Inv. $P_2 t$
LT: Ae

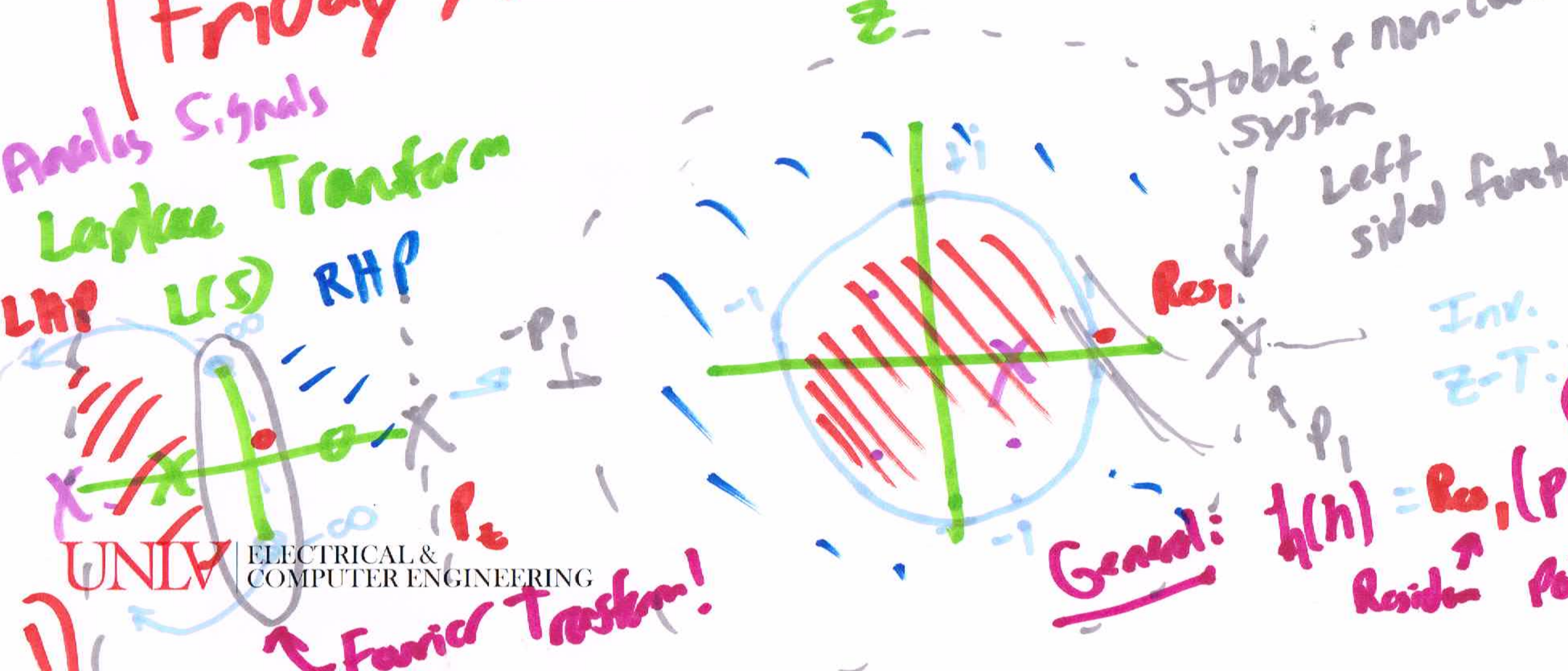
~~L14~~
L15

EE360

The Z-Transform

Digital Signals

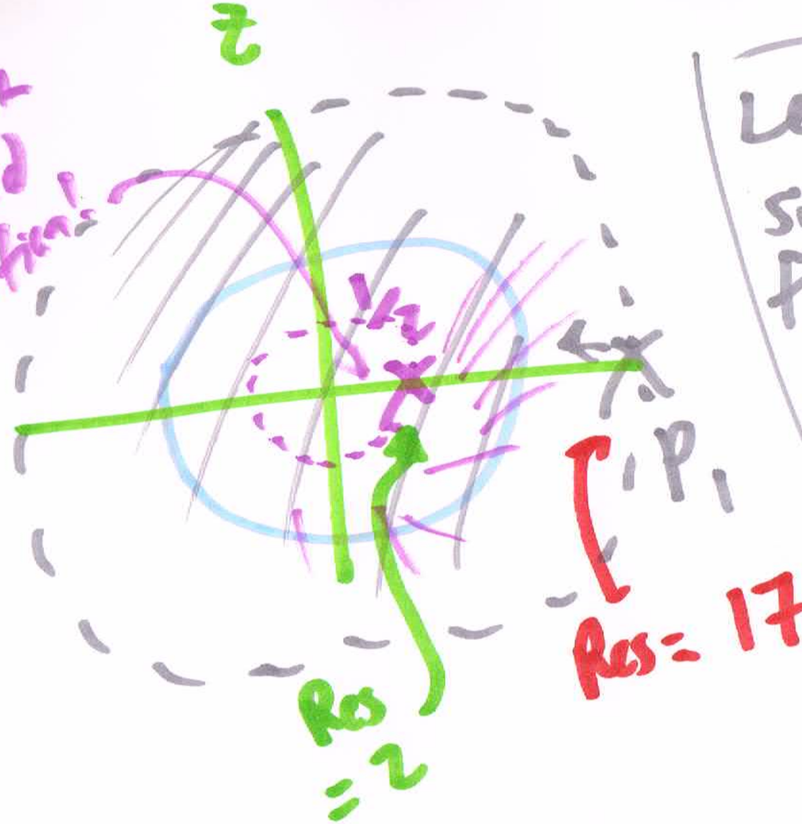
Friday, Nov. 19th, 2021



General: $h(n) = Res_1 (p_1)^{n} u(n)$

Residue Pole

Right sided function!



Left sided function!

Inv. z-T

$$g(n) = -17\,(p_1)^n\,u(-n-1)$$

Res

Don't forget!

General $X = \text{Res}\,(d)^n\,u(n)$

z-T: $X(z) = \text{Res}\,\dfrac{1}{1-(d)z^{-1}}$

Pole!

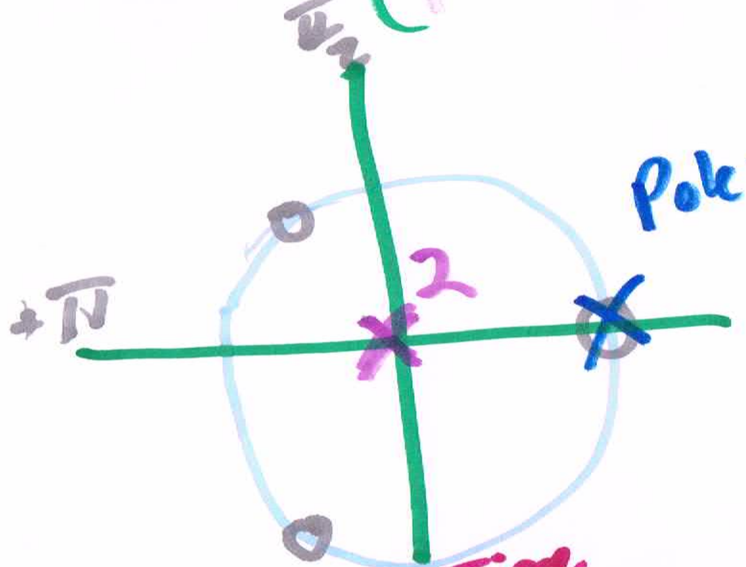
Inv. z-T: $h_1(n) = 2\left(\frac{1}{2}\right)^n u(n)$

$$h_T(n) = h_1(n) + g(n)$$

Delay

$g(n) \rightarrow \boxed{z^{-1}} \rightarrow g(n-1)$

$$H(z) = \frac{(1 - z^{-3})}{(1 - z^{-1})} \cdot \frac{z^3}{z^3} = \frac{z^{3} - 1}{z^3 - z^2} = \frac{z^3 - 1}{z^2(z - 1)}$$



Pole-zero Cancellation!

K is an Integer! memorize!

$$z^3 - 1 = 0$$

$$z^3 = 1 = e^{j0} = e^{j2\pi k}?$$

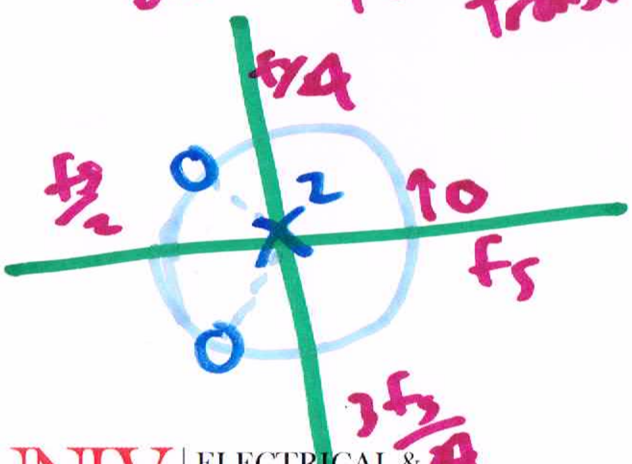
$$z = e^{j\frac{2\pi}{3}k}$$

k=0: $z_1 = e^{j0} = 1$

k=1: $z_2 = e^{j\frac{2\pi}{3}}$

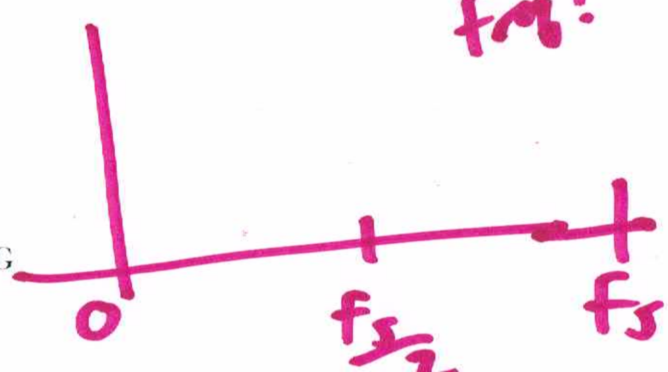
k=2: $z_3 = e^{j\frac{4\pi}{3}} = e^{-j\frac{2\pi}{3}}$

Discrete-Time Fourier Transform



DSP:

Nyquist freq!



3)

$$\frac{1-z^{-3}}{1-z^{-1}} = \frac{1-e^{-jw3}}{1-e^{-jw}}$$

$$z = e^{jw} = e^{\frac{j2\pi f}{Fs}}$$

$$z \Leftarrow \boxed{z = e^{st}}$$

$s = jw$??

ECG 722! thanks

$$\frac{e^{-jw\frac{3}{2}}\left(e^{+jw\frac{3}{2}} - e^{-jw\frac{3}{2}}\right)}{e^{-j\frac{w}{2}}\left(e^{+j\frac{w}{2}} - e^{-j\frac{w}{2}}\right)} \cdot \frac{2j}{2j}$$

$$= e^{-jw\left(\frac{3}{2} - \frac{1}{2}\right)} \cdot \frac{\sin\left(w\frac{3}{2}\right)}{\sin\left(\frac{w}{2}\right)} = \left| e^{-jw} \right| \left| \frac{\sin\left(w\frac{3}{2}\right)}{\sin\left(\frac{w}{2}\right)} \right|$$

phase!

Mag!

1!

$$|H(z)| = \left| \frac{\sin\left(w\frac{3}{2}\right)}{\sin\left(\frac{w}{2}\right)} \right| \qquad \angle H(z) = -w$$

$$|H(z)| = \left| \frac{\sin\left(2\pi \frac{f}{f_s} \cdot \frac{3}{2}\right)}{\sin\left(2\pi \frac{f}{f_s} \cdot \frac{1}{2}\right)} \right|$$

$\uparrow$ f

$$= \frac{\left|\sin\left(3\pi \frac{f}{f_s}\right)\right|}{\left|\sin\left(\frac{\pi f}{f_s}\right)\right|}$$

$$w = 2\pi \frac{f}{f_s}$$

$f = f_s$

$$\angle H(f) = -2\pi \frac{f}{f_s} \rightarrow -kf$$

Linear! Phase!

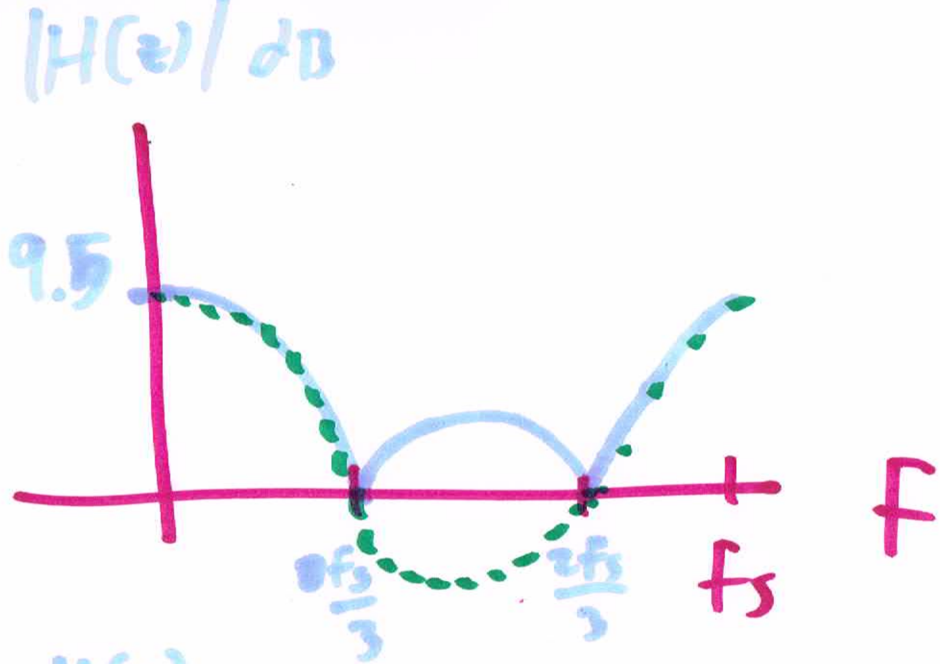
$f = 0$

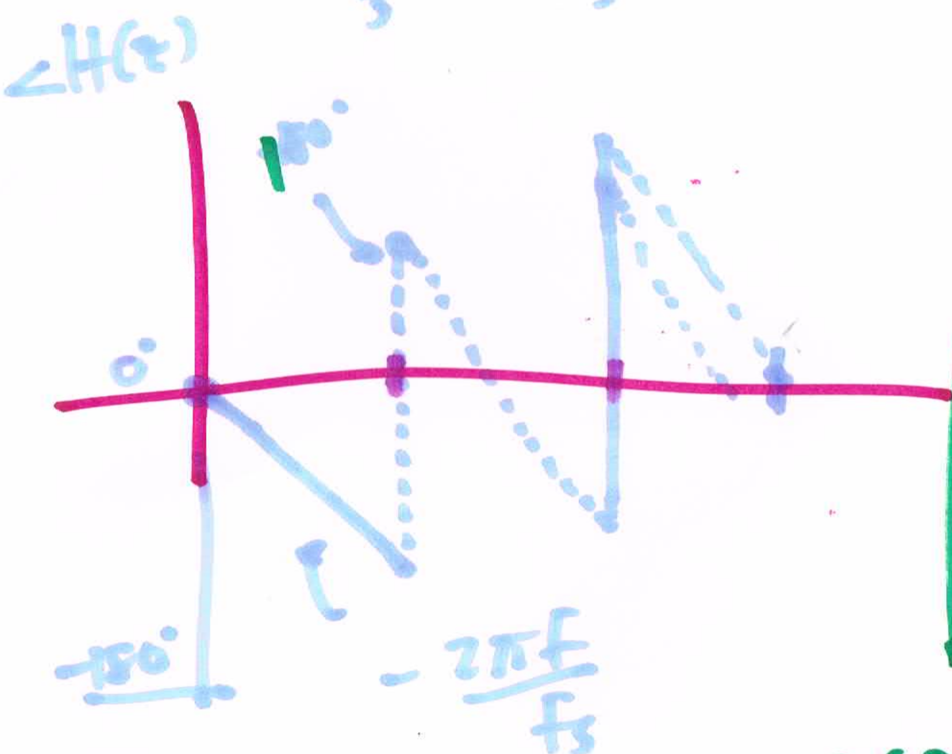
DC

L'H Rule!

$$\frac{d}{df}[\,] = \frac{\frac{3\cancel{\pi}}{\cancel{f_s}} \cos\left(3\pi \frac{f}{f_s}\right)}{\frac{\cancel{\pi}}{\cancel{f_s}} \cos\left(\frac{\pi f}{f_s}\right)} = 3 \cdot \frac{\cancel{\cos(0)}}{\cancel{\cos(0)}}^{\,1} = 3$$

DC: Power! $20\log_{10}(3) = 9.5\text{ dB}$



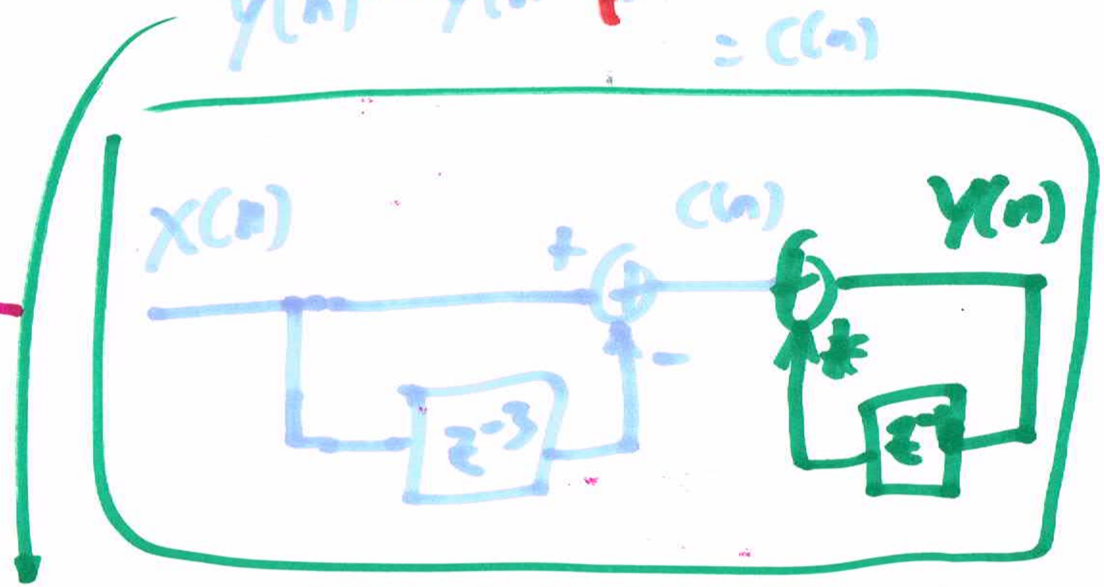


Block Diagram:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - z^{-3}}{1 - z^{-1}}$$

$$Y(z) - Y(z)z^{-1} = X(z) - X(z)z^{-3}$$

$$y(n) - y(n-1) = x(n) - x(n-3) = c(n)$$



$$y(n) = c(n) + y(n-1)$$

$$z^{-1} = \underline{1un}$$

$$\frac{1-z^{-3}}{1-z^{-2}} = H(z)$$

$x(n) = u(n)$

$$U(z) = \frac{1}{1-z^{-1}}$$

2-poles!

$Res_1(p_1)^n u(n) + Res_2(p_2)^n u(n)$

$$= \frac{z}{z-1}$$

$$Y(z) = H(z)X(z)$$

$$\underset{\text{2nd order} \rightarrow}{} \frac{1-z^{-3}}{(1-z^{-2})} \cdot \frac{1}{(1-z^{-1})}$$

conv(den1, den2)

3rd-order!

$$h(n) = R_1(p_1)^n + R_2(p_2)^n + R_3(p_3)^n$$