The Analog Fourier Series

April 1st 2021

$$f(t) = a_0 b_0(t) + a_1 b_1(t) \cdots a_{N-1} b_{N-1}(t)$$

$$= \sum_{n=0}^{n=N-1} a_n b_n(t)$$

$$b_k = e^{j\omega k t} \qquad c_k = \frac{\langle f(t), b_k(t) \rangle}{\langle b_k(t), b_k(t) \rangle}$$

$$\boxed{f(t) = \sum_{k=-\infty}^{k=\infty} c_k e^{-j\omega k t}}$$

$$f(t) = e^{t} \quad 0 \le t \le 1$$

$$c_k = \frac{\langle f(t), e^{jwkt} \rangle}{\langle e^{jwkt}, e^{jwkt} \rangle}$$

$$\langle e^{jwkt}, e^{jwkt} \rangle$$

$$\int_{t=0}^{t=1} e^{jwkt} \cdot e^{-jwkt} \, dt$$

$$\int_{0}^{1} e^{\overset{0}{\cancel{jwkt-jwkt}}} \, dt = \boxed{1}$$

$$\boxed{w = \frac{2\pi}{T}}$$

$$w = \frac{2\pi}{T}, \; T = 1$$

$$q = 1 - j2\pi k$$

$$\int_{t=0}^{t=1} e^{t} \cdot e^{-jwkt} \, dt = \int_{0}^{1} e^{(1-j2\pi k)t} \, dt = \int_{0}^{1} e^{qt} \, dt = \frac{1}{q} e^{qt} \Big|_{0}^{1}$$

$$\frac{1}{(1-j2\pi k)} \left(e^{1-j2\pi k} - e^{0} \right) = \boxed{\frac{1}{1-j2\pi k} \left(e^{1-j2\pi k} - 1 \right)}$$

2)

$$\text{Proj} \rightarrow f(t) = \ldots c_{-2} e^{j2\pi(-2)t} + c_{-1} e^{j(2\pi(-1)t} + C_0 e^{j2\pi(0)t} + C_1 e^{j2\pi(1)t} + C_2 e^{j2\pi(2)t} \ldots$$

$\text{Proj} = C(1)\, b(1) + \ldots$

$C(2)\, b(2) + \ldots$

$\ldots$

$t = 0 : 0.01 : 1$

1000

9

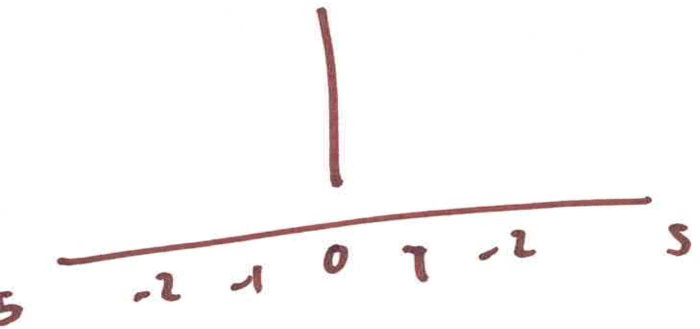
$b = \text{length}(t) \times (2N-1)$

$N = 5$

$C : (2N-1) \times 1$

$$b * c \quad \begin{bmatrix} c_a \\ c_2 \\ c_{\ldots} \end{bmatrix}$$

9×1

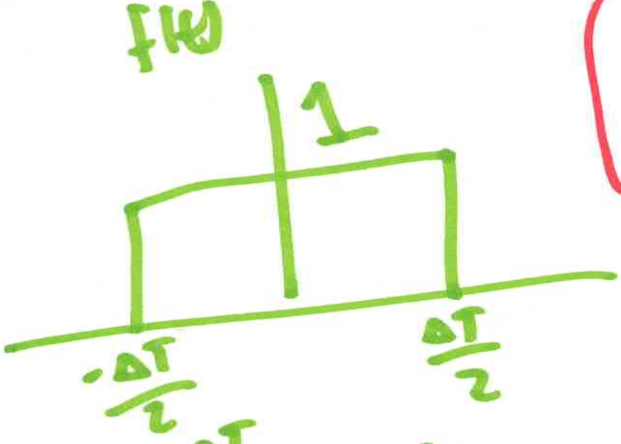


$$\text{Proj} = [1000 \times 9] * [9 \times 1]$$

$$= [1000 \times 1]$$

$$c_k = \frac{1}{1-j2\pi k}\left(e^{1-j2\pi k} - 1\right)$$

$$c_0 = \frac{1}{1-\cancel{0}}\left(e^{1-0} - 1\right) = \underline{e-1}$$



Fourier Transform:

$$F(j\omega) = \int_{t=-\infty}^{t=\infty} f(t)\, e^{-j\omega t}\, dt$$

$$F(j\omega) = \int_{-\frac{\Delta T}{2}}^{\frac{\Delta T}{2}} 1\cdot e^{-j\omega t}\, dt = \frac{-1}{j\omega} e^{-j\omega t}\Big|_{-\frac{\Delta T}{2}}^{\frac{\Delta T}{2}} = \frac{-1}{j\omega}\left(e^{-j\omega\frac{\Delta T}{2}} - e^{+j\omega\frac{\Delta T}{2}}\right)$$

$$\underline{\mathcal{F}(j\omega)} = \frac{f^1}{j\omega}\left(e^{+j\omega\frac{\Delta T}{2}} - e^{-j\omega\frac{\Delta T}{2}}\right)\frac{2}{2} \rightarrow \boxed{= \frac{2\sin\left(\frac{\omega\Delta T}{2}\right)}{\omega}}$$

$\int f \cdot \bar{b}^{-e}$

$-\infty$

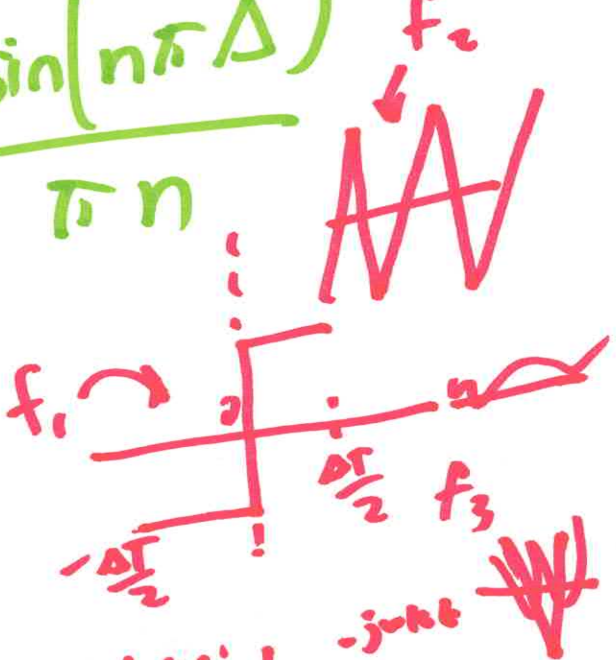
$$\mathcal{F}(j\omega) = \frac{2\sin\left(\frac{2\pi n}{T}\cdot\frac{\Delta T}{2}\right)}{\frac{2\pi n}{T}} = \frac{T\sin(n\pi\Delta)}{\pi n}$$

$$\omega = \frac{2\pi}{T}n$$

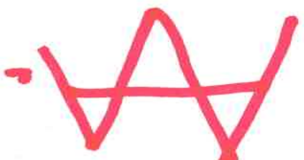
$\langle f, b\rangle$

$$c = \frac{\mathcal{F}(j\omega)\big|_{\omega=\frac{2\pi}{T}n}}{T} = \boxed{\frac{\sin(n\pi\Delta)}{\pi n}}$$

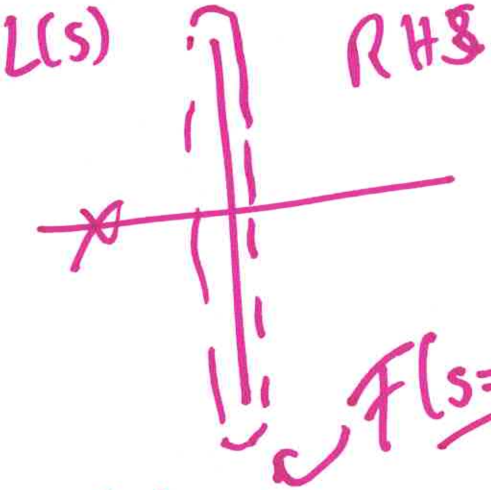
$\langle b . b\rangle \rightarrow T$



2nd way: $$c = \frac{\langle f, b\rangle}{\langle b, b\rangle} = \frac{\int_0^1 f e^{-j\omega k t}}{T}$$



$x(t) = e^{\alpha t}$ Laplace Transform $\mathcal{L}\{f(t)\}$



$$\mathcal{L}\{e^{\alpha t}\} = \frac{1}{s \mp \alpha}$$

$\mathcal{F}(s=j\omega)$

$$\mathcal{F}(j\omega) = L(s=j\omega) = \frac{1}{j\omega \mp \alpha}$$

$x(t) = e^{\alpha|t|}$

$x(t) = e^{\alpha t} u(t) + e^{-\alpha t} u(-t)$

$$L(x(t)) = \frac{1}{s-a} \overset{-}{\cancel{+}} \frac{1}{s+a}$$

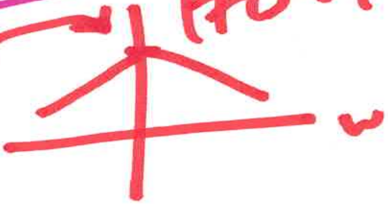
$a \le R_e(s)$ $R_e(s) \le -a$

$$\frac{2a}{s^2-a^2}$$

$$\mathcal{F}(j\omega) = L(s=j\omega) = \frac{2a}{(j\omega)^2 - a^2} = \frac{2a}{-\omega^2 - a^2}$$

$$\boxed{\mathcal{F}(j\omega) = \frac{-2a}{\omega^2 + a^2}}$$

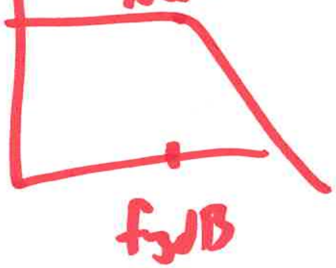
$$\mathcal{F}(0) = \frac{-2a}{a^2} = -\frac{2}{a}$$



c)

$RC\frac{dy(t)}{dt} + y(t) = x(t)$

3dB $H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$



$\mathcal{F}(j\omega) \Leftarrow$

$RC\left(sY(j\omega)\right) + Y(\overset{j\omega}{s}) = X(j\omega)$

$Y(j\omega)\left(sRC + 1\right) = X(j\omega)$

$\mathcal{F}\left[\mathcal{L}\left(\frac{dy}{dt}\right)\right] = \left\{sY(s)\right\}$

$s = j\omega$

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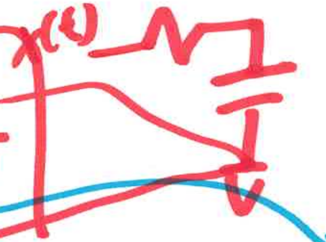
$$\boxed{H(j\omega) = \frac{Y(j\omega)}{X(j\omega)} = \frac{1}{sRC+1}}$$



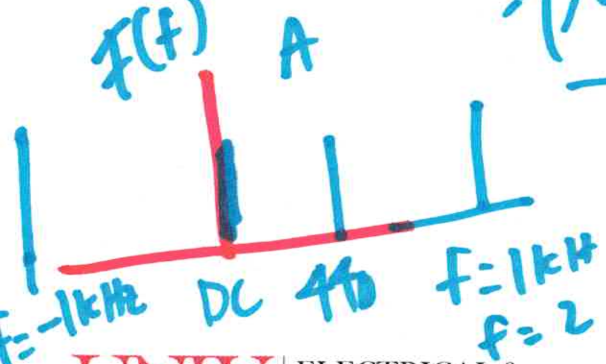
$\int e^{\pm 2t}\, e^{-2t}\, te$

$= j\omega Y(j\omega)$

$x(t) = 1 + 2\cos(2t)$ $f = 1kHz$

$$\boxed{X(j\omega) = 2\pi\delta(\omega) + 2\pi\left[\delta(\omega-2) \cancel{+} \delta(\omega+2)\right]}$$

$$\boxed{Y(j\omega) = H(j\omega)X(j\omega)} = 2\pi\left[\frac{\delta(\omega)+\delta(\omega-2)+\delta(\omega+2)}{1+j\omega RC}\right]$$



$$y(t) \overset{\sim}{=} L(s = \text{pole})\, e^{-st} = \text{Residue}\, e^{-\text{pole}\, t} + \dots$$

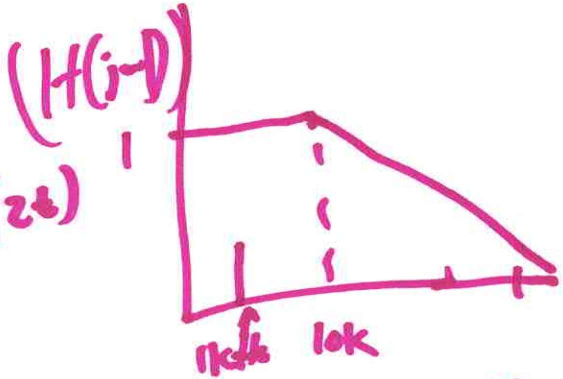
Laplace

$$f_{3dB} = \frac{1}{RC}$$

10KHz

$$X = \sin(1kHz)$$

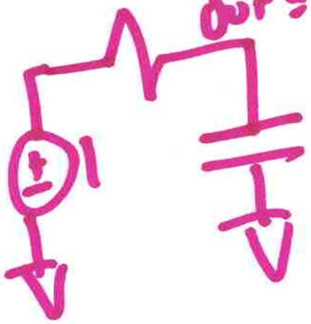
$$y(t) = x(t) \times |H(j\omega)|\, e^{j\angle H(j\omega)}$$



$$= \Big(1 + \underset{\omega = 2}{\underline{|H(j\omega)|\, e^{j\angle H(j\omega) + j(-2t)}}} + |H(j(-2))|\, e^{j\angle H(j(-2)) + j(2t)}\Big)$$

$$\frac{V_{out}}{V_{in}} = G \rightarrow V_{out} = G \cdot V_{in} \Big|_{f = f_0}$$

$$x = 1 + \underline{e^{-2t}} + e^{2t}$$



= [illegible]

$$= 1 + \frac{1 \angle \tan}{\sqrt{1 + 4(RC)^2}}$$

$$H(j\omega) = \frac{1}{1+j\omega RC}, \quad |H(j\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\sqrt{1+(\omega RC)^2}\right)$$

$$y(t) = 1 + \frac{1}{\sqrt{1+4(RC)^2}} < e^{-\tan\left(\sqrt{1+4(RC)^2}\right) \mp 2t} + \frac{1}{\sqrt{1+4(RC)^2}} < e^{+\tan\left(\sqrt{1+4RC^2} + 2t\right)}$$

$$\boxed{y(t) = 1 + \frac{1}{\sqrt{1+4(RC)^2}} \cos\left(\tan^{-1}\left(\sqrt{1+4(RC)^2}\right) + 2t\right)}$$

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