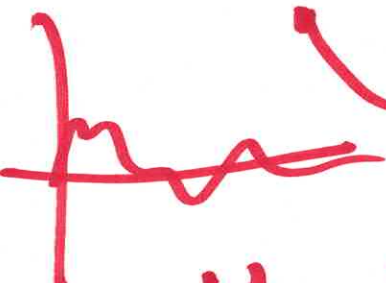
EE360
The Laplace Transform

Friday, November 5th, 2021

$x(t) = A e^{\alpha t}$ decay!

Amplitude



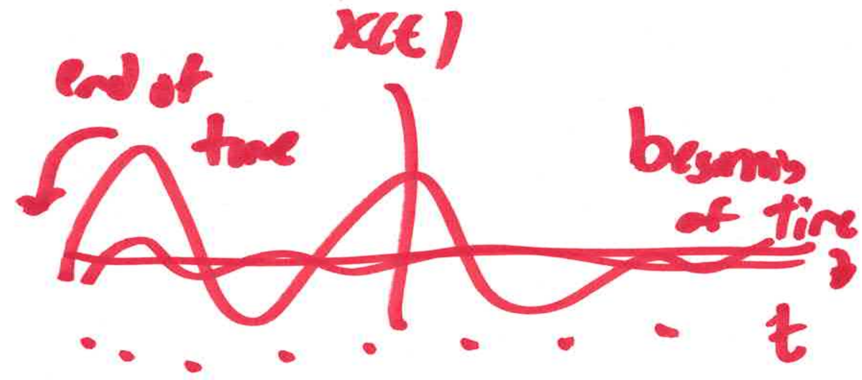
where α = Real + j Imaginary

$e^{j\omega t}$ ⇐ Periodic!

→ Sinusoidal

$$\boxed{\mathcal{L}(x(t)) = \int_{t=-\infty}^{t=+\infty} x(t)\, e^{-st}\, dt = X(s)}$$

Capital Letter



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Linearity: Scaling + additivity property

$c_1(y(t)) = (x(t) * h(t))c_1 + (x_2(t) * h(t))c_2$

Scaling!

$+ c_2(y_2(t))$

Convolution!

$y(t) = x(t) * h(t)$

$\Downarrow \mathcal{L}(y(t))$

$Y(s) = X(s)H(s)$

Laplacian Property: Multiplication!

$y(t) \rightarrow Y(s)$

$c_1 Y_1(s) + c_2 Y_2(s) = c_1(X(s)H(s)) + c_2(X_2(s)H(s))$

- Laplace is also linear! :)

$$Y(s) = X(s) H(s)$$

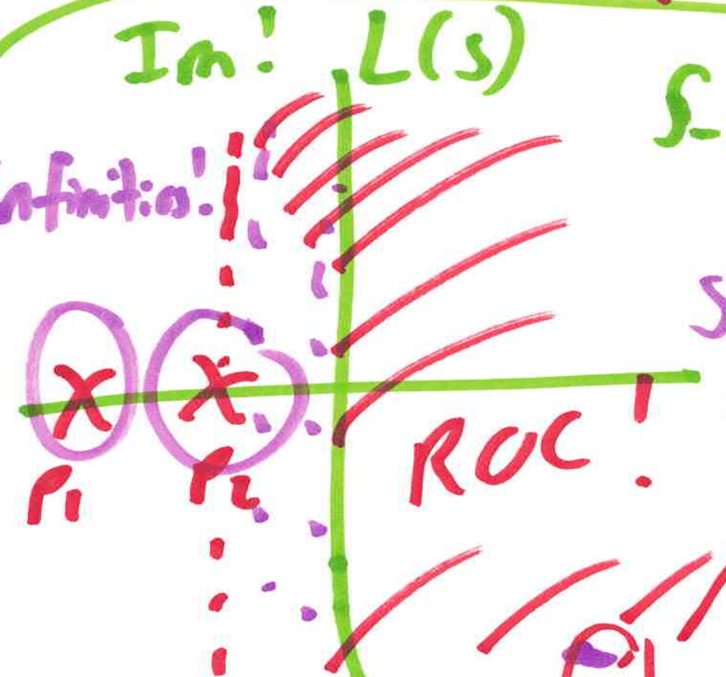
$$y(t) = x(t) * h(t) = \int x(\tau) h(t-\tau) \, d\tau$$

Impulse Response!

(can't dim $h(t)$!

$$H(s) = \frac{Y(s)}{X(s)} \quad \frac{V_{out}(s)}{V_{in}(s)}$$

$\boxed{\text{Transfer function.}}$



And all s points ??

Region of Convergence:

All poles must be on the LEFT side of the Im. Axis,

3) + ROC is on the Right side of Right most pole!

$$L(s) \to x(t) = \frac{1}{2\pi i} \int_{-\infty} H(s) e^{st} \, ds$$

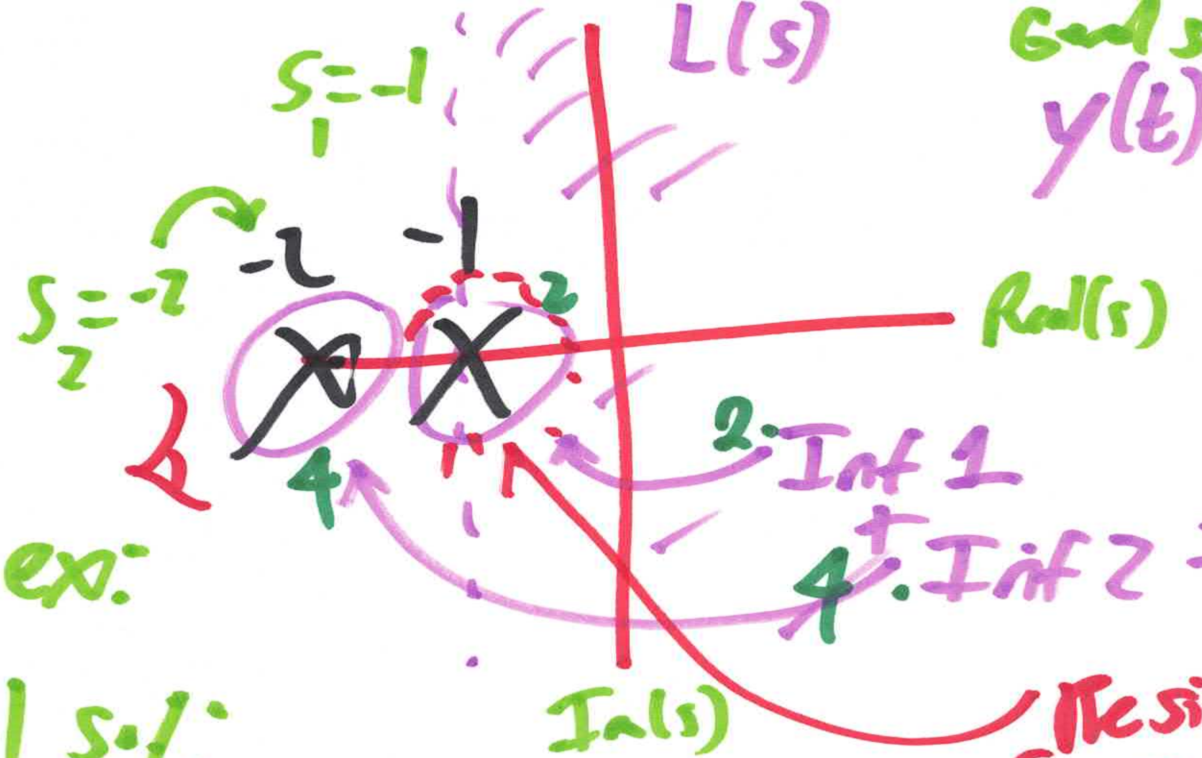
?? $L(t)$

Const + Infinity? = Infinity!

Look @ poles!

Gen'l sol:

$$y(t) = H(s = \text{pole})\, e^{s_{pole} t}$$

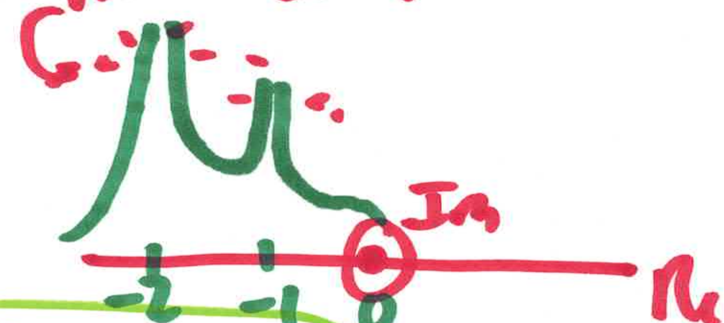


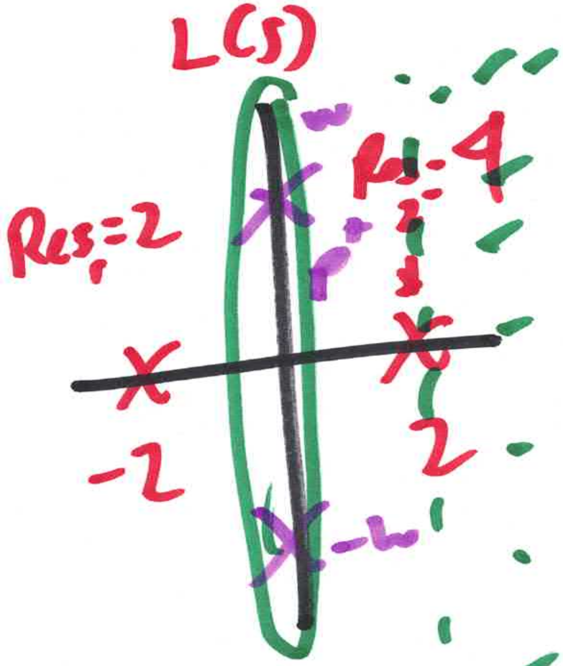
for this ex:

General sol:

$$y(t) = H(s_1)\, e^{s_1 t} + H(s_2)\, e^{s_2 t}$$

$$= \left(\underbrace{H(s_1)}_{\text{Residue}_1}\, e^{\overbrace{-t}^{\text{Pole}_1}} + \underbrace{H(s_2)}_{\text{Res}_2}\, e^{\overbrace{-2t}^{\text{Pole}_2}} \right) u(t)$$





$$y(t) = Res_1\, e^{-2t} + Res_2\, e^{+2t}$$

$t \to \infty$ Converges! Blows up!

$$y_p(t) = Res_1 \left[e^{+j\omega t} + e^{-j\omega t} \right] \cdot \frac{2}{2}$$

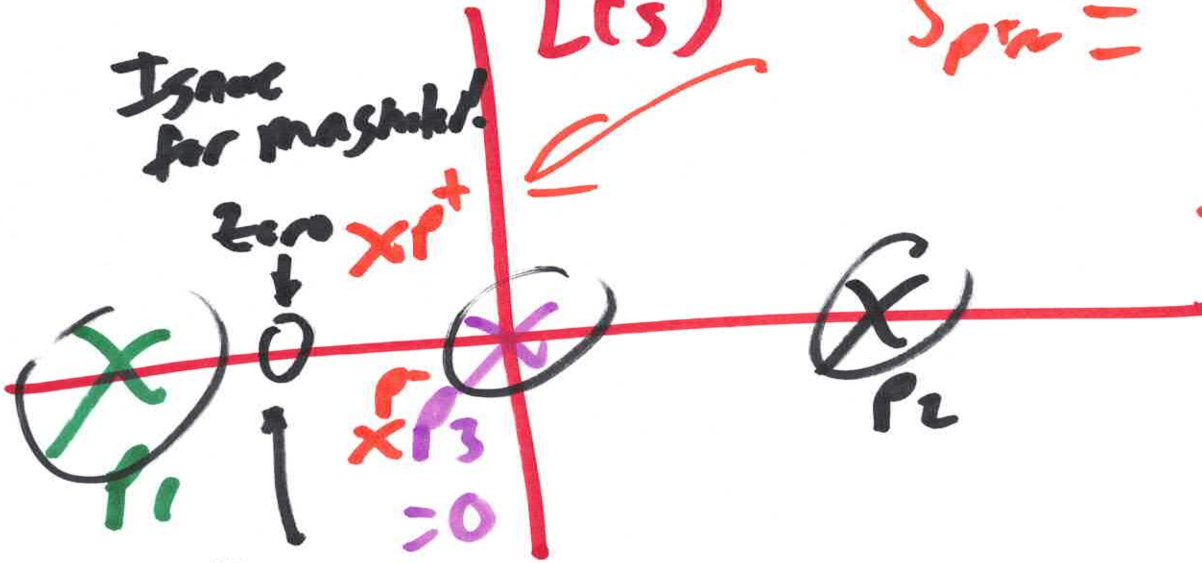
Im(s)

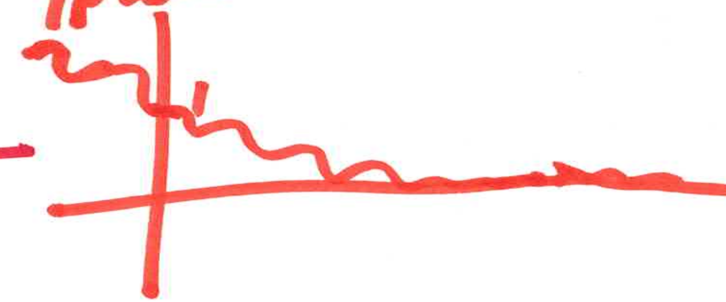
$s = Real + j\,Im.$

$$\boxed{y_p(t) = 2 Re_1 \cos(\omega t)}$$

IF RoC Does Not include Im(s), System is Unstable!

$$s_{p^\pm} = Re \pm j\,\underset{\omega}{Im}$$

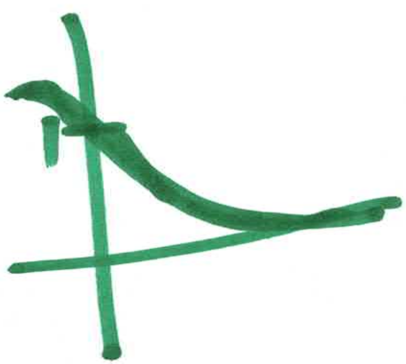


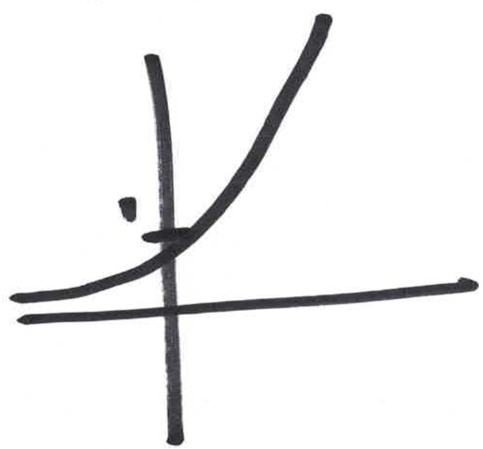


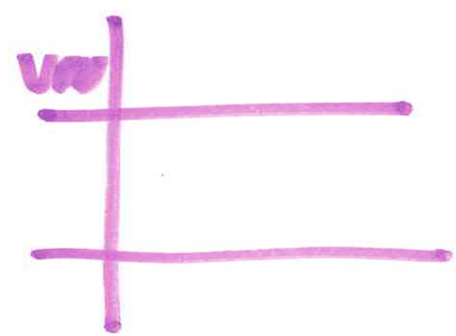
$$y_{p_1}(t) = e^{-p_1 t}$$

$$y_{p_2}(t) = e^{p_2 t}$$

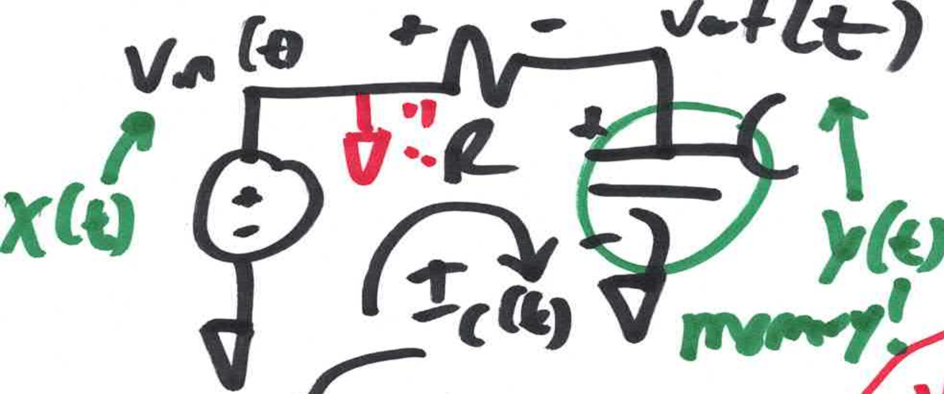
$$y_{p_3}(t) = e^{0t} \cdot VDD$$







c)



KVL:

$$V_{in}(t) - \underline{I_c(t)}R - V_{out}(t) = 0$$

$$= C\frac{dV_{out}(t)}{dt}$$

$$V_{in}(t) - RC\frac{dV_{out}(t)}{dt} - V_{out}(t) = 0$$

$$\frac{RC\frac{dV_{out}(t)}{dt} + V_{out}(t) = V_{in}(t)}{RC}$$

$$\boxed{\frac{dV_{out}(t)}{dt} + \frac{1}{RC}V_{out}(t) = \frac{1}{RC}V_{in}(t)}$$

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}X(t)$$

Memories!

$\rightarrow$ Unilateral Laplacian Differentiation Property

$t = 0^-$

$$\frac{d^N y(t)}{dt^N} \rightarrow S^N Y(s) - S^{N-1} y(t=0^-) + \cdots - S^{N-N}\frac{d^{N-1}y(t)}{d(t)}$$

?) $0^- < t \rightarrow \infty$

Causal!

$$\frac{dy(t)}{dt} + \frac{1}{\tau} y(t) = \frac{1}{\tau} x(t) \qquad \tau = RC$$

$\mathcal{L}(\ )$

$$\left[s^1 Y(s) - s^0 y(t=0^-)\right] + \frac{1}{\tau}\left[s^0 Y(s)\right] = \frac{1}{\tau} X(s)$$

$$Y(s)\left[s + \frac{1}{RC}\right] - y(t=0^-) = \frac{1}{\tau} X(s)$$

$$\left[s + \frac{1}{RC}\right]$$

Init. Condition!

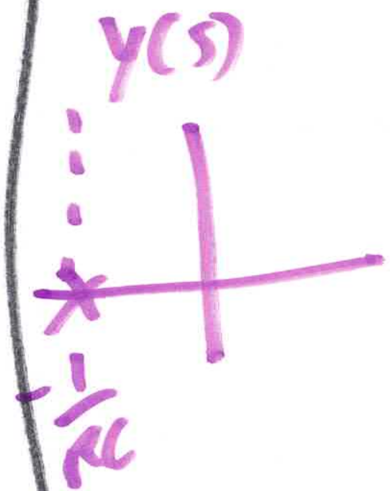
$$Y(s) = \underbrace{\frac{1}{RC}\frac{X(s)}{\left[s+\frac{1}{RC}\right]}}_{ZSR(t)} + \underbrace{\frac{y(t=0^-)}{\left[s+\frac{1}{RC}\right]}}_{ZIR(t)}$$

Y_{Total}!

$$\boxed{Y(s) = \frac{1}{\tau}\frac{X(s)}{[s+\frac{1}{\tau}]}} + \frac{y(t=0^-)}{[s+\frac{1}{\tau}]}$$

← Const!

$$s_p = -\frac{1}{\tau}$$



$$y(t) = y(t=0^-)\,e^{-t/\tau} + \frac{1}{\tau}x(t)*h(t)$$

Total Respnse!

Init. Cond!

$\cancel{X(s)}$

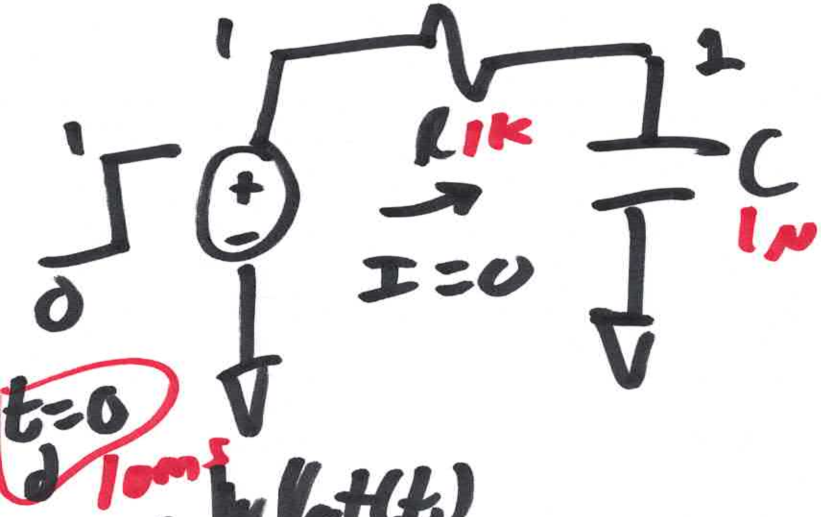
$V_{in}(t) = 1\,u(t)$

$$Y(s) = \frac{1}{\tau}\frac{X(s)}{[s+\frac{1}{\tau}]}$$

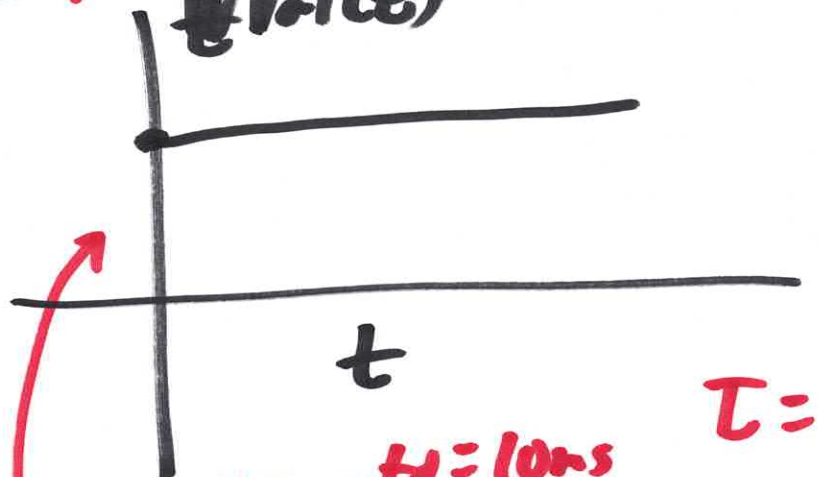
$$Y(s) = X(s)\cdot H(s)$$

$$y(t) = x(t)*h(t)$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{1}{\tau}\frac{1}{[s+\frac{1}{\tau}]}}$$

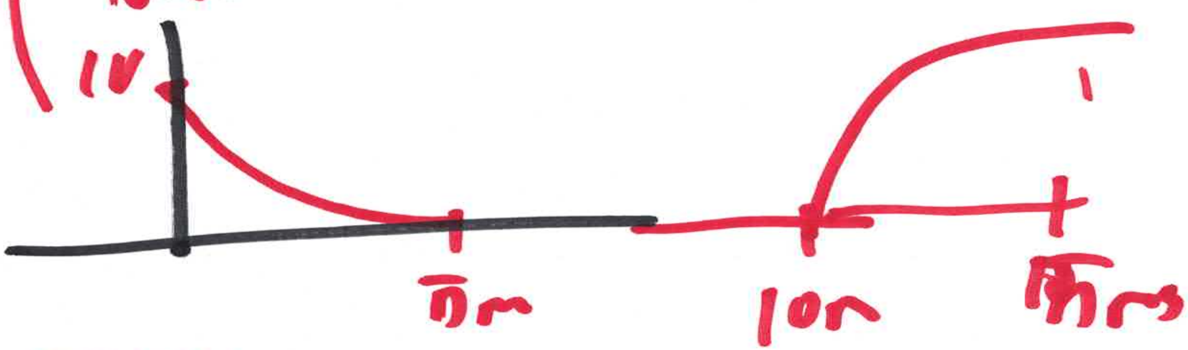


$V_{out}(t=0) = 1V$

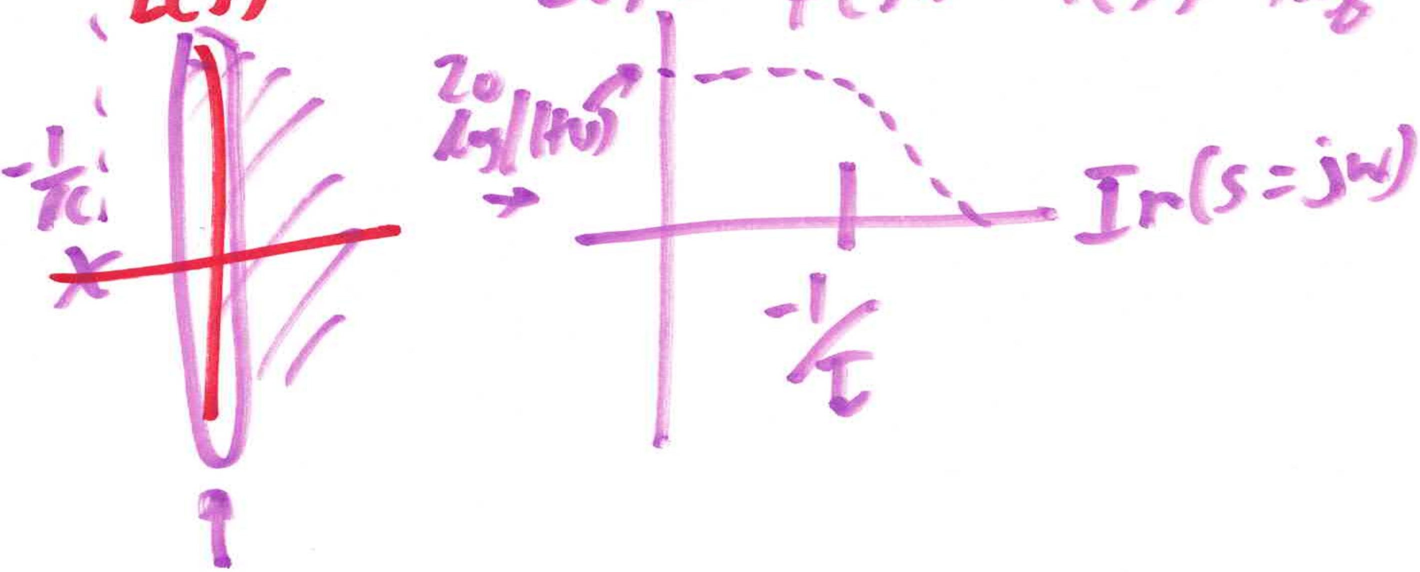


$\tau = 1m$

$V_{out}(t)$, $t_d = 10ms$



$L(j\omega) \to F(j\omega) \to F(s)$ freq



R = 0
cal

Look
@ Im!

Sinusoids
(Periodic!)

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