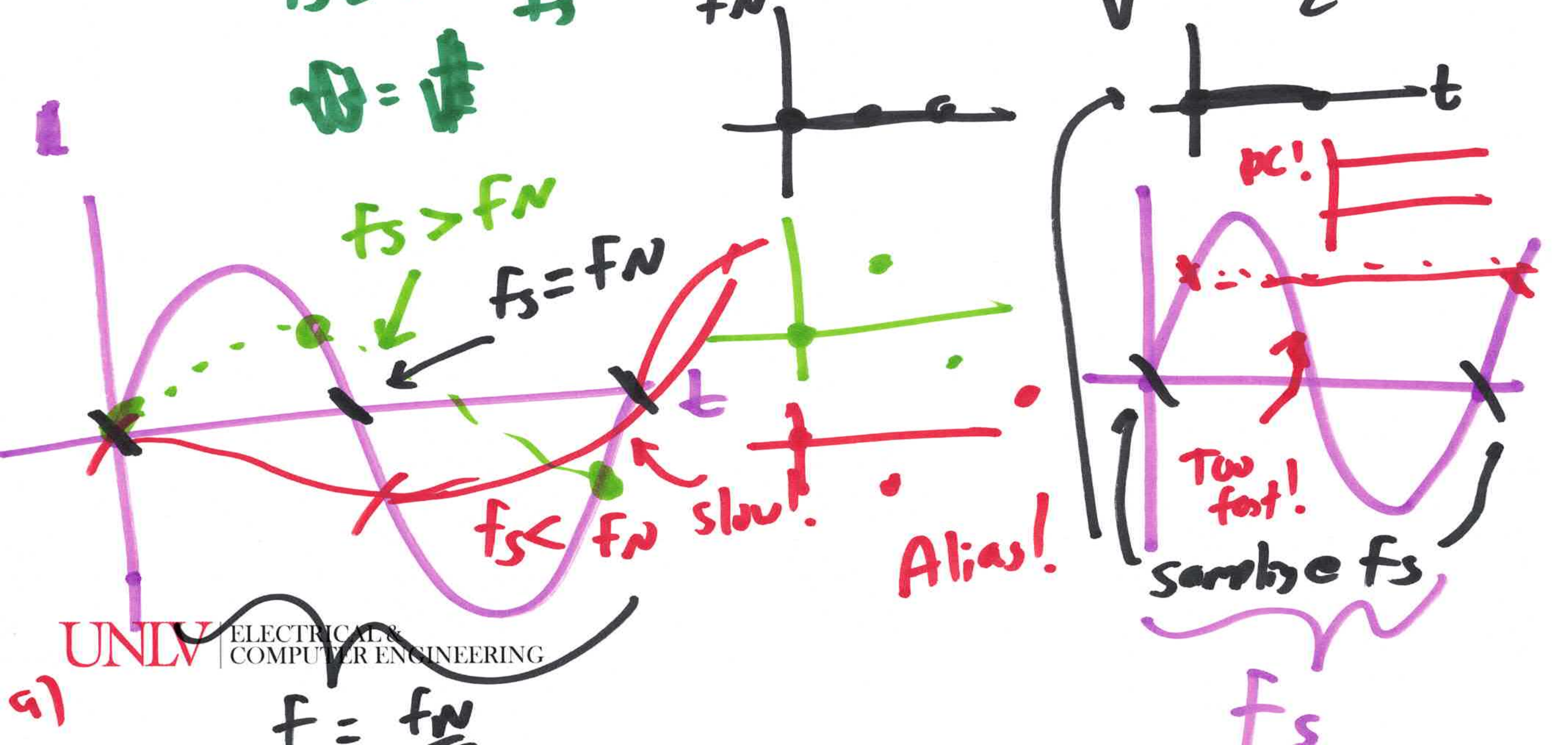
Sampling

$T_s = T = \frac{1}{f_s}$

Nyquist frequency:



$f_c < \frac{f_s}{2} = f_N$

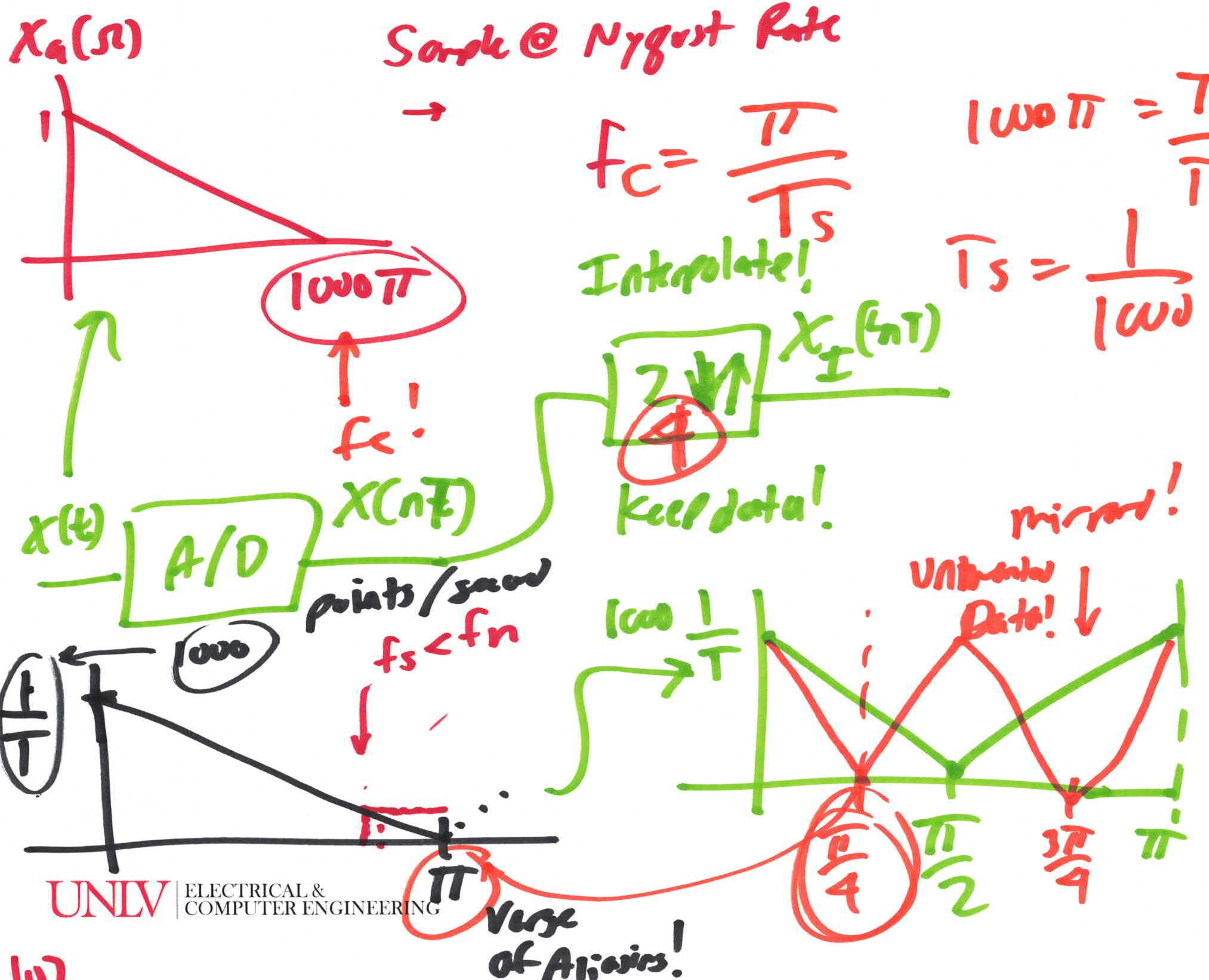


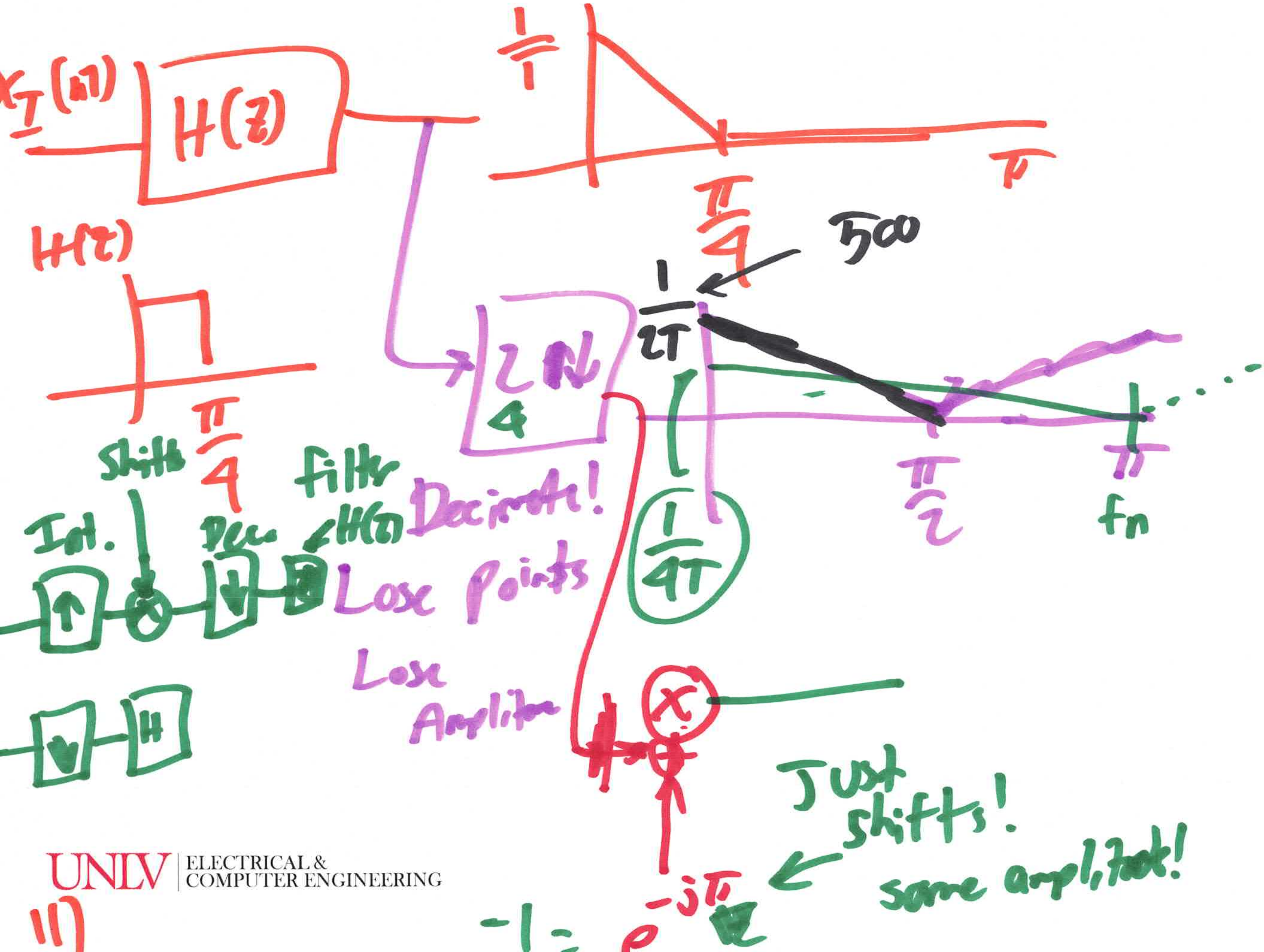
Sample @ Nyquist Rate

→ $f_c = \frac{\pi}{T_s}$

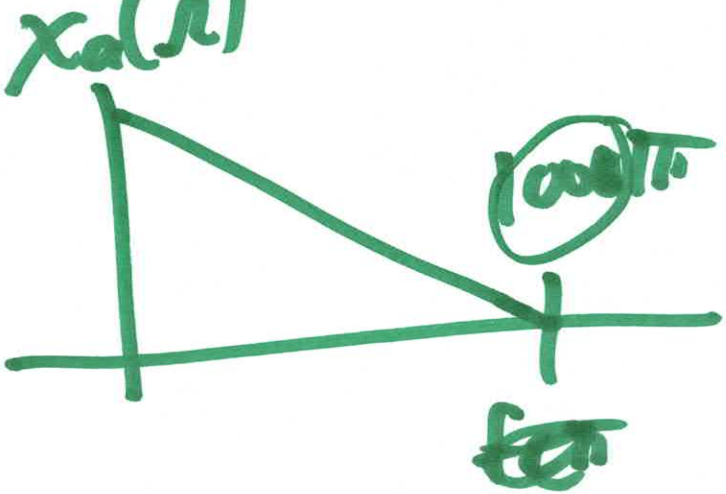
$1000\pi = \frac{\pi}{T_s}$

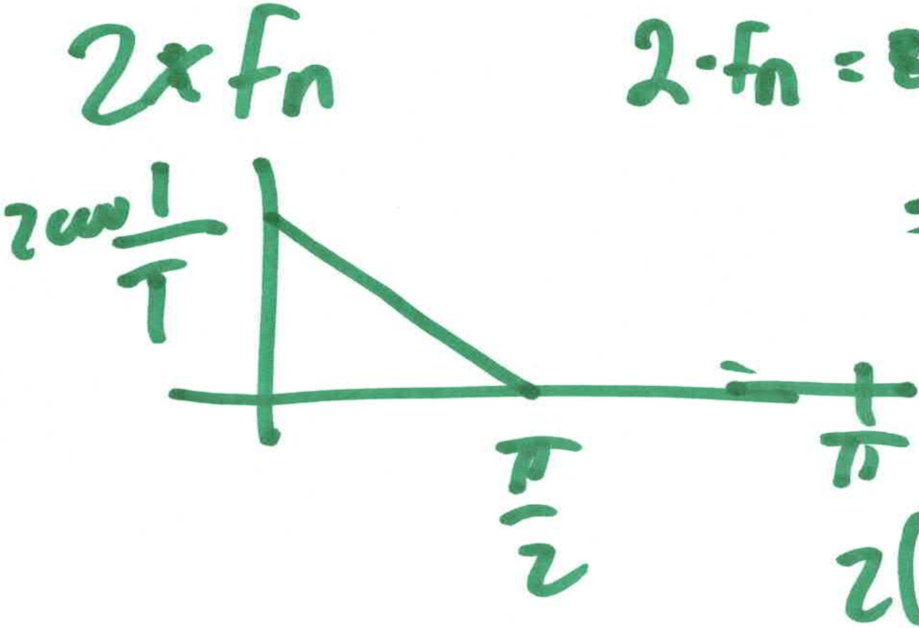
$T_s = \frac{1}{1000}$





x_T(nT)
H(z)
1
π/4
π
H(z)
π/4
Shifts
filter
Int.
Deci
H(z)
Decimate!
Lose Points
Lose Amplitude
↓
H
↓4
1/2T
500
1/4T
π/2
π
fn
Just shifts!
same amplitude!
-1 = e^{-jπ}





$2 \cdot f_n = \frac{\pi}{T}$

$2(1000\pi) = \frac{\pi}{T}$

$T = \frac{1}{2000}$

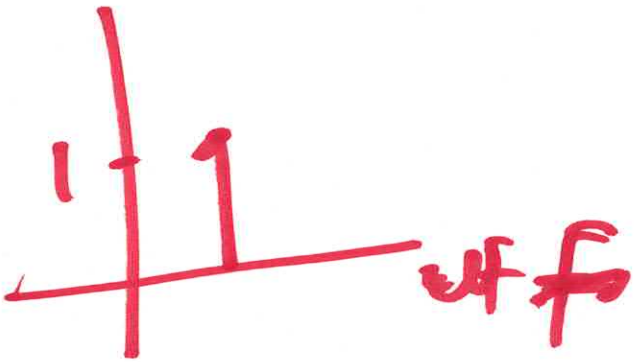
Fourier Transform ← Periodic

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)x(n-k) = \sum_{-\infty}^{\infty} h(k)\, e^{j\frac{\pi}{2}(n-k)}$$

$$x = e^{jw_0 n} \quad 2\pi f_0 \quad e^{j\frac{\pi}{2}n}$$

$$= e^{j\frac{\pi}{2}n} \sum_{-\infty}^{\infty} h(k)\, e^{-j\frac{\pi}{2}k}$$

w_0



$$y(n) = e^{j\frac{\pi}{2}n} H(e^{jw})$$

$x(n)$!

$w = \frac{\pi}{2}$!

$$y(n) = |H(e^{jw})|\, e^{j\frac{\pi}{2}n}\, e^{j\angle H}$$

magnitude phase

$$y(n)+a_1y(n-1)+a_2y(n-2)=b_0x(n)+b_1x(n-1)+b_2x(n-$$

$$H(e^{jw})=\frac{Y(e^{jw})}{X(e^{jw})}$$

$$\underbrace{\left(1e^{-j0}+a_1e^{-jw}+a_2e^{-j2w}\right)}_{Y}Y(e^{jw})$$

$$=\left(b_0e^{-j0}+b_1e^{-jw}+b_2e^{-j2w}\right)X(e$$

$$y(n-N)\rightarrow e^{-jNW}X(e^{jw})$$

Rule!

$$x(n-N)\rightarrow X(e^{jw})e^{-jNW}$$

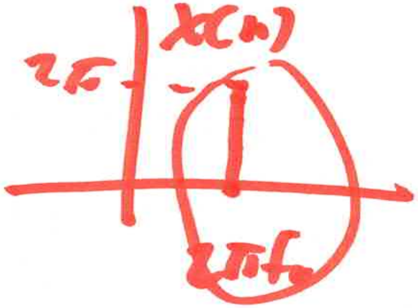
$$\frac{Y(e^{jw})}{X(e^{jw})}=\frac{b_0+b_1e^{-jw}+b_2e^{-j2w}}{1+a_1e^{-jw}+a_2e^{-j2w}}=H(e^{jw})$$

$$y(e^{j\omega}) = H(e^{j\omega})\Big|_{\omega=\frac{\pi}{2}} \underline{X(e^{j\omega})}$$

$$= \left(\frac{b_0 + b_1 e^{-j\frac{\pi}{2}} + b_2 e^{-j\frac{2\pi}{2}}}{a1 + a_1 e^{-j\frac{\pi}{2}} + a_2 e^{-j\frac{2\pi}{2}}}\right) 2\pi\delta(\omega - \tfrac{\pi}{2})$$

Comes from Input frequency

???



Memorize!

$$X(e^{j\omega}) = 2\pi\delta(\omega - \tfrac{\pi}{2})$$

easy $e^{+j\frac{\pi}{2}n}$

T → F

hard

F → T

Davids way

$$y(n) = \underset{\text{mag!}}{|H(e^{jw})|}_{w=\frac{\pi}{2}} x(n) \cdot e^{j\frac{\pi}{2} + \angle H(e^{jw})_{w=\frac{\pi}{2}}}$$

Phase!

Residues!

~~Things to expect:~~

z-Transform

Same as FT:

$$y(n-N) \rightarrow Y(z) z^{-N}$$

Partial fraction Expansion:

$$\cancel{\frac{\cdots}{(1+z^{-1})(1-z^{-1})}} \quad \frac{(1+b_1 z^{-1})}{(1+a_1 z^{-1})(1+g z^{-1})}$$

$$= \frac{A}{(1+c z^{-1})} + \frac{B}{(1+g z^{-1})}$$

mimic

$$z(u(n)) = \frac{z}{z-1} \rightarrow \frac{1}{1-z^{-1}}$$

$$\mathcal{Z}\left(\alpha^n u(n)\right) \rightarrow \left(\frac{1}{1-\boxed{a}z^{-1}} \right) \leftarrow \text{Unit step}$$

time domain

$$\downarrow$$

$$\underline{(a)^n u(n) + b^n u(n)} \quad \ldots$$

Expect

ZT, FT, Impulse Response / Sampling (Drawn!) / Convolution?
ZSR ???