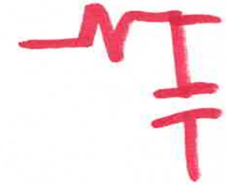
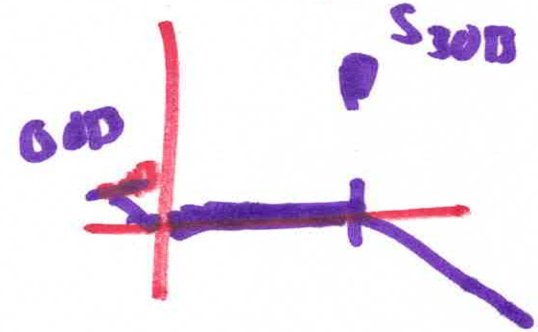
EE360

L19

Review Laplace Transforms

Friday, Nov. 12^{th}, 2021





$$\mathcal{L}\{f(t)\} = \int_{t=-\infty}^{t=+\infty} f(t)e^{-st}\,dt$$

Diff. property: w/ I.C.

$$\ddot{y} \overset{\mathcal{L}()}{\Longrightarrow} \boxed{s^2 Y(s)} - sy(t=0^-) - \frac{dy(t=0^-)}{dt}$$

No I.C.'s

I.C.

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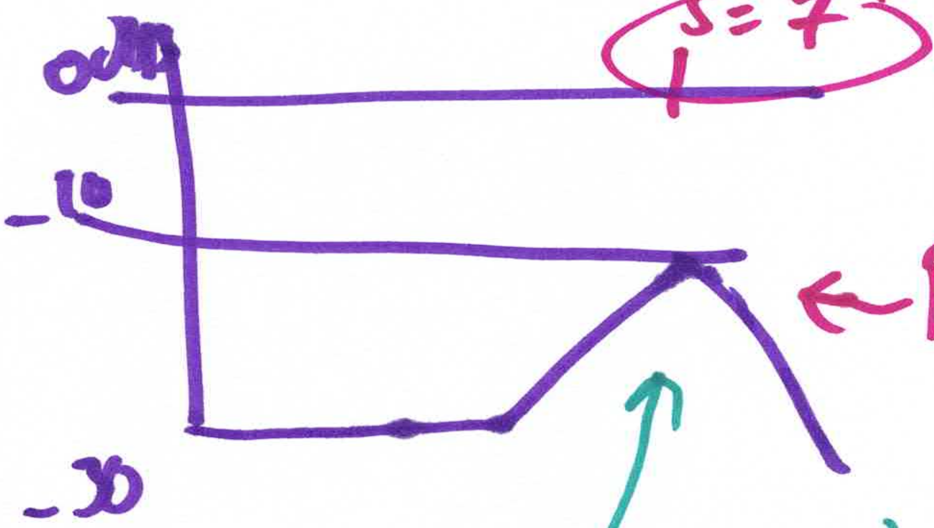
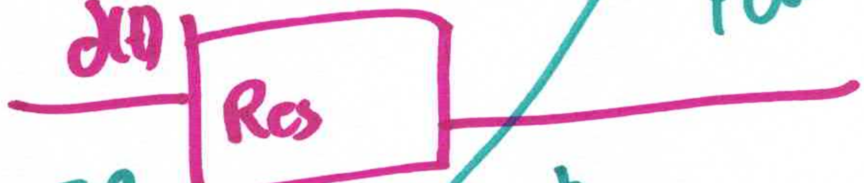


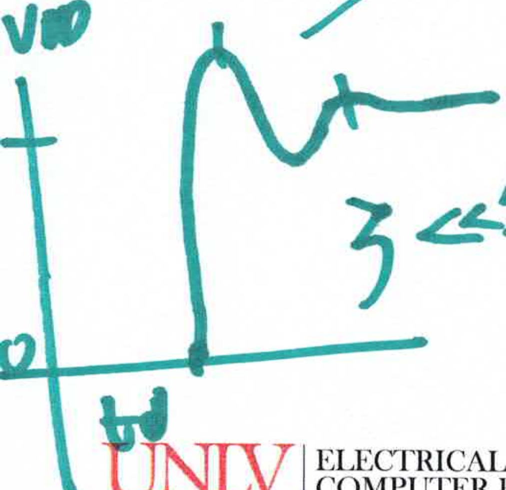
$\rightarrow f = \frac{s}{2\pi} = \frac{7}{2\pi}$

← Resonators!

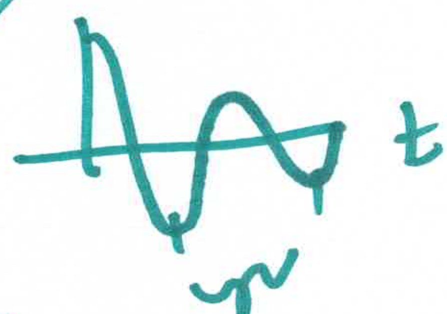


Poor damping factor! (EE370!)

STEP

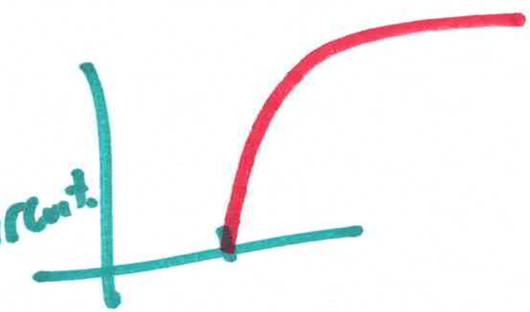


$\zeta \ll 1$



f_{Res}

RC Circuit



$\underline{\zeta = 1}$

clean Response!

2)

Ch. 3 SS-Eqn.

Time Domain!

$$\dot{Q}(t) = \overset{\text{Const}}{A}\,\underline{Q(t)} + B\,X(t)$$

$$Y(t) = C\,Q(t) + D\,X(t)$$

$\mathcal{L}(s)$

$$s\,Q(s) = A\,\underline{Q(s)} + B\,X(s)$$

$$Y(s) = C\,\underline{Q(s)} + D\,X(s)$$

$$\underset{m\times m}{s}\,\overset{1\times 1}{}\underline{Q(s)} - \overset{m\times m}{A}\,\underline{Q(s)} = B\,X(s)$$

$$I = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & & & \\ \vdots & & \ddots & & \\ 0 & & & & 1 \end{bmatrix}$$

$$\cancel{Q(s)}\,(sI - A)\,\cancel{Q(s)} \overset{\text{a col.}}{=} B\,X(s)$$

Verify! multiplication

3)

$$(sI-A)^{-1}(sI-A)Q(s) = (sI-A)^{-1}BX(s)$$!!! Review!

$$Q(s) = (sI-A)^{-1}BX(s)$$

$$Y(s) = C\{(sI-A)^{-1}BX(s)\} + DX(s)$$

$$\boxed{\begin{aligned} Y(s) &= \left[C\{(sI-A)^{-1}B\} + D\right]X(s) \\ \frac{Y(s)}{X(s)} = H(s) &= C\{(sI-A)^{-1}B\} + D \end{aligned}}$$

T.F!

$$\dot{q}(t) = \underbrace{\begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}}_{A} q(t) + \begin{bmatrix}1\\ -3\end{bmatrix} x(t) \qquad \dot{q}(t) = \begin{bmatrix}\dot{q}_1(t)\\ \dot{q}_2(t)\end{bmatrix}$$

$$y(t) = \underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C} q(t) \qquad D = 0!$$

$$Y(s) = \left[C(sI-A)^{-1}B + D\right]X(s)$$

$$= \left[\underbrace{\begin{bmatrix}1 & 0\end{bmatrix}}_{C}\Big(\underbrace{\begin{bmatrix}s & 0\\ 0 & s\end{bmatrix}}_{sI} - \underbrace{\begin{bmatrix}-3 & 1\\ -2 & 0\end{bmatrix}}_{A}\Big)^{-1}\underbrace{\begin{bmatrix}1\\ -3\end{bmatrix}}_{B}\right]X(s)$$

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$= \left[\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}s+3 & -1\\ +2 & s\end{bmatrix}^{-1}\begin{bmatrix}1\\ -3\end{bmatrix}\right]X(s)$$

$$= \left[\begin{bmatrix}1 & 0\end{bmatrix}\frac{1}{(s^2+3s+2)}\begin{bmatrix}s & +1\\ -2 & s+3\end{bmatrix}\begin{bmatrix}1\\ -3\end{bmatrix}\right]X(s)$$

5) $(s+1)(s+2)$ stable!

all poles on LHS

$$Y(s) = \underbrace{\frac{1}{(s+1)(s+2)}}_{H_{poles}} \left[\underbrace{\underbrace{[1 \quad 0]}_{1 \times 2} \underbrace{\begin{bmatrix} s & 1 \\ -2 & s+3 \end{bmatrix}}_{2 \times 2}}_{1 \times 2} \underbrace{\begin{bmatrix} 1 \\ -3 \end{bmatrix}}_{2 \times 1} \right] X(s)$$

$$= H_{poles}(s) \left[s(1) + (-2)(0) \qquad (1)(1) + 0(s+3) \right]$$

$$= H_{poles}(s) \underbrace{\overset{1 \times 2}{\begin{bmatrix} s & 1 \end{bmatrix}} \overset{2 \times 1}{\begin{bmatrix} 1 \\ -3 \end{bmatrix}}}_{1 \times 1} X(s)$$

$$= H_{poles}(s) \left[s(1) + (1)(-3) \right]$$

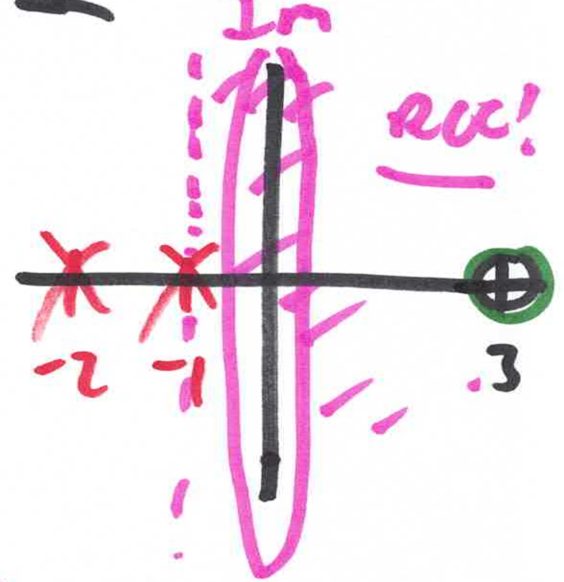
$$= \frac{\left[H_{poles}(s) \; (s-3) \right]}{H(s)} X(s)$$

$$\boxed{Y(s) = \frac{(s-3)}{(s+1)(s+2)} X(s)}$$

Impulse Response:

$\delta(t) \xrightarrow{\mathcal{L}(s)} 1$

$X(s) = \mathcal{L}(\delta(t)) = 1$



$$Y(s) = H(s)\,X(s) = H(s) = \frac{(s-3)}{(s+1)(s+2)}$$

$$= \frac{A}{s+1} + \frac{B}{s+2} = \frac{-4}{s+1} + \frac{5}{s+2}$$

Residue

pole

Stable since ROC includes $j\omega$!

Gen eqn: $y(t) = \text{Res}\, e^{\text{pole}(t)}$

$j\omega \rightarrow \text{Im}$!

Heaviside:

$$A = \left.\frac{(s-3)}{(s+2)}\right|_{s=-1} = \frac{(-1-3)}{(-1+2)} = \frac{-4}{1} = -4$$

t-domain $= \left(-4e^{-t} + 5e^{-2t}\right)u(t)$

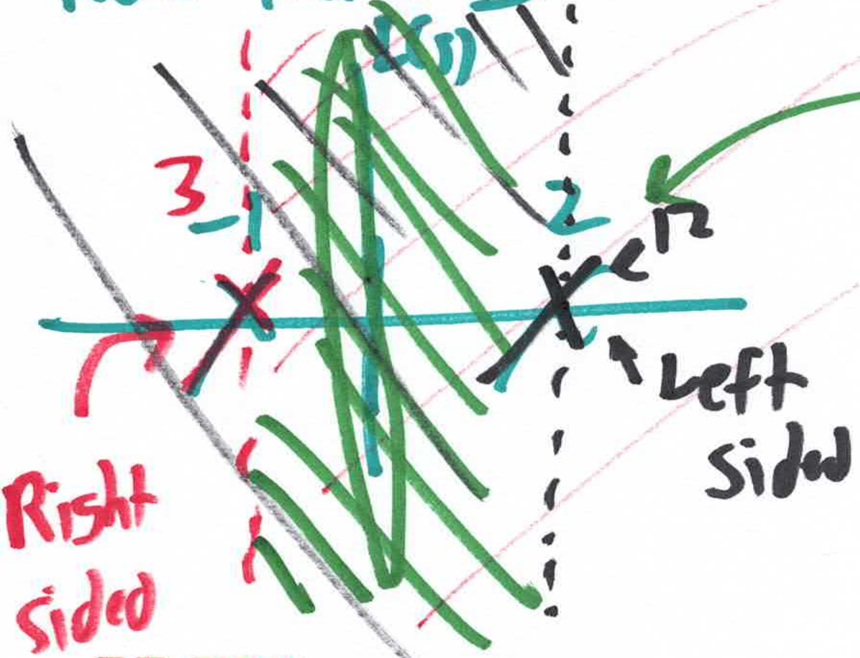
Don't Forget!

7) $$B = \left.\left(\frac{s-3}{s+1}\right)\right|_{s=-2} = \frac{-2-3}{-2+1} = \frac{-5}{-1} = +5$$

$$H(s) = \frac{0s^2 + 1s - 3}{1s^2 + 3s + 2} = \frac{s-3}{s^2+3s+2} = \frac{s-3}{(s+1)(s+2)}$$

~~8I~~ Stable System

ROC includes Im. axis

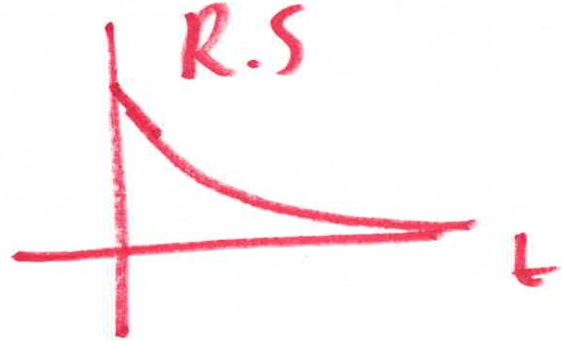


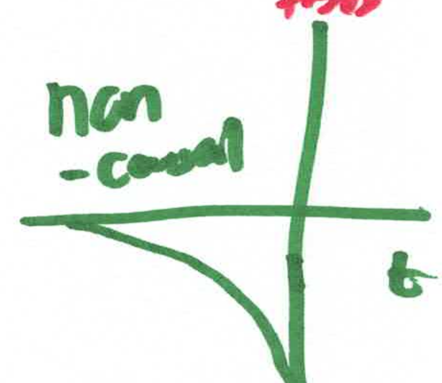
$$L(s) = \underbrace{\frac{3}{s+1}}_{\text{R.S.}} + \underbrace{\frac{12}{s-2}}_{\text{L.S.}}$$

+2 Right ~~Left~~ pole

$$l(t) = \underbrace{3e^{-t}u(t)}_{\text{R.S.}} - \underbrace{12e^{2t}u(-t)}_{\text{L.S.}}$$

Don't forget

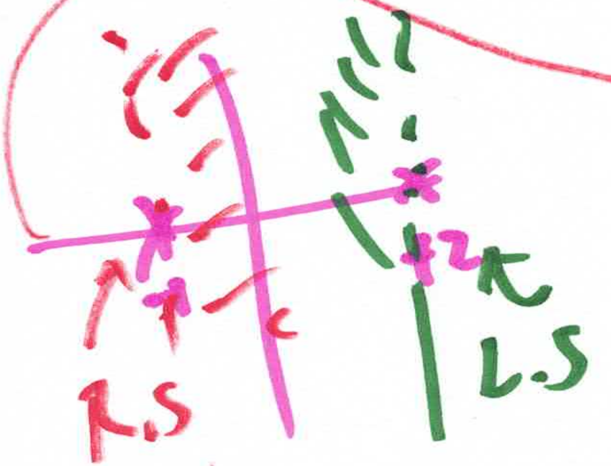




Stable

8)

$$D(s) = H(s) = \frac{3}{s+1} + \frac{12}{s-2} + 2$$



$$h(t) = 3e^{-t}u(t) - 12e^{+2t}u(-t) + 2\delta(t)$$

R. Side

Sim!

L. Side

Don't Sim!

Don't Forget!

$\boxed{S}$ $2u(t)$

$x(t) = \delta(t)$ $\boxed{b_0}$ $y(t) = 2\delta(t)$

Physically means nothing!

Use for math!

$b_0 = 2$ Direct path from input to output!

a)