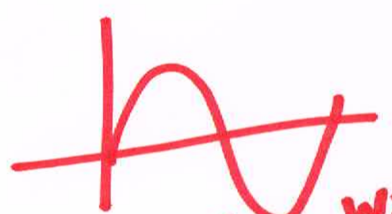
The Discrete Fourier Transform

April 17th, 2021

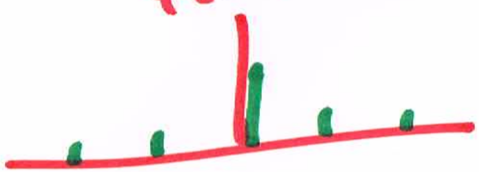
Cont. (Analog) FS

f(t)



$w = \frac{2\pi n}{N}$

$F(e^{jw})$



f(n)

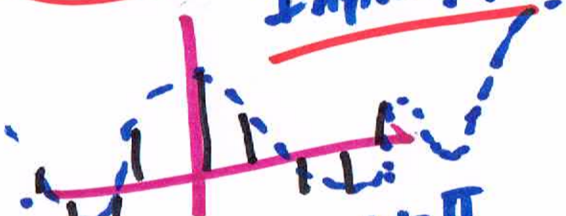
$X(e^{jw}) = \sum_{n=-\infty}^{n=\infty} x(n) e^{-jwn}$

$w = \frac{2\pi K}{N}$

Related

DTFT

Infinite/Aperiodic



Related to Z-Transform

Fourier Transform

$2\pi K$

$-jw\frac{2\pi k}{N}$

$n = N-1$

$f_p(t) = \sum_{k=-\infty} d_k e$

f(n)

$w = -\pi$ $w = \pi$

2π

$F(e^{jw})$

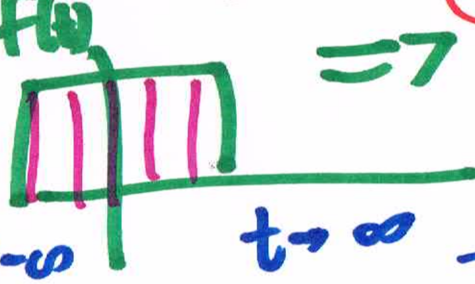
DFT

Finite/Periodic

$X(e^{jwn}) = \sum_{n=0}^{N-1} x(n)$

$W = \frac{2\pi}{N}$

f(t)



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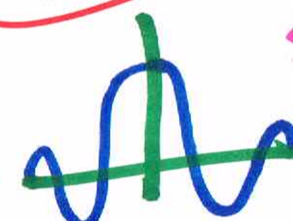
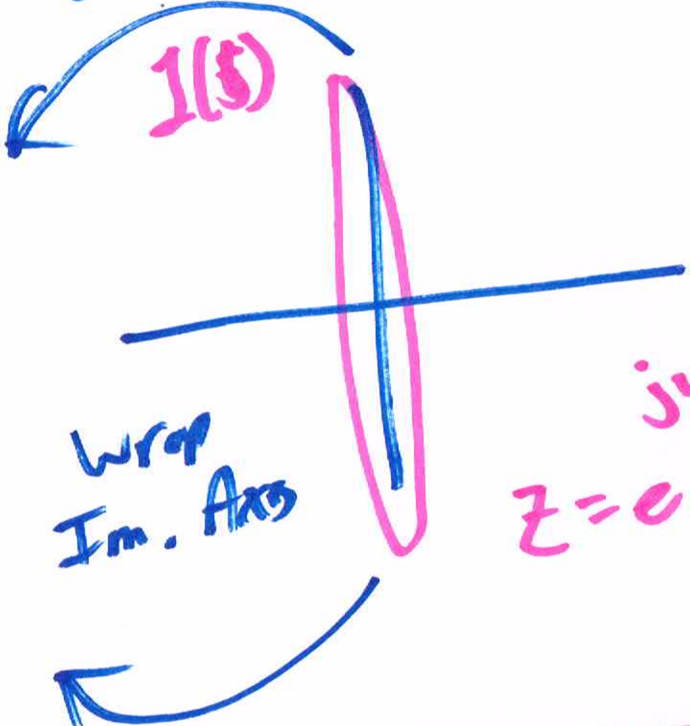


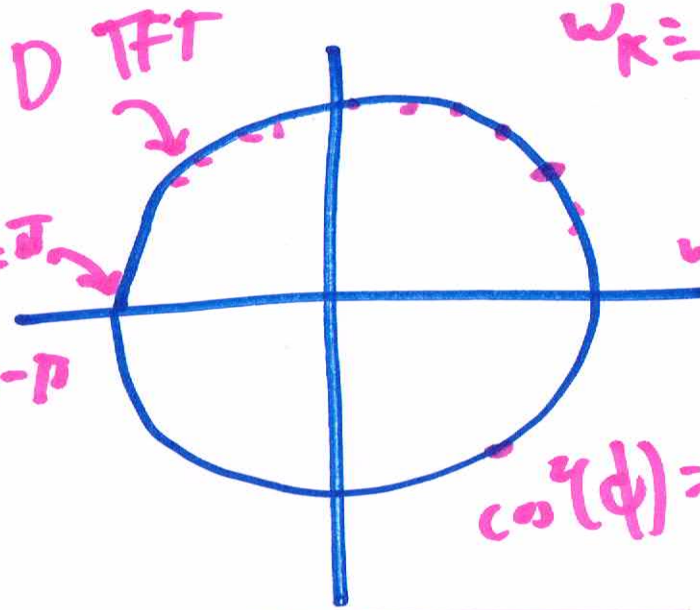
$t \to -\infty$ $t \to \infty$

Infinite/Aperiodic

1)

DTFT $\to \mathcal{L}(s) \Rightarrow$ ROCs $F(s=j\omega) \to z$





$w_K = \frac{2\pi K}{N}$

$\cos^2(\phi) = \frac{1}{2}[1+\cos(2\phi)]$

$b_K^{(n)} = \cos\left(\frac{2\pi}{N} K n\right) \quad 0 \le n \le N-1$

$\langle b_0, b_0 \rangle = \sum_{n=0}^{N-1} (1)\cdot(1) = N$

$\langle b_K, b_K \rangle = \sum_{n=0}^{N-1} \cos\left(\frac{2\pi k n}{N}\right)\cos\left(\frac{2\pi k n}{N}\right)$

$= \sum_{n=0}^{N-1} \frac{\left(1 + \cos\left(\frac{4\pi k n}{N}\right)\right)}{2}$

$$\begin{bmatrix} \langle b_0, b_0 \rangle & \langle b_1, b_0 \rangle & \cdots & \langle b_{N-1}, b_0 \rangle \\ \langle b_0, b_1 \rangle & \langle b_1, b_1 \rangle & \cdots & \\ & & \cdots & \langle b_{N-1}, b_{N-1} \rangle \end{bmatrix}$$

1)

$$\sum_{n=0}^{N-1} \frac{\left[1+\cos\left(\frac{4\pi kn}{N}\right)\right]}{2} = \frac{N}{2} + \frac{1}{2}\sum_{n=0}^{N-1} \frac{1}{2}\left[e^{j\frac{4\pi kn}{N}} + e^{-j\frac{4\pi kn}{N}}\right]$$

$$= \frac{N}{2} + \frac{1}{4}\underbrace{\left[\frac{1-e^{j4\pi k}}{1-e^{j\frac{4\pi k}{N}}} + \frac{1-e^{-j4\pi k}}{1-e^{-j\frac{4\pi k}{N}}}\right]}_{0}$$

$1!$ $\quad$ $!$

$$\therefore \frac{N}{2}$$

$$\begin{array}{ccc} N\times N & N\times 1 & N\times 1 \\ \begin{bmatrix} N & 0 & \cdot \\ \cdot & \frac{N}{2} & 0 \\ \cdot & \cdot & \ddots \\ & & N \end{bmatrix} & \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{N-1} \end{bmatrix} = & \begin{bmatrix} N\alpha_0 \\ \frac{N}{2}\alpha_1 \\ \vdots \end{bmatrix} \end{array}$$

(For Cosine)

ex $\begin{bmatrix} N & \cdots & \vdots \\ \cdot & N & \\ \cdot & \sim & N \end{bmatrix}$

Sine $\begin{bmatrix} N/2 & \cdot & 0 & \cdot \\ 0 & \ddots & & 0 \\ \cdot & \cdot & & N/2 \end{bmatrix}$

$$\langle b_k, b_k\rangle = \sum_{n=0} b_k\cdot b_k$$

$$d_k = \frac{\langle f, b_k\rangle}{\langle b_k, b_k\rangle}$$

3)

$f(n) = \alpha_0 \underline{b_0(n)} + \alpha_1 b_1(n) \ldots \alpha_{15} b_{15}(n)$

Into Matlab

N=10

α_K

$= b_N \, [N-1 \times K] * [K \times 1]$

errnis = 1

Kom Time

$= f_p \, [N-1 \times 1]$

Exp. basis:

$\langle b_K, b_K \rangle = \sum_{n=0}^{N-1} e^{\frac{j2\pi kn}{N}} \cdot e^{-\frac{j2\pi kn}{N}}$