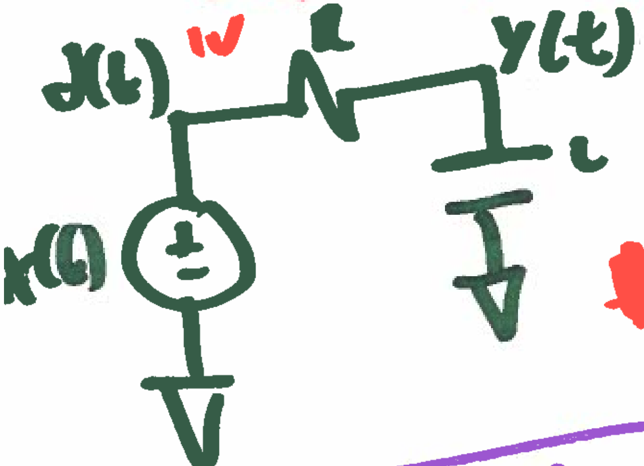
The Laplace Transform

Practical! SS equations

Wed. November 10^{th}, 2021



$$\frac{dy(t)}{dt} + \frac{1}{RC}\,y(t) = \frac{1}{RC}\,x(t)$$

$s_{sup} = 7k$

HW: $\frac{d}{dt}\ddot{y} + a_1\dot{y} + a_0 y = b_1\dot{x} + b_0 x \overset{L(s)}{\Rightarrow} s^2 Y(s) + a_1 Y(s) + Y(s) = b_1 s X(s) + b_0 X(s)$

Memorize!

1) $\frac{d^N y}{dt^N} \Rightarrow s^N Y(s) - s^{N-1} y(t=0^-) - \cdots s^{N-N} \frac{d^{N-1} y}{dt^{N-1}}(t=0^-)$

I.C.

$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

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General RC Filter TF

Res

$$H(s) = \frac{1}{[1+sRC]} = \frac{\frac{1}{RC}}{[s+\frac{1}{RC}]}$$

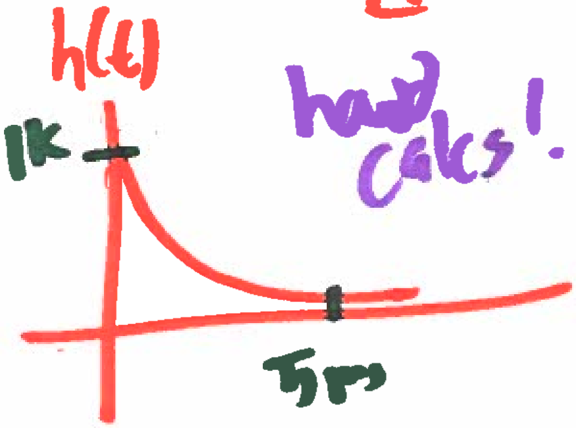
Pole $= -\frac{1}{RC}$

$$s_{pole} = -\frac{1}{RC} = -1k$$

Pole Location!

$\tau = 1ms$

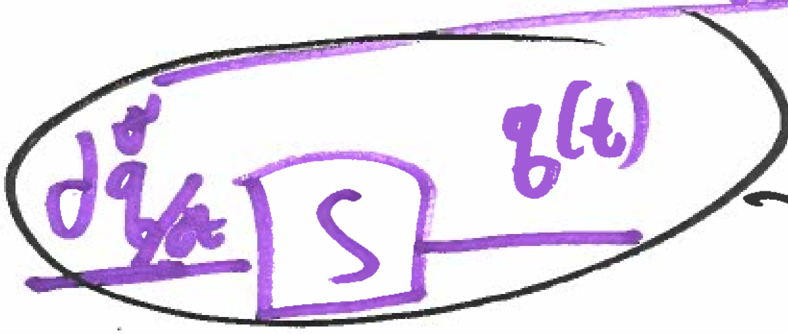
hand calcs!



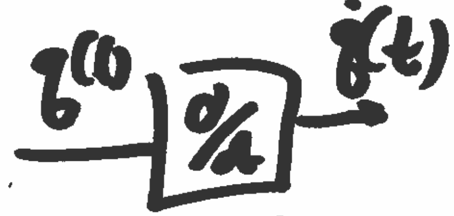
$$\mathcal{L}^{-1}\{H(s)\} = \frac{1}{RC} e^{-\frac{1}{RC}\cdot t} = \boxed{1k\, e^{-\frac{t}{1ms}}}$$

2)

State Space Eqns



from Ch. 2/3?



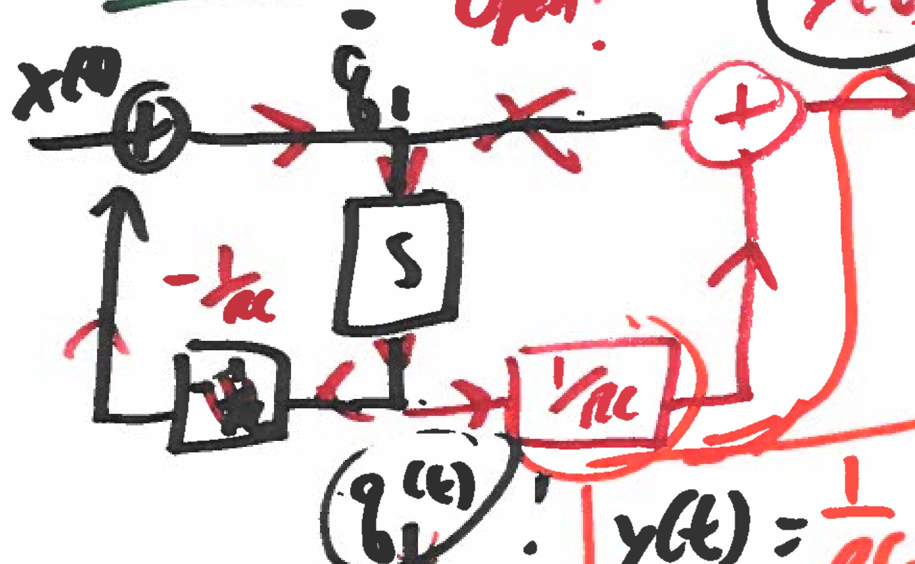
diff. eqns!

DFII:

$$\int \frac{dy(t)}{dt} + \int \frac{1}{RC} y(t) = \int \frac{1}{RC} x(t)$$

$$y(t) + \frac{1}{RC}\int y = \frac{1}{RC}\int x$$

$$y(t) = \frac{1}{RC}\int x - \frac{1}{RC}\int y \; !$$



$$\boxed{y(t) = \frac{1}{RC} q_1(t)}$$

$$\boxed{\dot{q}_1(t) = x(t) - \frac{1}{RC} q_1(t)}$$

SS Eqns:

$$\dot{q} = Aq + Bx$$

$$y = Cq + Dx$$

$$\dot{q} = [\dot{q}_1(t)] = \left[-\frac{1}{RC}\right][q_1(t)] + [1]\,x(t)$$

$$y = [y(t)] = \left[\frac{1}{RC}\right][q_1(t)] + 0$$

← EE 370's SS. Eqns!

$$H(s) = \frac{1}{RC}\,\frac{1}{\left(s+\frac{1}{RC}\right)}$$ ← Use Controls ToolBox!

$s = j\omega$

$\omega = 2\pi F \rightarrow F = \frac{\omega}{2\pi}$!

$y(t) = h(t) * x(t)$

freq! 10kHz

$x = e^{-s_0 t}$

$$y(t) = |H(s=s_0)|\, e^{-s_0 t + \angle H(s=s_0)}$$

10kHz, 10kHz, Phase

$x = 1V e^{-j10k}$?

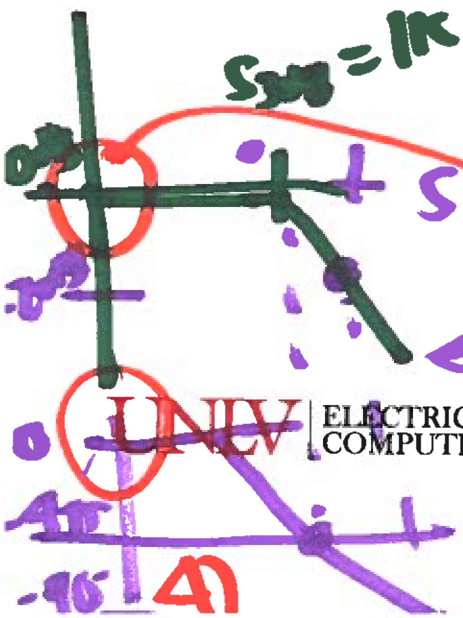
$y = \, .1V e^{-j10k + \phi}$

-20dB

$s_{3dB} = 1k$

$s = 10k$

-20dB/dec



$y(t) = |H(s=0)| = |1V|$!

$s = DC = 0$!

$\angle = 0$!

DC Gain of RC Filter = 1!

$$H(s) = \frac{1}{RC}\frac{1}{s+\frac{1}{RC}}$$

DC Gain!

$$H(s=0) = \frac{1}{RC}\frac{1}{0+\frac{1}{RC}} = \frac{1}{RC\left(0+\frac{1}{RC}\right)} = \boxed{1}$$

$$Y(s) = H(s)X(s)$$

$U(s) \rightarrow u(t)$! Step Response!

$$\boxed{U(s) = \frac{1}{s}}$$

@ t=0

$$Y(s) = H(s)U(s)$$

$$\boxed{Y(s) = H(s)\cdot\frac{1}{s}} = \frac{1}{RC}\cdot\frac{1}{s+\frac{1}{RC}}\cdot\frac{1}{s} = \frac{A}{s} + \frac{B}{s+\frac{1}{RC}}$$

Check for stability

- Damping factor
  - Set. time, etc.

Heaviside Theorem:

$H(s)X(s)$

$$A = H(s)\,s\Big|_{s=0} = \frac{1}{RC}\frac{1}{s+\frac{1}{RC}}\cdot\frac{1}{\cancel{s}}\cdot\cancel{s} = \frac{\frac{1}{RC}}{\frac{1}{RC}} = \boxed{1}$$

b)

$$B = \left. \frac{1}{RC} \cdot \frac{1}{\cancel{s+\frac{1}{RC}}} \cdot \frac{1}{s} \cdot \cancel{\left(s+\frac{1}{RC}\right)} \right|_{s=-\frac{1}{RC}} = \left. \frac{1}{RC} \cdot \frac{1}{s} \right|_{s=-\frac{1}{RC}} = \frac{-RC}{RC} = \boxed{-1}$$

Step Response:

$$Y(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{RC}}$$

Assume Stable!

$\text{Res} = 1 \qquad \text{pole} = -\frac{1}{RC}$

$$y(t) = \underline{u(t)} - 1\, e^{\left(-\frac{1}{RC}\right)t}\, \underline{u(t)}$$

$$\boxed{y(t) = \left[1 - e^{-t/RC}\right] u(t)}$$