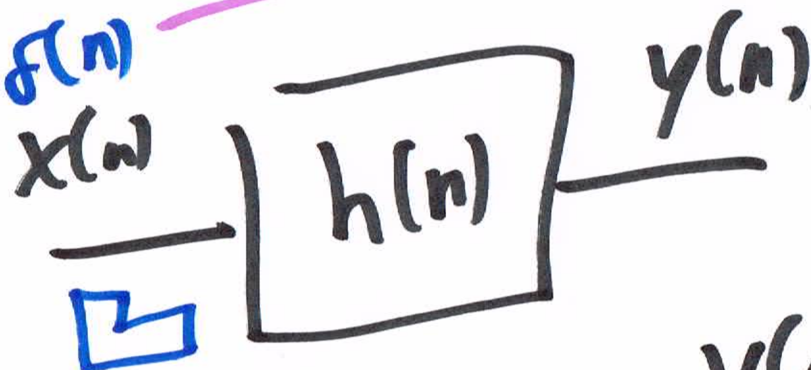
EE360D

Friday, Oct. 22nd, 2021

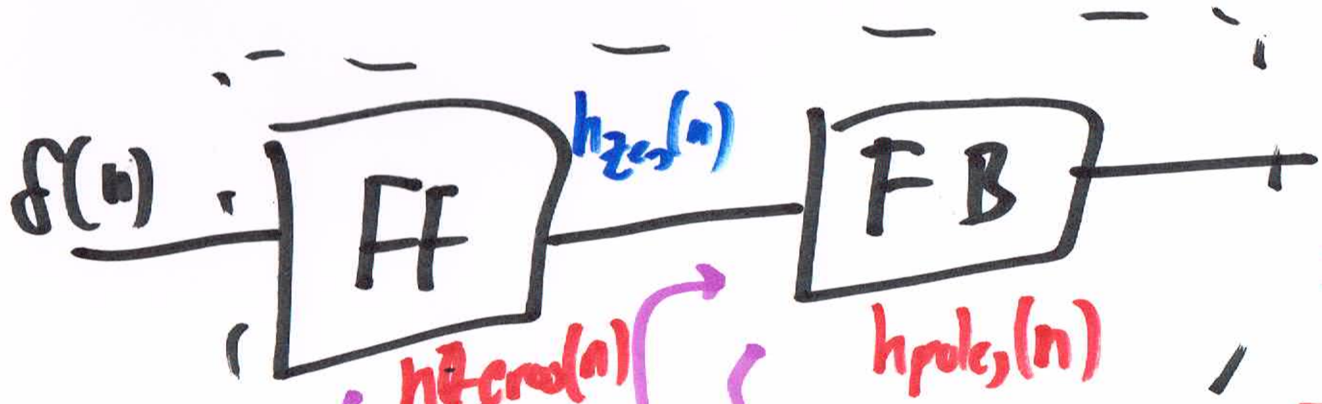
Total Response



Anything $* \delta(n)$

$$y(n) = h(n) * x(n)$$

$$= \underbrace{h(n)}_{\text{Impulse Response}}$$

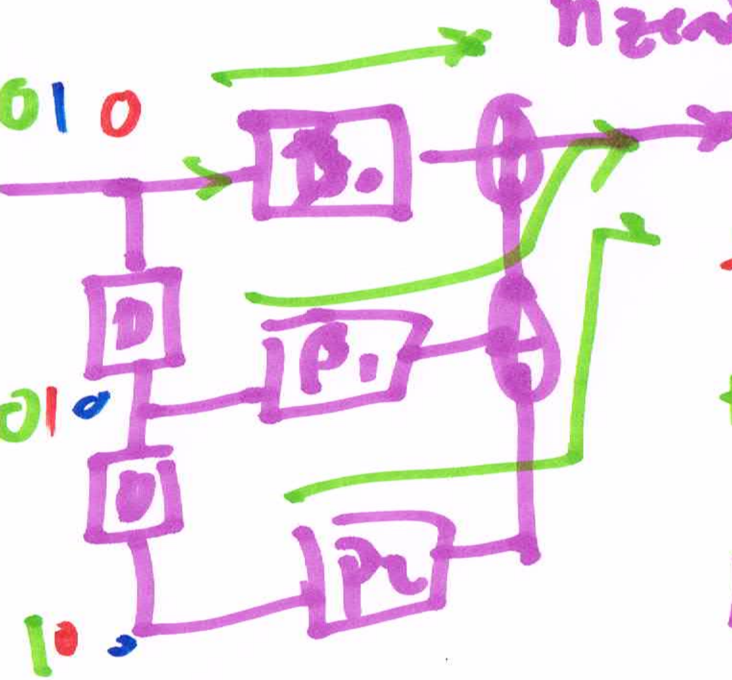


$$y(n) = h_{zeros}(n) * h_{poles}(n)$$

$$= \underline{h(n)}$$

$\underline{h(n)}$

$$y_{ZSR} = h(n) * x(n)$$



$$h_{zeros}(n) = B_0\,\delta(n) + B_1\,\delta(n-1) + B_2\,\delta(n-2)$$

$$h_{zero}(n) = \sum_{i=0}^{i=N-1} b_i\,\delta(n-i)$$

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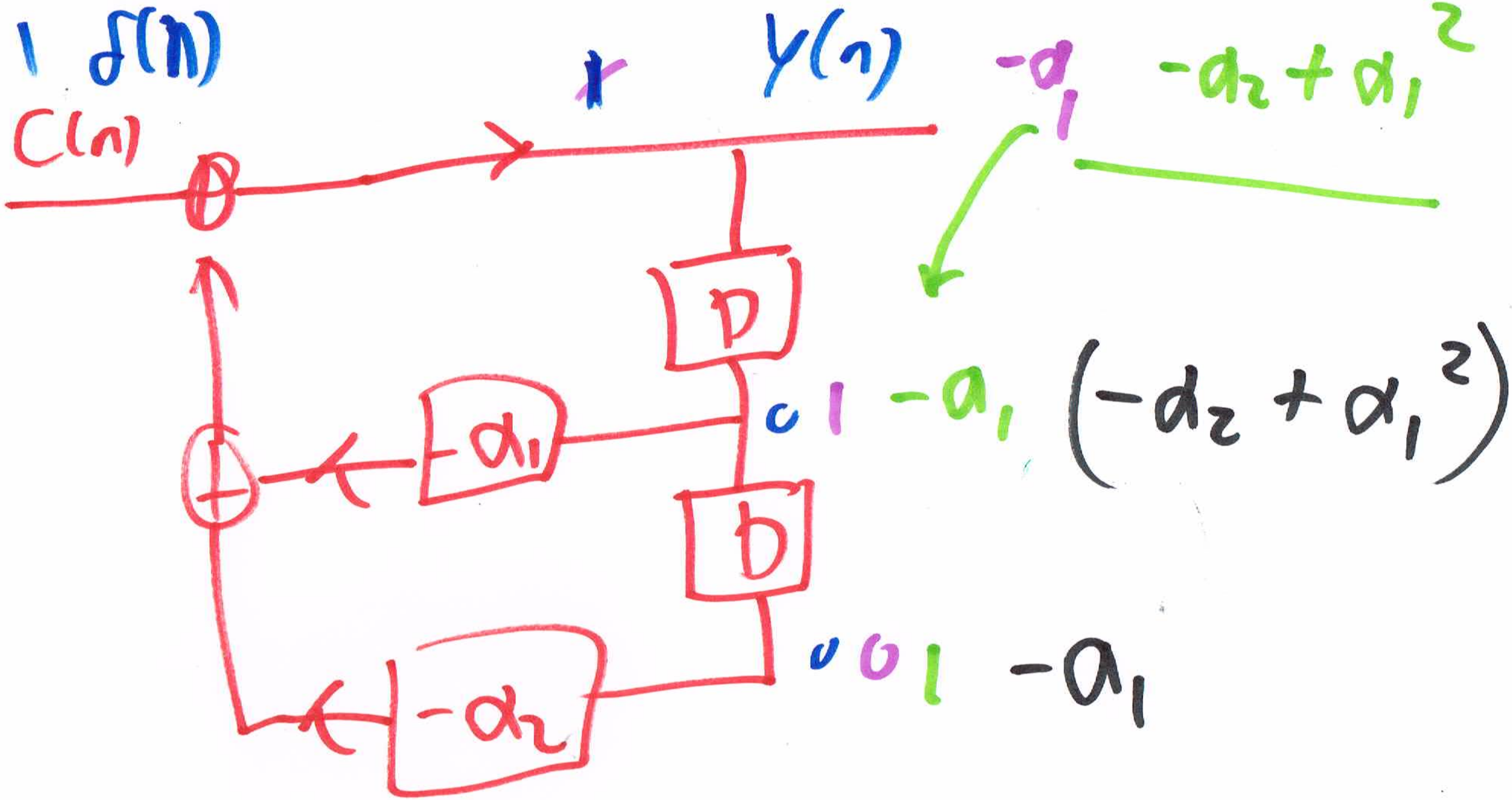
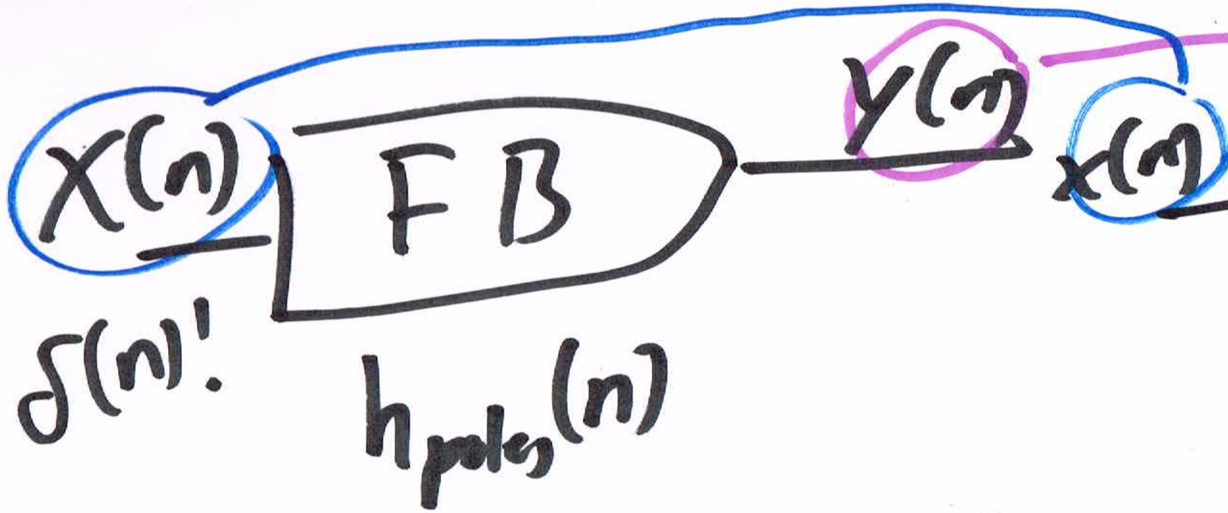
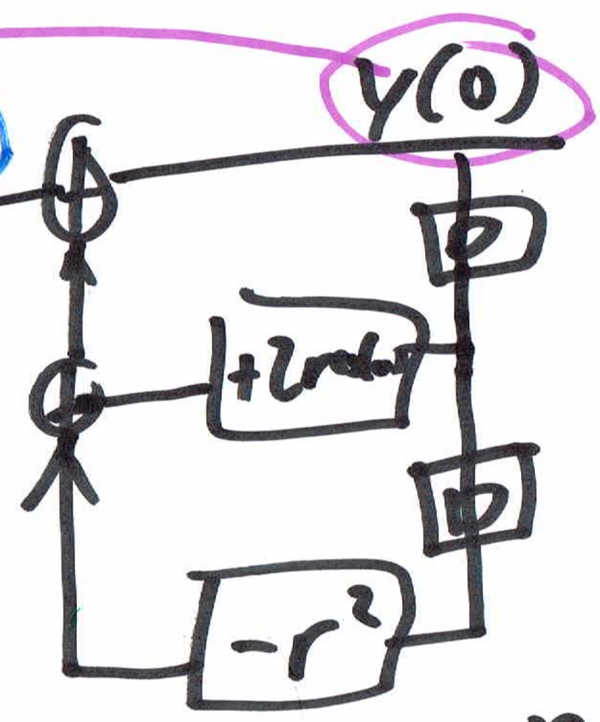


$$y(n) - 2r\cos(\omega_0)\,y(n-1) + r^2 y(n-2) =$$





$$\lambda^2 - 2r\cos(w_0)\lambda + r^2 = 0$$

$$\boxed{\lambda = re^{\pm jw_0}} \rightarrow y_{poles}(n) = C_1\left(re^{+jw_0}\right)^n + C_2\left(re^{-jw_0}\right)^n$$

$$y(0) = 1 = C_1\left(\cancel{re^{jw_0}}\right)^0 + C_2\left(\cancel{re^{-jw_0}}\right)^0$$

$$\boxed{1 = C_1 + C_2}$$

$$y(n=-1) = 0 = C_1 (re^{j\omega})^{-1} + C_2 (re^{-j\omega})^{-1}$$

$$= (1-C_2)(re^{j\omega})^{-1} + C_2 (re^{-j\omega})^{-1}$$

from $y(0)$: $\underline{C_1 = 1 - C_2}$

$$= (re^{j\omega})^{-1} \underline{- C_2 (re^{j\omega})^{-1} + C_2 (re^{-j\omega})^{-1}}$$

$$C_2 \left[\cancel{r^{-1}} e^{-j\omega} - \cancel{r^{-1}} e^{+j\omega}\right] = \cancel{r^{-1}} e^{-j\omega}$$

$$C_2 = \left[\frac{e^{-j\omega}}{e^{-j\omega} - e^{+j\omega}}\right] \frac{e^{j\omega}}{e^{j\omega}} = \frac{1}{1 - e^{2j\omega}}$$

$$\cancel{C_2 = \frac{1}{2j - e^{2jw}}}$$

$$C_2 = \left[\frac{-e^{-jw}}{-e^{-jw} + e^{+jw}}\right]\frac{2j}{2j} = \frac{-e^{-jw}}{2j\sin(w_0)}$$

$$C_1 = 1 - C_2 = 1 + \frac{e^{-jw}}{2j\sin(w_0)} = 1 + \frac{e^{-jw}}{\left(e^{+jw} - e^{-jw}\right)}$$

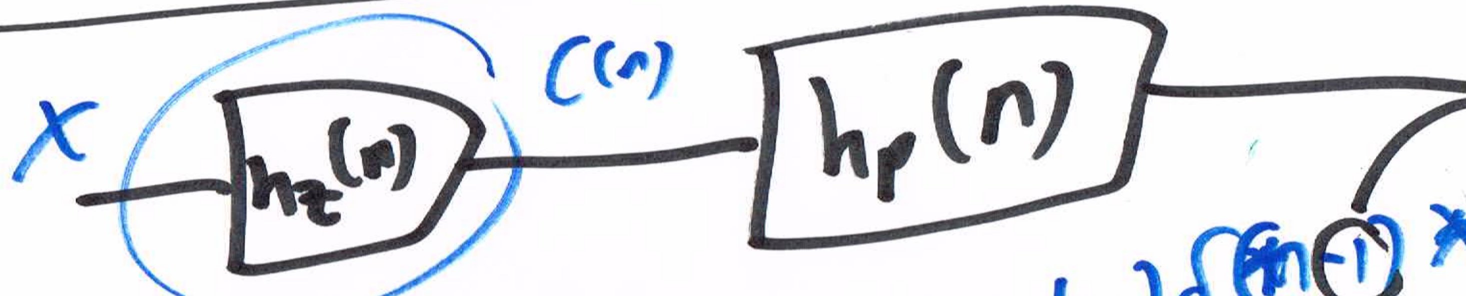
$$= \frac{e^{+jw} - e^{-jw}}{e^{+jw} - e^{-jw}} + \frac{e^{-jw}}{e^{+jw} - e^{-jw}}$$

$$= \left[\frac{e^{+jw}}{e^{+jw} - e^{-jw}} \right] \frac{2j}{2j} = \boxed{\frac{e^{+jw}}{2j\sin(w)} = C_1}$$

$$y^{\text{pole}}_{ZSR}(n) = \left(\frac{e^{+jw}}{2j\sin(w)} \right) \left(r e^{+jw} \right)^n - \left(\frac{e^{-jw}}{2j\sin(w)} \right) \left(r e^{-jw} \right)^n$$

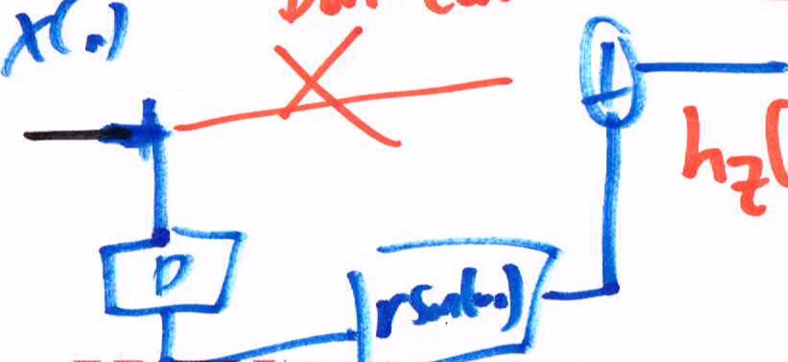
$$= r^n \frac{1}{2j\sin(w)} \left[e^{jw(n+1)} - e^{-jw(n+1)} \right] \frac{\cancel{2j}}{2j}$$

$$\left[\frac{r^n \sin(w_0(n+1))}{\sin(w_0)}\right]u(n) = h_{poles}(n)$$



$$h_T(n) = h_z * h_p = \frac{r\sin(w_0)}{\text{Const}}\,\delta(n-1) * r^n\,\frac{\sin(w(n+1))u(n)}{\sin(w_0)}$$

$$= r\,\cancel{\sin(w_0)}\cdot r^{n-1}\,\frac{\sin(wn)}{\cancel{\sin(w_0)}}\,u(n-1)$$



$$h_z(n) = r\sin(w_0)\,\delta(n-1)$$

$$\boxed{h(n) = r^n \sin(wn)\,u(n-1)}$$

$$|y(n)| \leq \left|\sum_{k=\infty}^{k=\infty} h(n)\, x(n)\right|$$

$$\leq \sum_{-\infty}^{\infty} |h(n)|\,|x(n)|$$

Bounded

$$= \sum_{-\infty}^{\infty} \left| r^{n} \sin(wn)\, u(n-1) \right| \, |x(n)|$$

$$\leq \sum_{0}^{n \to \infty} |r^{n}|\,|\sin(wn)|\,|x(n)|$$

$$1 \qquad \boxed{r \leq 1}$$

CA3
pt2.

for ...

$x * h$

shift

circshift

CA4

$$\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$$

$y(0) = -1$

memory!

ZIR:

$x = 0$

$$\lambda' + 3\frac{dy(t)}{dt} = 0$$

$$y_{ZIR}(t) = C_1 e^{\lambda t} = C_1 e^{-3t}$$

$$\boxed{\lambda = -3}$$

$$y(0) = -1 = C_1 e^{0}$$

$$\boxed{C_1 = -1}$$

Analog

$$\frac{d^N y(t)}{dt^N} \rightarrow \lambda^N$$

$$\boxed{y_{ZIR}(t) = -e^{-3t}\,u(t)}$$

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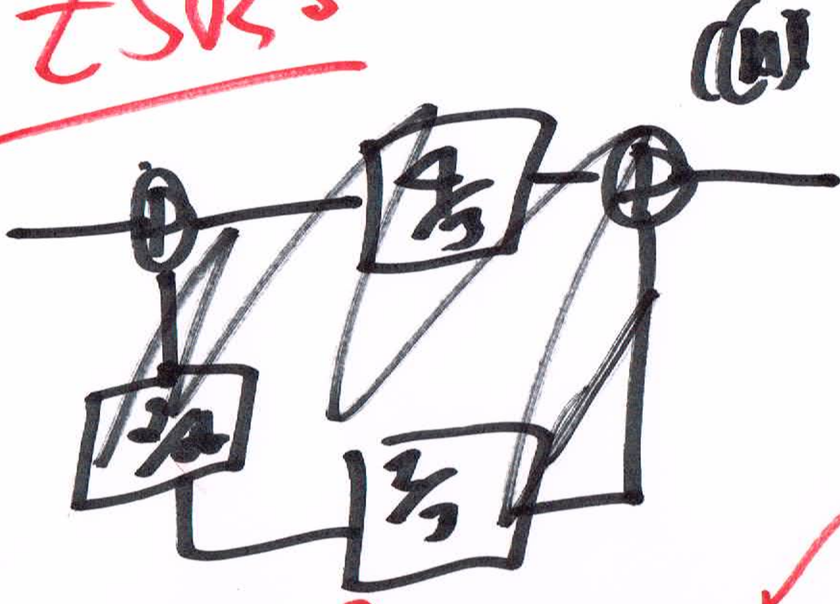
Rewrite:

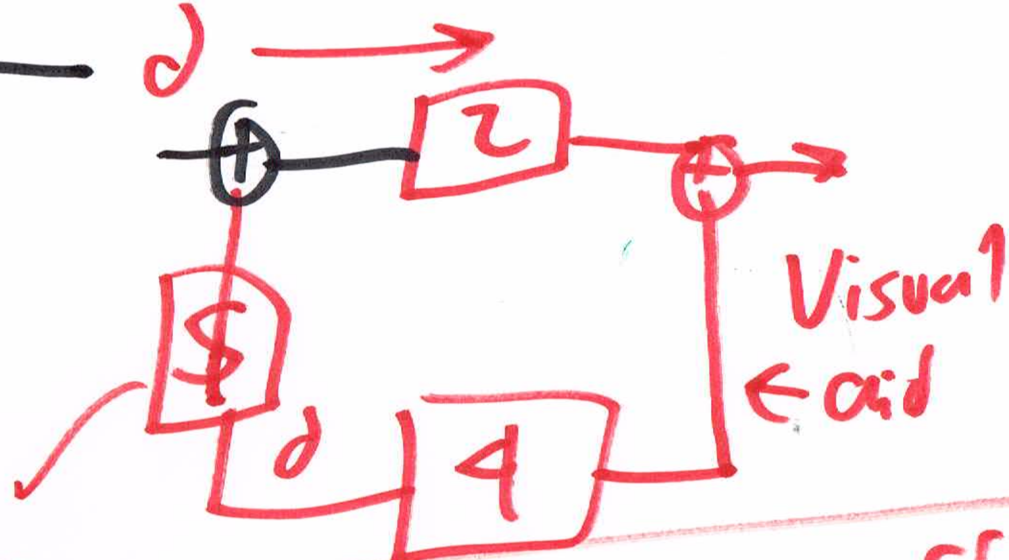
$$\cancel{y(t) = \frac{2}{3}\dot{x} + \frac{4}{3}x - \dot{y}}$$

$$y(t) + 3\int y = 2x + 4\int x$$

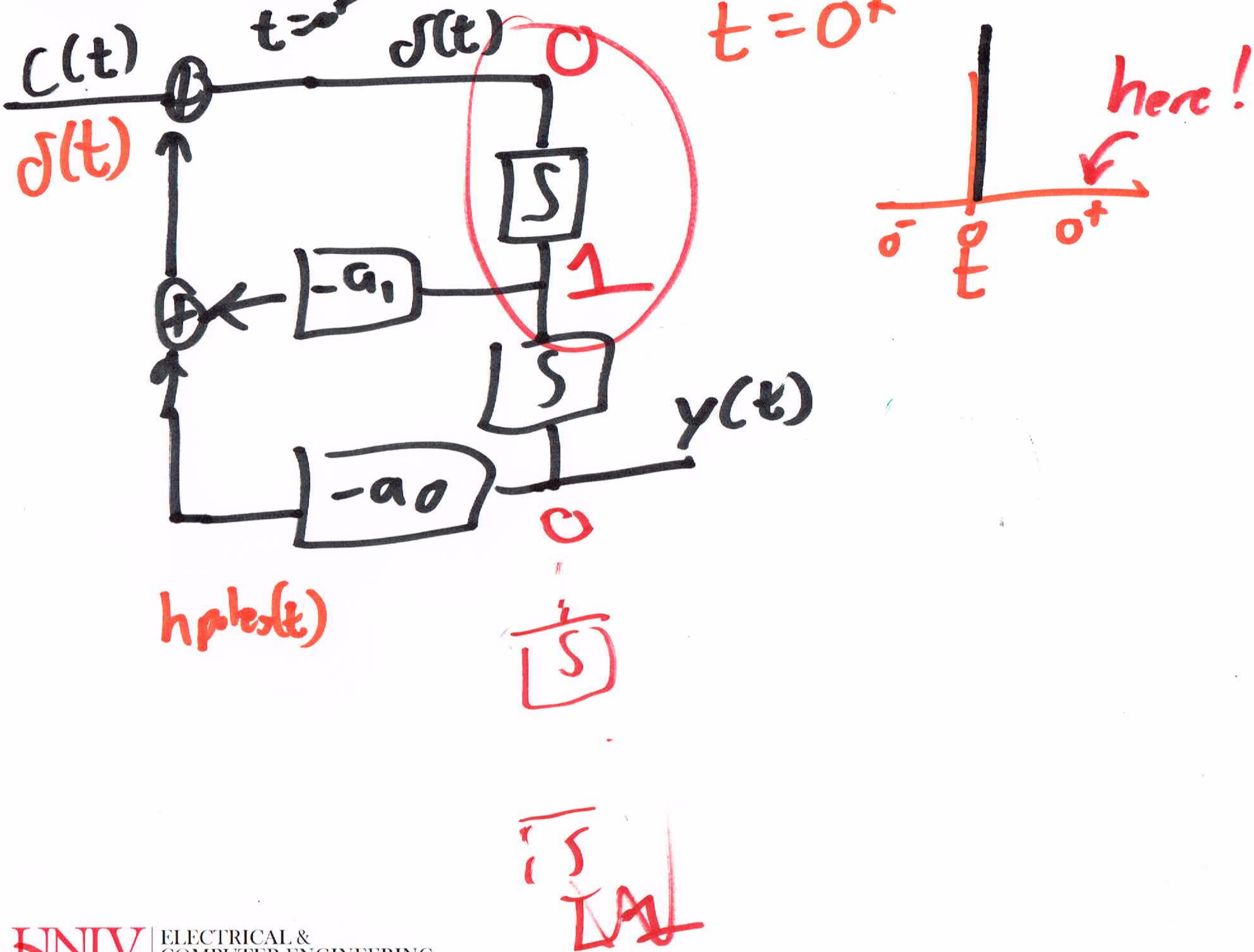
ZSR:

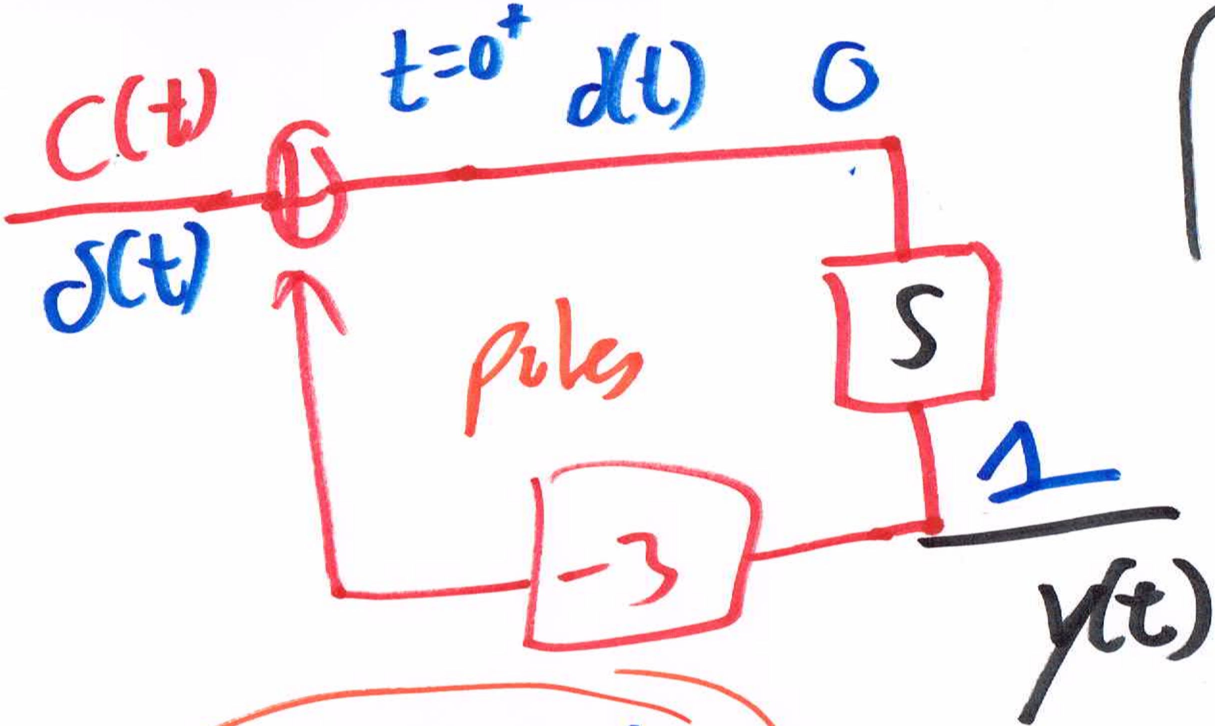


hz



$$\boxed{h_{zero}(t) = 2\frac{d\delta(t)}{dt} + 4\delta(t)}$$


c(t)
δ(t)
t = 0+
δ(t)
0
1
0
−a1
−a0
y(t)
t = 0+
here!
0−
0
0+
t
hpoles(t)



$$\int \frac{dy}{dt} + 3y(t) = 0$$

$$y = -3 \int y$$

$$y(t=0^+) = 1$$

$$\frac{dy}{dt} = 0$$

$$y^{?}_{ZIR}(t) = C_1 e^{-3t}$$

$$y_{poles}(t) = 1 = C_1 e^{0}$$

$t=0$

$$C = 1$$

$$\boxed{y_{poles}(t) = e^{-3t} u(t)}$$

$$\delta(t) \longrightarrow \boxed{h_z(t)} \longrightarrow \boxed{h_p(t)} \longrightarrow$$

$$h_T(t) = h_z * h_p(t)$$

$$= \left[4\delta(t) + 2\frac{d\delta(t)}{dt}\right] * \left[e^{-3t}u(t)\right]$$

$$= \left[4e^{-3t}u(t)\right] + 2\int_{\tau=-\infty}^{\tau=+\infty} e^{-3(t-\tau)}u(t-\tau)\frac{d\delta(\tau)}{d\tau}$$

Dirac derivative sifting property!

$$= 2\left\{ -\frac{\partial}{\partial\tau}\left[e^{-3(t-\tau)} u(t-\tau) \right] \right|_{\tau=0}$$

$$= 2\left\{ -\left[-3(t-\cancel{\tau}) e^{-3(t-\cancel{\tau})} u(t-\cancel{\tau}) + e^{-3(t-\cancel{\tau})} \delta(t-\cancel{\tau}) \right] \right|_{t=0}$$

$$= +6e^{-3t} - 2e^{-3t}\delta(t)$$

$$h(n) = h_z * h_p = 4e^{-3t}u(t) + 6e^{-3t}u(t) - 2e^{-3t}\delta(t)$$

$$\boxed{h(n)_{ZSR} = 10e^{-3t}u(t) - 2e^{-3t}\delta(t)}$$

~~Total~~ Impulse Response

$$y_{Total} = h_{Total} = \overset{\text{memory}}{h_{ZIR}} + h_{ZSR}$$

$$= -e^{-3t}u(t) + 10e^{-3t}u(t) - 2e^{-3t}\delta(t)$$

$$\boxed{y_{Tot} = 9e^{-3t}u(t) - 2e^{-3t}\delta(t)}$$