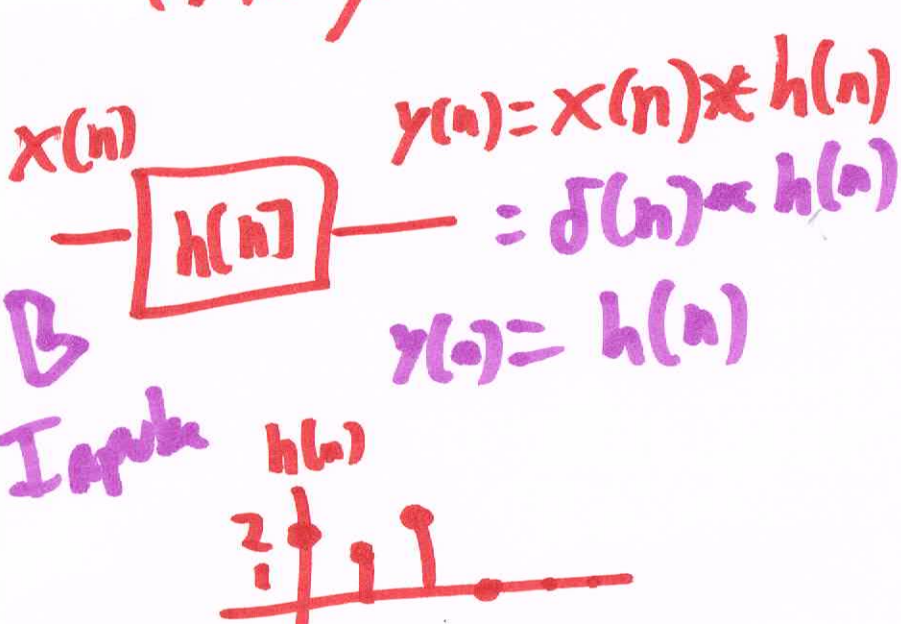
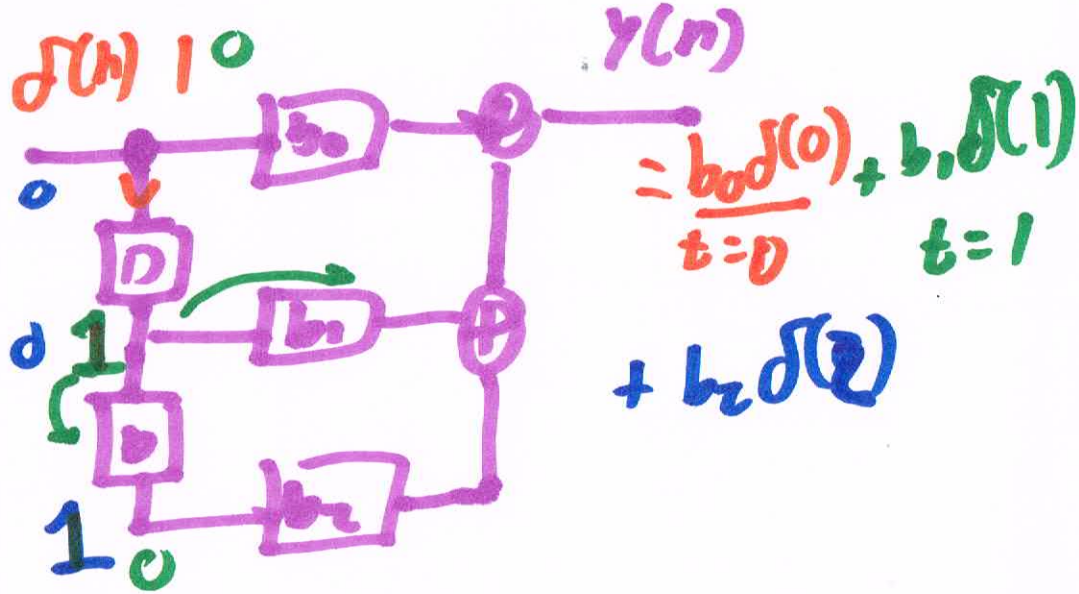
EE360D Lec 9

Convolution pt. 2

Friday Oct. 8th, 2021 DFI





for i = pL+1 : 1 : pL+1+eL

$c(n) = b_0 x(i) + b_1 x(i-1) \cdots b_N x(i-N)$;

$y(n) = c(n) - a_1 c(n-1) - a_2 c(n-2) \cdots a_M c(n-M)$

DFI

DFII/TDFII

Scope

$y(n)$

$q_0(n) = b_N x(n) + a_M$

$$x(n) * h(n) = \sum_{k=-\infty}^{k=\infty} h(k)\,x(n-k) = y(n)$$

constant!

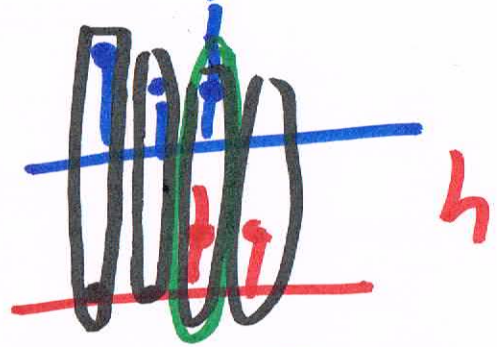
flips!

DFI

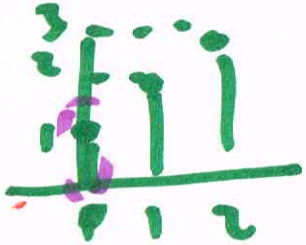
$x[1, 2, 3]$ input!

$xF[3, 2, 1] = x\,flip!$

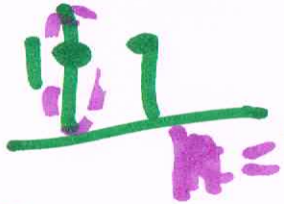
Graph way xflip



x



h

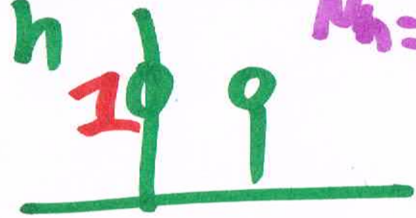


Math way!

$h(-1)x(1)$ $\quad h(0)x(0) + \quad h(1)x(0-1) = y(0)$

$= h(0)x(0)$

$k=-1$ $\quad k=0$ $\quad k=1$



$n = 1$

$$\sum_{k=0}^{\infty} h(k)\,x(n-k)$$

$$h(0)\,x(1-0) + h(1)\,x(1-1)$$

$x(1)$, $x(0)$

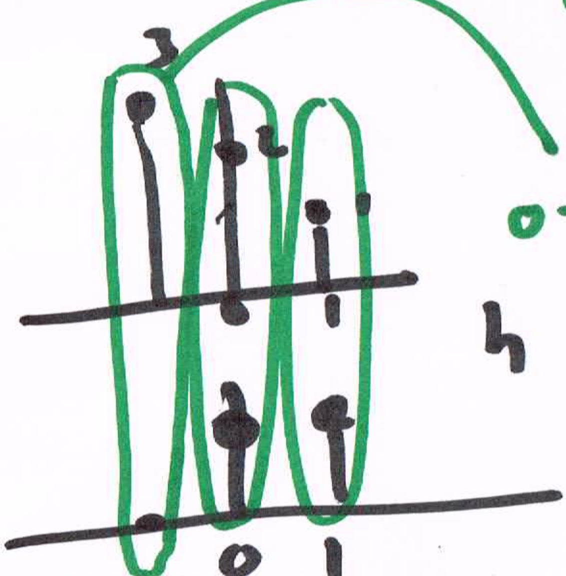
$h(0) = 1$, $x(1) = 2$, $k = 0$

$h(1) = 1$, $x(0) = 1$, $k = 1$

$2 + 1 = y(1) = 3$

Math way!

$\vec{x}$ Flip(−1)



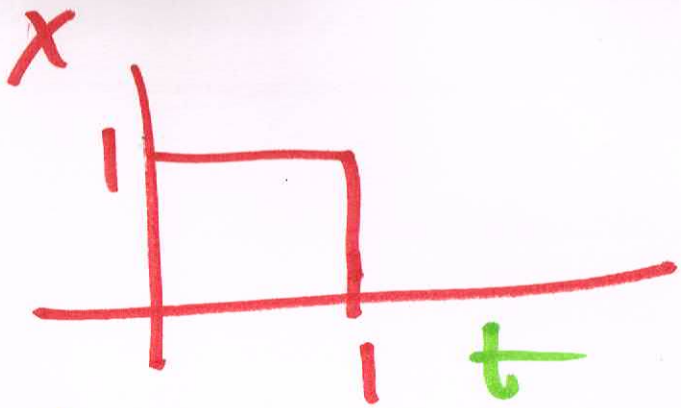
Graph way!

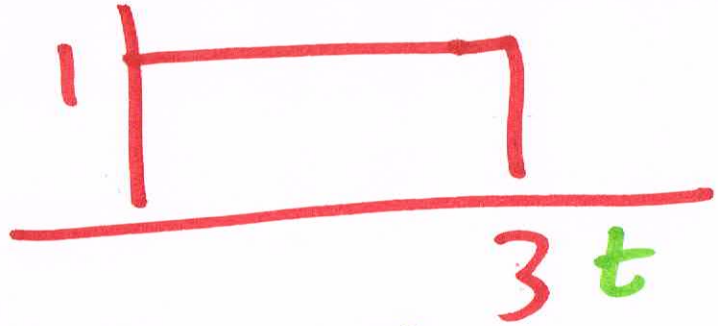
$0 \cdot 3 + 1 \cdot 2 + 1 \cdot 1 = 3 = y(1)$

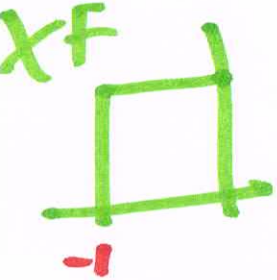
Length Property:

$$N_y = N_x + N_h - 1 = 3 + 2 - 1 = 4$$

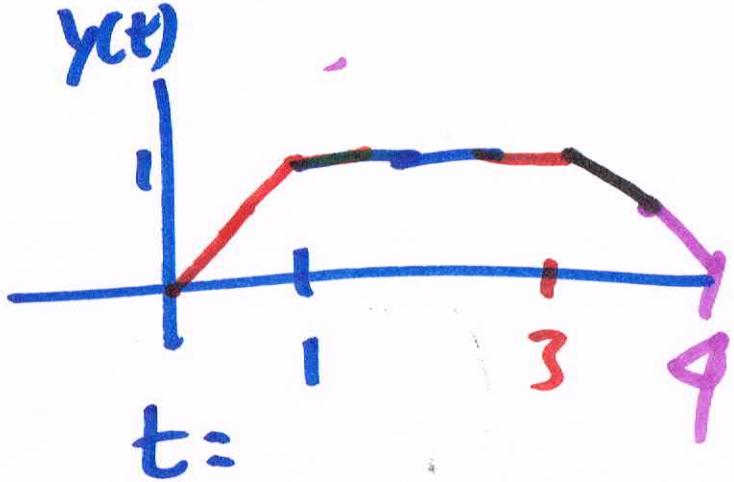
DSP Length Rule: $N_y = 2N - 1$, X + H are same Length

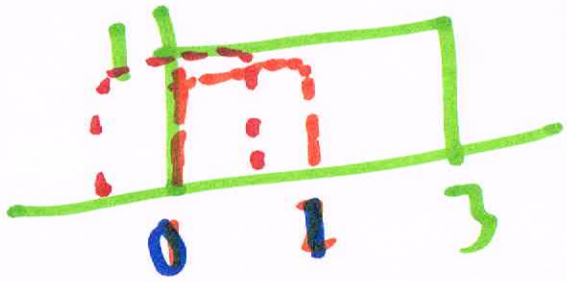


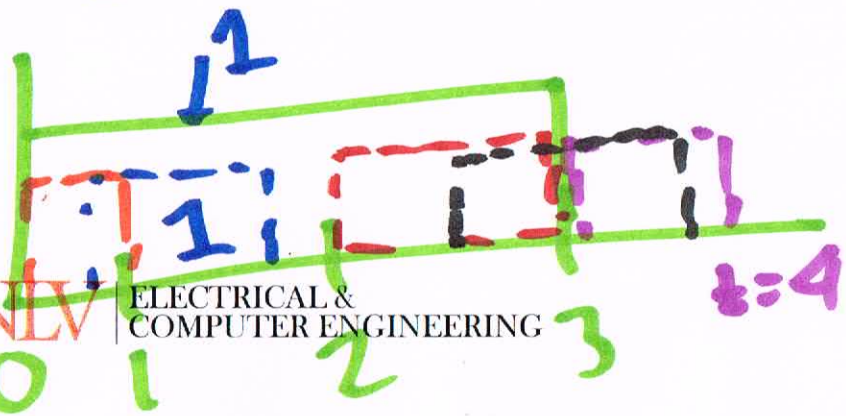


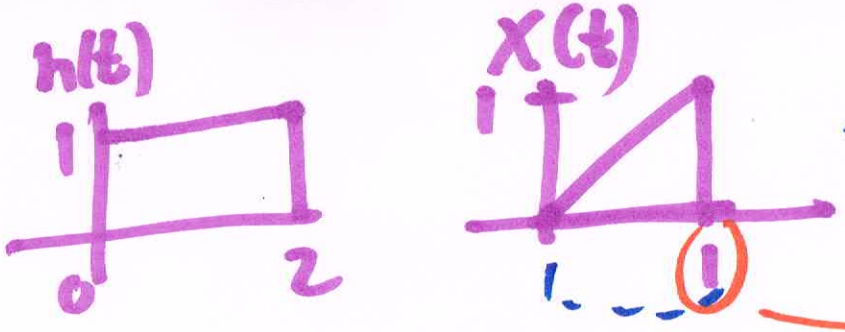


τ=0









$$x(t) = t\,[u(t) - u(t-2)]$$

$$= u(t) - u(t-2)$$

$$x(t) * h(t) = \int_{\tau=-\infty}^{\tau=\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{+\infty} [\tau\,[u(\tau) - u(\tau-2)]] \cdot [u(t-\tau) - u(t-2-\tau)]\, d\tau$$

$$= \int_{-\infty}^{\infty} \tau\, u(\tau)\, u(t-\tau) - \tau\, u(\tau)\, u(t-2-\tau) - \tau\, u(\tau-2)\, u(t-\tau) + \tau\, u(\tau-2)\, u(t-2-\tau)$$

$-(t-2)$

$$= \int_0^t \tau\, d\tau - \int_0^{t-2} \tau\, d\tau - \int_1^t \tau\, d\tau + \int_1^{t-2} \tau\, d\tau$$

$\Delta = \text{Top} - \text{Bot}$

$= t - 0$

$= t$

$u(\Delta)$

$\Delta = t-2$

$u(t-2)$

$\Delta = t-1$

$u(t-1)$

$\Delta = t-2-1$

$= t-3$

$u(t-3)$

$$\times \frac{\tau^2}{2}\Big|_0^t$$

$$= \frac{t^2}{2} u(t) -$$

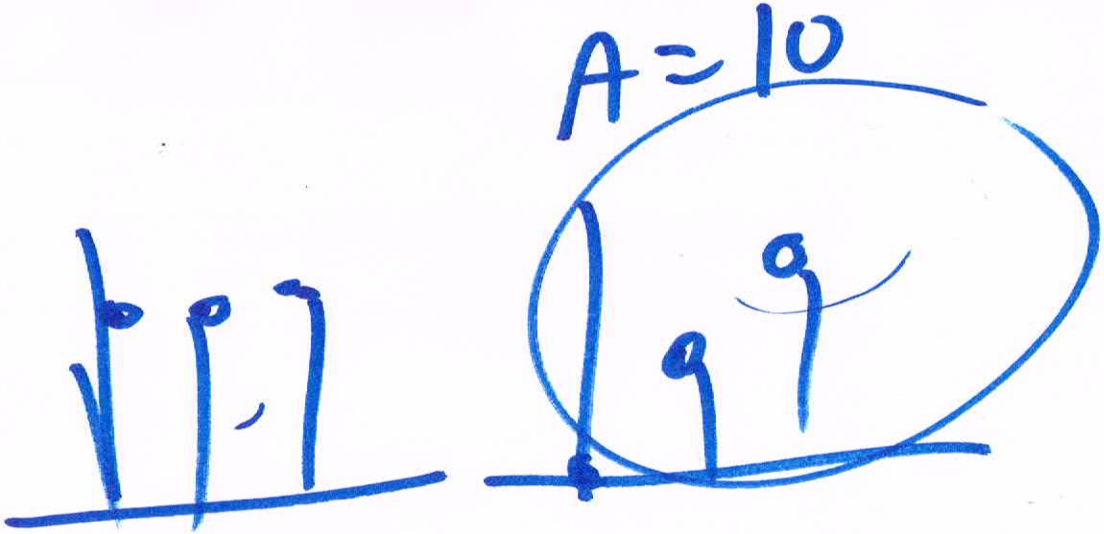
$$\frac{\tau^2}{2}\Big|_0^{t-2}$$

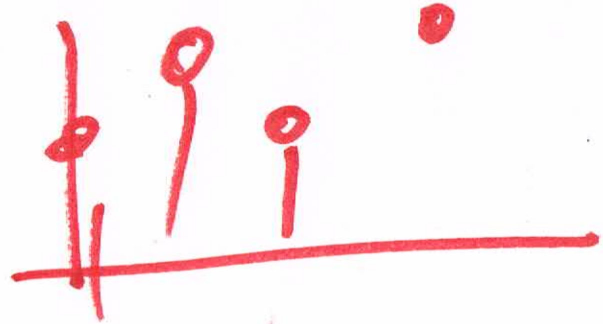
$$\frac{(t-2)^2}{2} u(t-2)$$

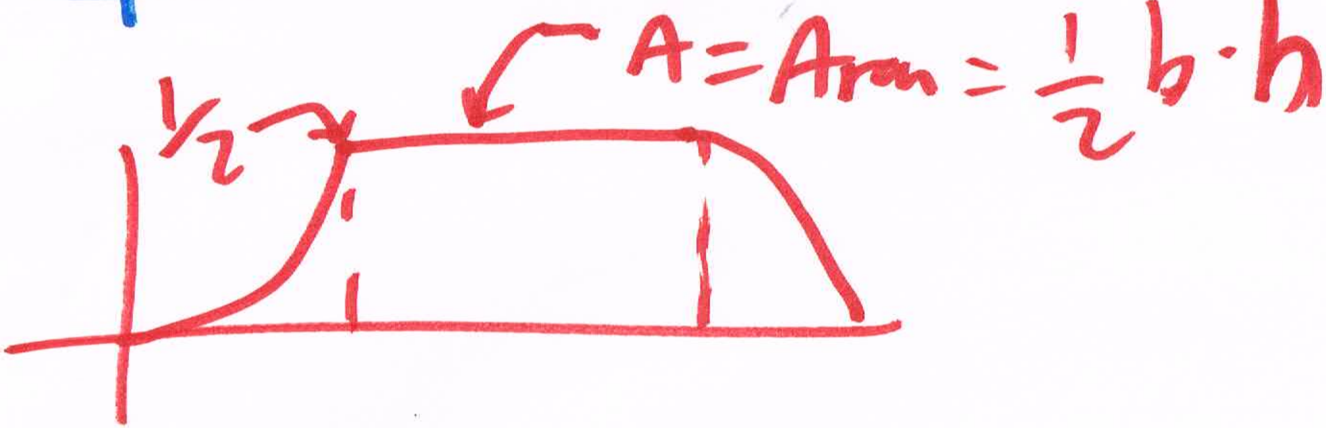
$$\frac{\tau^2}{2}\Big|_1^t$$

$$-\left[\frac{t^2}{2} - \frac{1}{2}\right] + \left[\frac{(t-2)^2}{2} - \frac{1}{2}\right]$$

$$\times \; u(t-1) \qquad u(t-3)$$







h = [h0, h1 ... hN

X[1 : : length(h)

Q[1 : length(h)