# EE 360

# The Z-Transform Pt. 2

Monday, Nov. 22nd, 2021

Initial: Laplace Derivative Property

$$\frac{d^N y(t)}{dt^N} \to s^N Y(s) - s^{N-1} y(t=0^-) - \dots - s^{N-N} \frac{d^{N-1} y(t=0^-)}{dt^{N-1}}$$

I.C's

Unilateral Z-T. "Delay" Property:

$$y(n-N) \to z^{-N} Y(z) + z^{-N+1} y(-1) + z^{-N+2} y(-2) \dots + z^{-N+N} y(n-N)$$

I.C!

Impulse Response:

$$H(z) = \frac{Y(z)}{X(z)}$$

Laplace!

1)


$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = 3x(n) - \frac{1}{3}x(n-1) - \frac{1}{6}x(n-2)$$

$$Y(z) - \frac{1}{6}\left[z^{-1}Y(z) + z^{-1+1}y(-1)\right] - \frac{1}{6}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] =$$

$$= 3X(z) - \frac{1}{3}\left[z^{-1}X(z) + x(-1)\right] - \frac{1}{6}\left[z^{-2}X(z) + z^{-1}x(-1) + x(-2)\right]$$

$$Y(z)\left[1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}\right] + y(-1)\left[-\frac{1}{6} - \frac{1}{6}z^{-1}\right] - \frac{1}{6}y(-2)$$

$$= X(z)\left[3 - \frac{1}{3}z^{-1} - \frac{1}{6}z^{-2}\right] + x(-1)\left[-\frac{1}{3} - \frac{1}{6}z^{-1}\right] - \frac{1}{6}x(-2)$$

$$Y(z)\left[1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}\right] = X(z)\left[3-\frac{1}{3}z^{-1}-\frac{1}{6}z^{-2}\right] + x(-1)\left[-\frac{1}{3}-\frac{1}{6}z^{-1}\right] - \frac{1}{6}x(-2)$$

$$+ -y(-1)\left[-\frac{1}{6}-\frac{1}{6}z^{-1}\right] + \frac{1}{6}y(-2)$$

$$Y(z) = \frac{\left[3-\frac{1}{3}z^{-1}-\frac{1}{6}z^{-2}\right]}{\left[1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}\right]}X(z) + \frac{x(-1)\left[-\frac{1}{3}-\frac{1}{6}z^{-1}\right]-\frac{1}{6}x(-2)-y(-1)\left[-\frac{1}{6}-\frac{1}{6}z^{-1}\right]+\frac{1}{6}y(-2)}{\left[1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}\right]}$$

$H(z)$ — ZSR, Impulse Response!

$H_{ZIR}(z)$ !!

$H_{Total}(z) = H(z) + H_{ZIR}(z)$ !

$$H(z) = \frac{3 - \frac{1}{3}z^{-1} - \frac{1}{6}z^{-2}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}} \quad \times 2 \cdot 3$$

$$\left|\frac{1}{2}\right| > \left|\frac{1}{3}\right|$$

$$\text{Pole} = \frac{+\frac{1}{6} \pm \sqrt{\frac{1}{36} + 4\left(\frac{1}{6}\right)}}{2} \quad \frac{25}{36}$$

$$= \frac{\frac{1}{6} \pm \sqrt{\frac{25}{36}}}{2}$$

$$= \frac{1}{12} \pm \frac{\frac{5}{6}}{2} = \frac{1}{12} \pm \frac{5}{12}$$

$$= \frac{6}{12}, \frac{-4}{12} \rightarrow \boxed{\frac{1}{2}, -\frac{1}{3}}$$

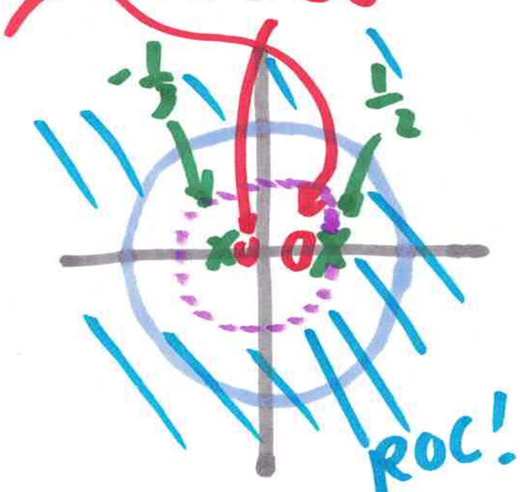
Rewrite: $1 - \frac{1}{9}z^{-1} - \frac{1}{18}z^{-2}$

$$\text{Zero} = \frac{\frac{1}{9} \pm \sqrt{\frac{1}{81} + \frac{4}{18}}}{2}$$

Using matLab:

Roots([3 $-\frac{1}{3}$ $-\frac{1}{6}$])

= 0.2977, −0.1866




A)

$$H(z) = \frac{3 - \frac{1}{3}z^{-1} - \frac{1}{6}z^{-2}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}} \rightarrow \frac{Res(1)}{1 - pole(1)z^{-1}} + \frac{Res(2)}{1 - pole(2)z^{-1}} \ldots$$

Partial Fraction Expansion!

$$\begin{array}{r} 1 \\ -\frac{1}{6}z^{-2} - \frac{1}{6}z^{-1} + 1 \;\big)\; -\frac{1}{6}z^{-2} - \frac{1}{3}z^{-1} + 3 \\ -\;\;\; + \left(+\frac{1}{6}z^{-2} + \frac{1}{6}z^{-1} - 1\right) \\ \hline -\frac{1}{6}z^{-1} + 2 \end{array}$$

$$H(z) = 1 + \frac{2 - \frac{1}{6}z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{3}z^{-1}\right)}$$

$$= 1 + \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + \frac{1}{3}z^{-1}}$$

$$\frac{2-\frac{1}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}}$$

$z = \frac{1}{2}$

$z^{-1} = 2$

$$A = \frac{2-\frac{1}{6}z^{-1}}{\left(1+\frac{1}{3}z^{-1}\right)} = \frac{\frac{12}{6}-\frac{1}{6}\cdot 2}{\frac{3}{3}+\frac{2}{3}} = \frac{\frac{10}{6}}{\frac{5}{3}} = \frac{10}{6}\cdot\frac{3}{5} = \boxed{1}$$

$z = -\frac{1}{3}$

$z^{-1} = -3$

$$B = \frac{2-\frac{1}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)} = \frac{\frac{12}{6}+\frac{3}{6}}{\frac{2}{2}+\frac{3}{2}} = \frac{\frac{15}{6}}{\frac{5}{2}}$$

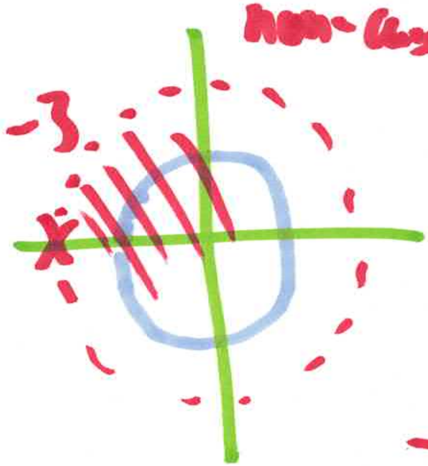
$$= \frac{15}{6}\cdot\frac{2}{5} = \boxed{1}$$

$$H(z) = 1 + \frac{1}{1-\frac{1}{2}z^{-1}} + \frac{1}{1+\frac{1}{3}z^{-1}}$$

$$h(n) = \delta(n) + 1\left(\frac{1}{2}\right)^n u(n) + 1\left(-\frac{1}{3}\right)^n u(n)$$

stable ← (Left side)!

non-causal!




$$\frac{Res}{1+3z^{-1}}$$

$\rightarrow -Res\,(-3)\,u(-n-1)$

$X(z) =$ Step Respons:

$$U(z) = \frac{1}{1-z^{-1}}$$

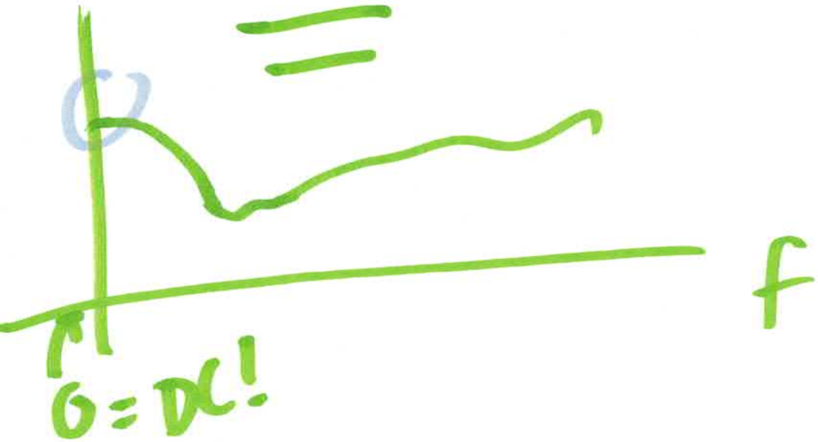
$$Y(z) = H(z)\,U(z)$$

$$H(z) = \frac{3 - \frac{1}{3}z^{-1} - \frac{1}{6}z^{-2}}{1 - \frac{1}{6}z^{-1} - \frac{1}{6}z^{-2}} = \frac{\frac{18}{6} - \frac{2}{6} - \frac{1}{6}}{\frac{6}{6} - \frac{1}{6} - \frac{1}{6}} = \frac{\frac{15}{6}}{\frac{4}{6}}$$


$z = 1$

$$= \frac{15}{6} \cdot \frac{6}{4} = \boxed{3.75}$$

DC Gain!




$20 \log_{10}(3.75$

Laplace: $\dot{Q}(t) = A Q(t) + B X(t)$

$y(t) = C Q(t) + D x(t)$

$q(n-1)$

$\rightarrow z^{-1} q(z)$

$\dot{Q}(t) \rightarrow s Q(s)$

$Q(s)$

Z-T: $Q(n+1) = A Q(n) + B x(n)$

$y(n) = C Q(n) + D X(n)$

$z Q(z)$

Prove: $y(z) = \left[C(zI - A)^{-1} B + D\right] X(z)$

CA 5

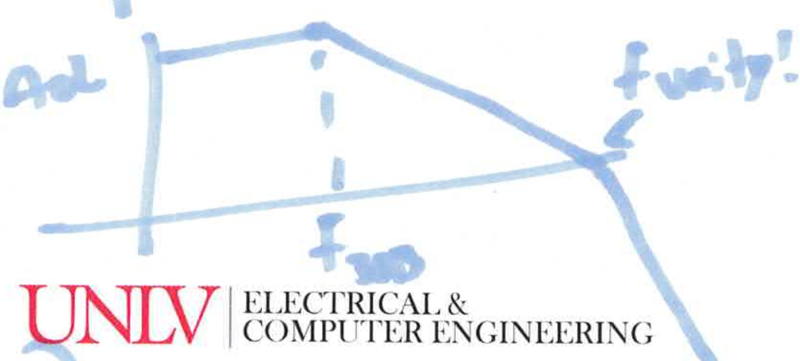
Analog

EE370

Bode plots!

circuits EE420

Op-amps!




CA 6

Digital

EE480/ECG 722

mixed signal-circuit design!


D Time Fourier Transform

ADC's