EE 361

Stochastic Signals & Probability

(PART) II !!

MAY 2^{nd}, 2021

Sample Space $= \{ X, O, \Delta, \square \}$

$P(x) = \frac{1}{4} P(\Delta)$

$P(o) = \frac{1}{8} P(\Delta)$

$P(\square) = \frac{3}{8} P(\Delta)$

$P(x) + P(o) + P(\Delta) + P(\square) = 1$

$P(x) + P(o) + P(\Delta) + \frac{3}{8} P(\Delta) = 1$

$\frac{1}{4} P(\Delta) + \frac{1}{8} P(\Delta) + P(\Delta) + \frac{3}{8} P(\Delta)$

$P(\Delta) \left\{ \frac{1}{4} + \frac{1}{8} + 1 + \frac{3}{8} \right) = 1$

$P(\Delta) \left(\frac{7}{4} \right) = 1$

$\boxed{P(\Delta) = \frac{4}{7}}$

Event $\longrightarrow X\{P(\cdot)\}$

$P(X) = \frac{1}{7} +$

$P(o) = \frac{1}{14} +$

$P(\square) = \frac{3}{14} +$

$P(\Delta) = \frac{4}{7}$

$= \frac{7}{7}$

$X\{P(\Delta)\} = -10$

$X(P(o)) = 10$

$X(P(x)) = 5$

$X(P(\square)) = -5$

1)

$$\text{Expected Value} = P(x)\cdot X(P(x)) + P(O)X(O) + \underline{P(\Delta)X(\Delta)} + P(\Box)X(\Box)$$

$$= \frac{1}{7}(5) + \frac{1}{14}\cdot 10 + \frac{4}{7}(-10) + \frac{3}{14}\cdot(-5)$$

$$= \frac{5}{7} + \frac{10}{14} - \frac{40}{7} - \frac{15}{14}$$

$$= \frac{-35}{7} - \frac{5}{14}$$

$$\boxed{= \frac{-70}{14} - \frac{5}{14} = \frac{-75}{14}}$$

2)

$$SNR = \frac{\text{Power Signal}}{\text{Power Noise}} = \frac{\sigma_S}{\sigma_N}$$

p(x)

$$y(t) = \cos(t) + X(t)$$

$$E(x) = \int_a^b p(x) \cdot x \, dx$$

$$= \alpha \int_{-1}^{1} x \, dx = \alpha \frac{x^2}{2} \Big|_{-1}^{1}$$

$$= \alpha \left(\frac{1}{2} - \frac{(-1)^2}{2} \right) = 0$$

$$\boxed{\int_a^b p_x(x)\, dx = 1}$$

$$\alpha \int_{-1}^{1} dx = 1$$

$$\alpha x \Big|_{-1}^{1} = \alpha(1-(-1))$$

$$= 2\alpha = 1 = p(x)$$

$$\boxed{\alpha = \frac{1}{2}}$$

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$$y(t) = \cos(t) + X(t)$$

$$P_{avr} = \frac{1}{T}\int_{-\Delta T}^{+\Delta T} \sin^2(x)\,dt = \frac{1}{T}\int_{-\Delta}^{\Delta} X^2(t)\,dt$$

mean

squared

$$m_y = m_x \sum_{-\infty}^{\infty} h(n) = 0$$

$$\sigma_y = E(y^2) - m_y^2$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos^2(t)\,dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1+\cos(2t)}{2}\,dt$$

$$= \cancel{\frac{1}{2\pi}\left[\frac{t}{2} + \cos(2t)\right]_{-\pi}^{\pi}}$$

$$= \frac{1}{2\pi}\left[\frac{\pi-(-\pi)}{2}\right] = \frac{1}{2\pi}\left[\frac{2\pi}{2}\right] = \boxed{\frac{1}{2}}$$

$$+ \left(\frac{\overset{0}{\sin(2\pi)} - \overset{0}{\sin(-2\pi)}}{2}\right)$$

$$= \sigma_y$$

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4)

$$SNR = \frac{\sigma_y}{\sigma_N} = \frac{\frac{1}{2}}{\frac{1}{3}} = \frac{1}{2}\cdot\frac{3}{1} = \boxed{\frac{3}{2}}$$

$$\sigma_N^2 = \frac{1}{2}\int_{-1}^{1} x^2\,dx$$

$$= \frac{1}{2}\,\frac{x^3}{3}\Big|_{-1}^{1}$$

$$= \frac{1}{6}\left(1+(+1)^3\right)$$

$$= \boxed{\frac{1}{3}}$$

$X(t) = $

$y(t) = x(t) - c$

$P_x(x)$ α

0 1

$$1 = 1\cdot\alpha = 1$$

$$E(x) = 1\int_0^1 x\,dx$$

$$= 1\,\frac{x^2}{2}\Big|_0^1 = \boxed{\frac{1}{2}}$$

$$\sigma_x^2 = E(x^2) - m_x^2$$

$$\int_0^1$$

$$\sigma_y^2 = E(y^2) - [E(y)]^2$$

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~~A~~)

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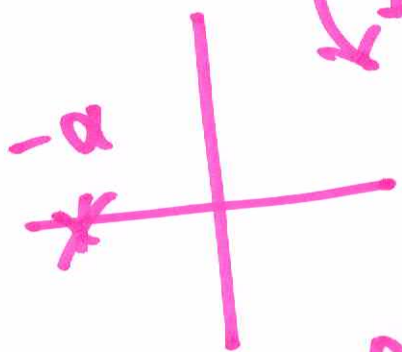
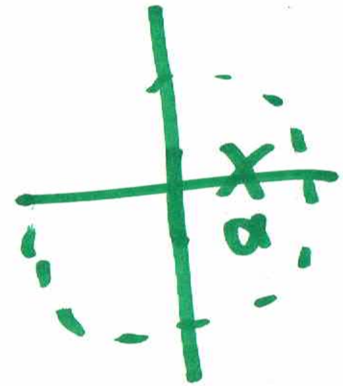
LaPlace Transform $L(s) = \frac{Re}{s + \alpha}$

Inv: $f(t) = e^{-\alpha t} u(t)$



Z transform $F(z) = \frac{1}{1 - \alpha z^{-1}}$



$f(n) = (\alpha)^n u(n)$

$$\frac{1}{(s-p_1)(s-p_2)} \to \frac{A}{s-p_1} + \frac{B}{s-p_2}$$

or $\sigma_y^2 = \sigma_x^2 \int_{-\infty}^{\infty} h^2(t)$

LAPlace TRANSform
Analog/Continuous

$$\sigma_y^2 = \sigma_x^2 \sum_{\infty}^{\infty} h^2(n)$$

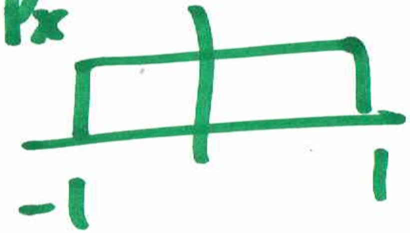
Z-TRANSform
Discrete

$h(t) = e^{-t} u(t)$

$$\int_{-\infty}^{\infty} h^2(t) = \int_{-\infty}^{\infty} e^{-2t} u(t) = \int_{0}^{\infty} e^{-2t}$$

$h(t) = e^{-t} u(t)$

$$\boxed{m_y = m_x \int_{-\infty}^{\infty} h(t)}$$



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DFTS/DTFTs

z-TRANsfm