EE360
Total Response

Monday, Oct. 25th 2021

CA1	CA2	CA3	CA4
Intro to Matlab	Definitions	Convolution!	Total Response
	$\delta[n]$ $u[n]$	$h * x$[bespoke]	
	Diff Eqns	Const.	
	for i: ...		
	$y = b_0 x(n) + b_1 x(n-1)$		
	!!! ...		

$(\delta[n]$ ZSR$)$ + (memory! ZIR) = Total Resp!

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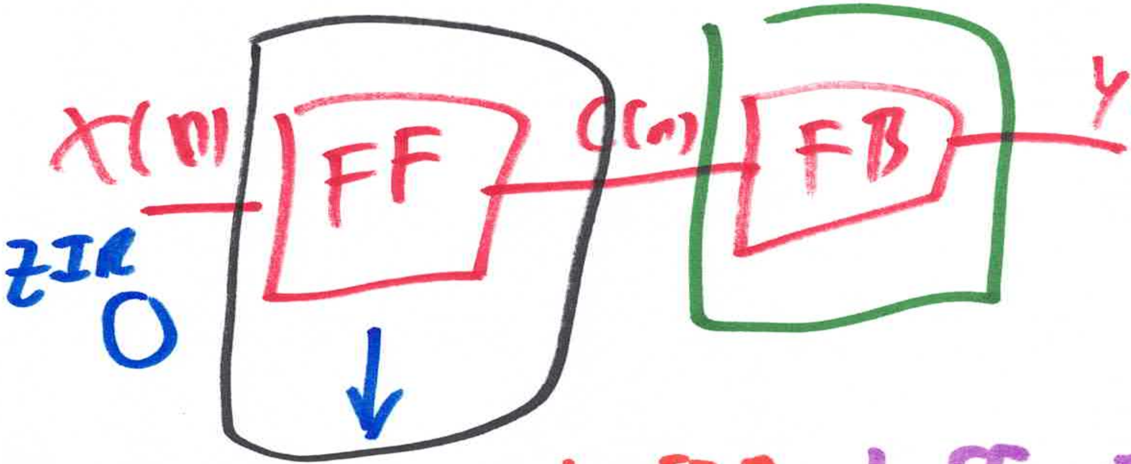
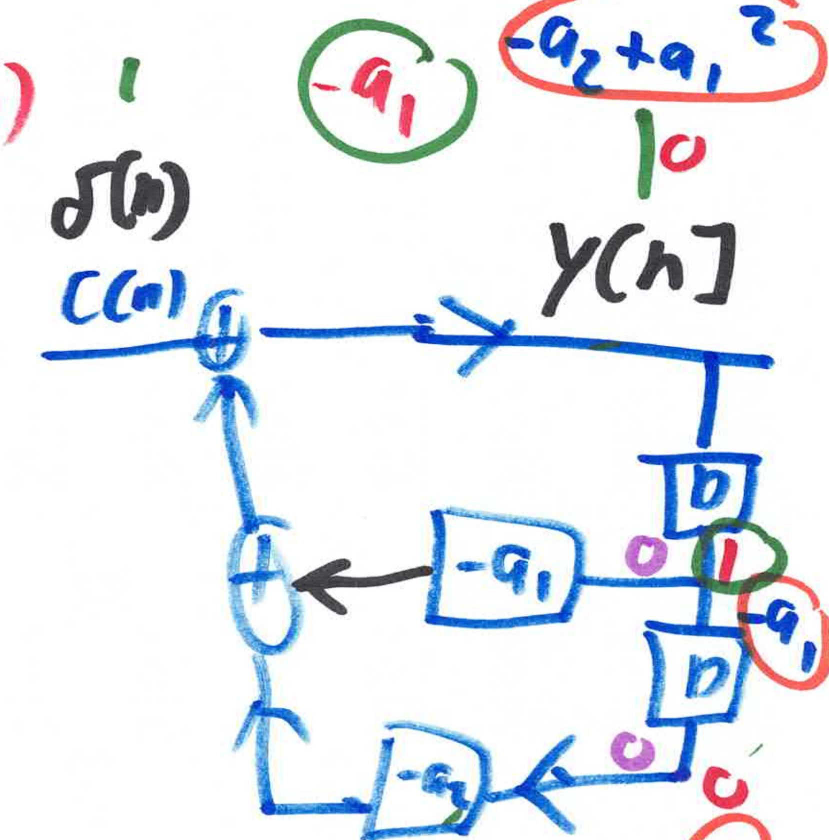
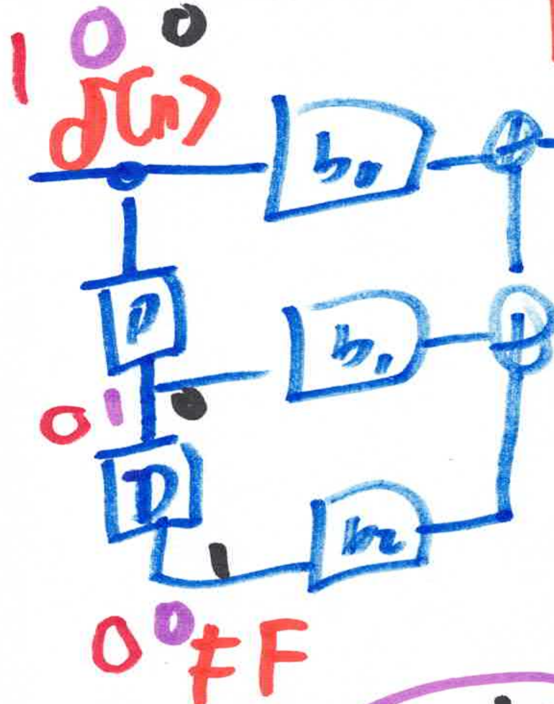






$b_0\delta[n] + b_1\delta[n-1]$
$+ b_2\delta[n-2]$
$= h_{zero}[n]$

Building Part #1!

Infinite Impulse Response (IIR)

Finite Impulse Response

Response to delta function δ[n]

2)

EE221:

$H(s) = \dfrac{(s+1) \leftarrow \text{zeros}}{s+2 \leftarrow \text{poles}}$

$y[n] + a_1 y[n-1] + a_2 y[n-2] = 0$?

$y[n] = -a_1 y[n-1] - a_2 y[n-2] + C[n]$

$\lambda^2 + a_1 \lambda + a_2 = 0$

$$y[n] - 2r\cos(w_0)\,y[n-1] + r^2 y[n-2] = 0$$

$$\left(\lambda^0 - 2r\cos(w_0)\,\lambda^{-1} + r^2\lambda^{-2} = 0\right)\lambda^2 \qquad y[n-N] \to \lambda^{-N}$$

Euler's Eqn.:

$$\cos(w_0) = \frac{e^{+jw} + e^{-jw}}{2}$$

$$\lambda^2 - \cancel{2}r\lambda\left[\frac{e^{+jw} + e^{-jw}}{\cancel{2}}\right] + r^2 = 0$$

$$\lambda^2 - r\lambda e^{+jw} - r\lambda e^{-jw} + r^2 = 0$$

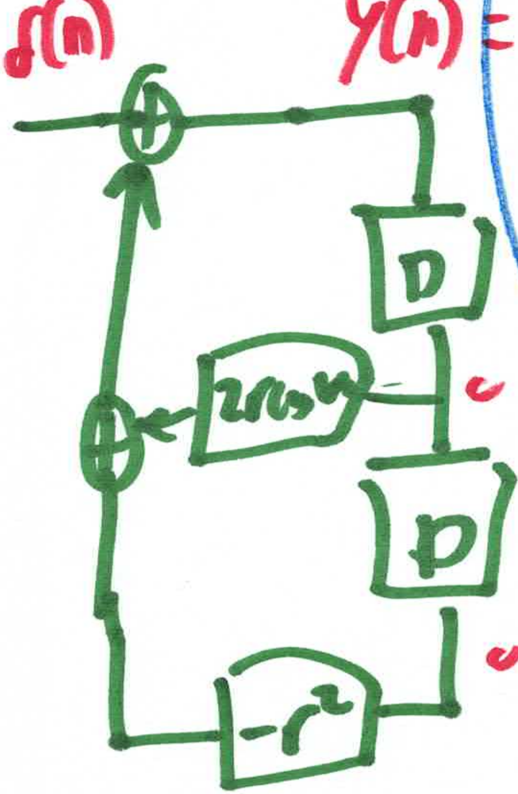
Decomposition (Algebra)

$$\lambda(\lambda - re^{+jw}) \cancel{- re^{+jw}(\lambda - e^{+j})}$$

$$- re^{-jw}(+\lambda - re^{+jw})$$

$$(\lambda - re^{+jw})(\lambda - re^{-jw}) = 0$$

$$\boxed{\lambda = re^{\pm jw}}$$



$$Y_{pulse}(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$n = 0$

$$Y(n) = 1$$

$$\underline{Y_{pulse}(n=0)} = 1 = C_1(\lambda_1)^0 + C_2(\lambda_2)^0$$

$$= C_1 + C_2$$

Condition for Impulse Response of FB (Digital)

$$\underline{C_1 = 1 - C_2}$$

$$Y_p(n) = C_1\left(re^{+j\omega}\right)^n + C_2\left(re^{-j\omega}\right)^n$$

$$Y_p(n=-1) = 0 = \underline{C_1 \cancel{r}^{-1} e^{-j\omega}} + C_2 \cancel{r}^{-1} e^{+j\omega}$$

$$0 = \left(e^{-j\omega} - \underline{C_2} e^{-j\omega}\right) + \underline{C_2} e^{+j\omega}$$

$$-e^{-j\omega} = C_2\left(-e^{-j\omega} + e^{+j\omega}\right) \qquad \sin\phi = \frac{e^{j\phi} - e^{-j\phi}}{2j}$$

$$C_2 = \frac{-e^{-j\omega}}{e^{+j\omega} - e^{-j\omega}} \cdot \frac{2j}{2j} = \frac{-e^{-j\omega}}{2j\sin(\omega_0)}$$

$$C_1 = 1 - C_2 = 1 - \left(\frac{-e^{-j\omega}}{2j\sin(\omega_0)}\right)$$

$$= \frac{1}{1} - \left(\frac{-e^{-j\omega}}{\left(e^{+j\omega} - e^{-j\omega}\right)}\right)$$

$$c_1 = \frac{\left(e^{+j\omega} \cancel{- e^{-j\omega}}\right) + \left(\cancel{+ e^{-j\omega}}\right)}{e^{+j\omega} - e^{-j\omega}}$$

$$= \frac{e^{+j\omega}}{e^{+j\omega} - e^{-j\omega}} \cdot \frac{2j}{2j} = \boxed{\frac{e^{+j\omega}}{2j\sin(\omega_0)} = c_1}$$

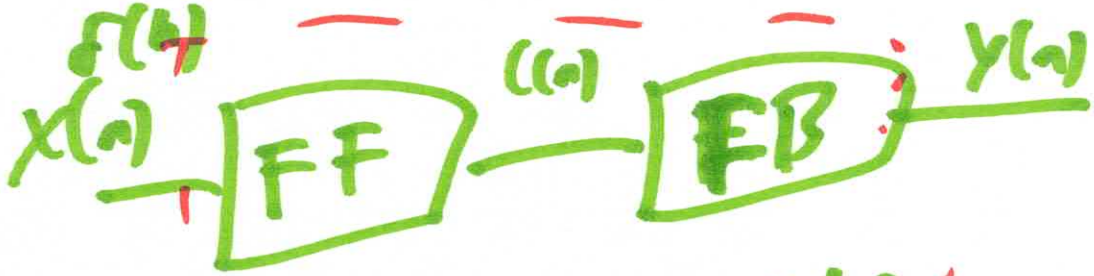
$$Y_{poles}(n) = \left(\frac{e^{+j\omega}}{2j\sin(\omega_0)}\right)\left(re^{+j\omega}\right)^n + \left(\frac{-e^{-j\omega}}{2j\sin(\omega_0)}\right)\left(re^{-j\omega}\right)^n$$

$$y_{pd}(n) = \frac{r^n}{2j\sin(w_0)}\left(e^{jw(n+1)}\right) - \frac{r^n}{2j\sin(w_0)}\left(e^{-jw(n+1)}\right)$$

$$= \frac{r^n}{2j\sin(w_0)}\left[e^{+jw(n+1)} - e^{-jw(n+1)}\right]$$

$$\boxed{y_{poles}(n) = \frac{r^n}{\sin(w_0)}\sin\left(w(n+1)\right)u(n)}$$

$$= h_{poles}(n)$$



$h_{zeros}(n)$ $\quad$ $h_{poles}(n)$

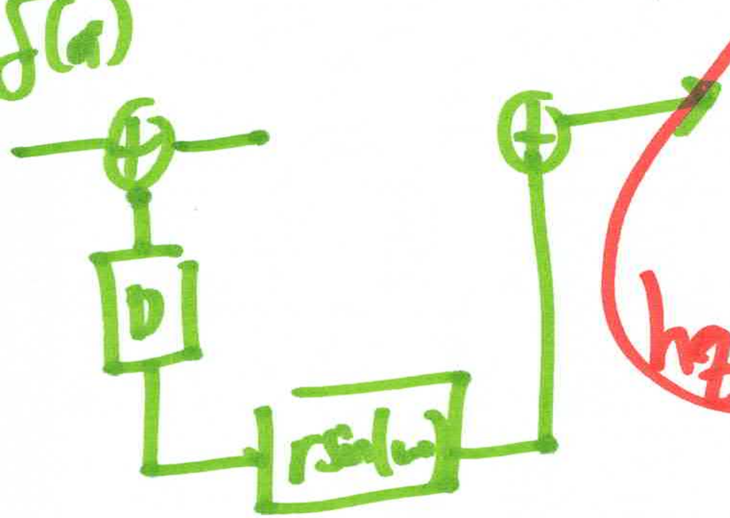
$$y(n) = h_{zeros}(n) * h_{poles}(n)$$

$$r\sin(\omega_0)\,\delta(n-1) * \frac{r^n}{\sin(\omega_0)}\sin(\omega_0(n+1))u(n)$$

$$C(n) = h_{zeros}(n)$$

$$= \frac{r\sin(\omega_0)}{\sin(\omega_0)}\left(r^{n-1}\sin(\omega_0(n))u(n-1)\right)$$

$$h_{zeros}(n) * h_{poles}(n) = r^n\sin(\omega_0 n)u(n-1)$$



$$= h(n)$$

$$h_{zeros}(n) = r\sin(\omega_0)\,\delta(n-1)$$

ZSR (Impulse ~~Impulse~~ Response!!!)

a)

$$|y(n)| \leq \left| \sum_{k=-\infty}^{k=+\infty} h(k)\, x(n-k) \right|$$

$$\left| \sum_{k=-\infty}^{\infty} h(k) \right|$$

$$= \left| \sum_{k=-\infty}^{\infty} r^{k} \sin(wk)\, u(k-1) \right|$$

$$= \sum_{1}^{k=\infty} |r^{k}|\, |\sin(wk)|$$

$$\boxed{r \leq 1}$$

Total Response: ZSR + ZIR

the "kick" into your System (No memory)

memory!

$y_{ZIR}(n) = C_1\lambda_1^n + C_2\lambda_2^n$

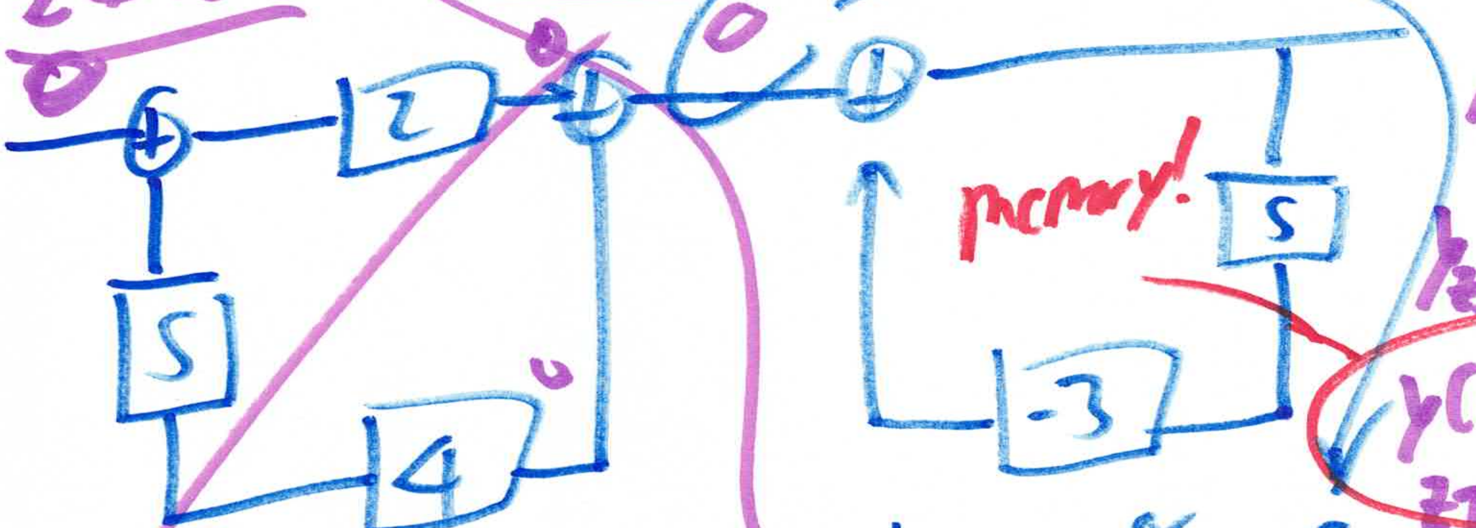
$y(0) =$ memory!

whatever Dr. Stubberud Gives!

Analog: $\frac{dy(t)}{dt} + 3y(t) = 2\frac{dx(t)}{dt} + 4x(t)$

model: $y(t) + 3\int y = 2x + 4\int x$ Guess!

ZIR



$y_{ZIR}(t) = C_1 e^{\lambda t}$

$y_{ZIR}(t) = C_1 e^{-3t}$

$y(0) = -1 = C_1 e^{-3(0)}$

$C_1 = -1$

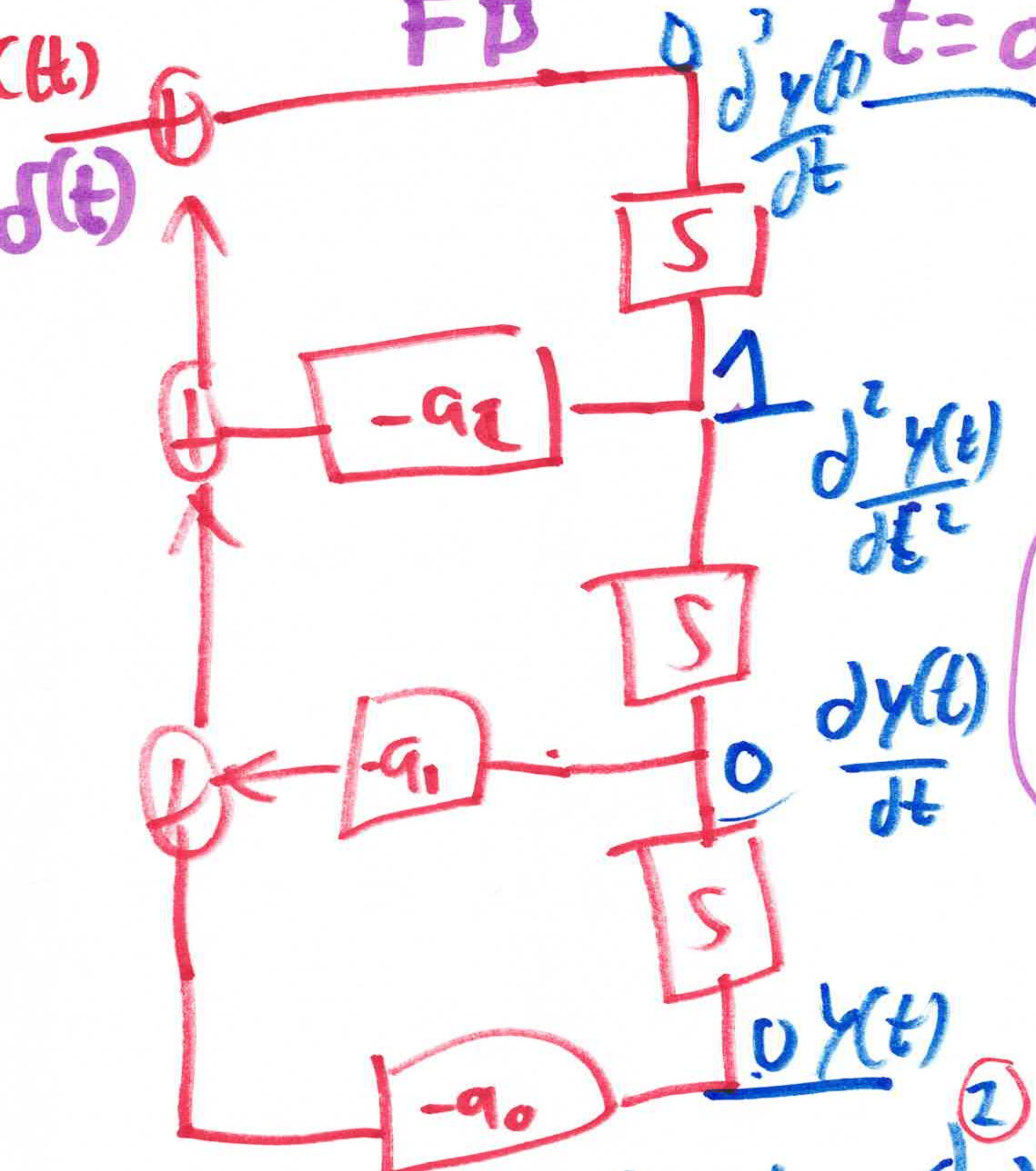
$\lambda^1 + 3\lambda^0 = 0$

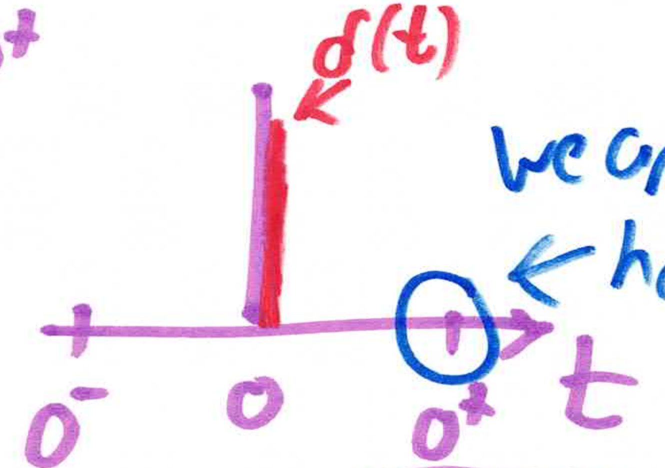
$\frac{d^N y(t)}{dt^N} \to \lambda^N$

$\lambda = -3$

DONT FORGET

$y_{ZIR}(t) = -e^{-3t}\, u(t)$





we are here!

$$\boxed{\text{Nth order Condition for ZSR}: \frac{d^{N-1}y(t)}{dt^{N-1}} = 1}$$

$N = 1$

$$\frac{d^{1-1}y(t)}{dt^0}$$

$$\boxed{= y(t) = 1}$$

3rd order system $\frac{d^2 y(t)}{dt} = 1$!

2nd order: $\frac{d^{2-1}y(t)}{dt} = \frac{dy(t)}{dt} = 1$

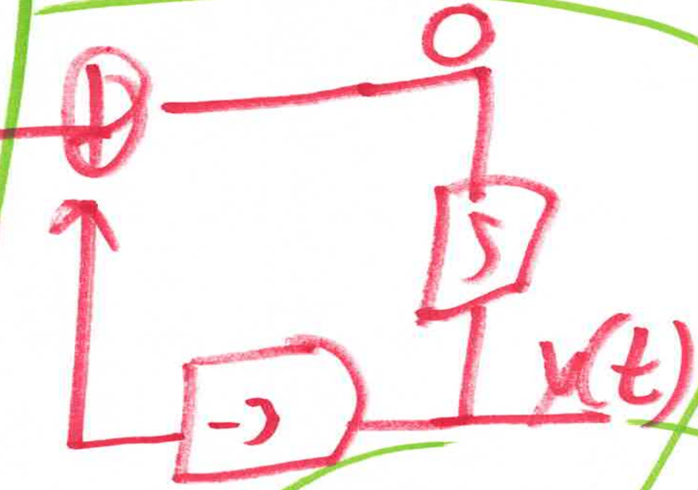
ZSR

Analog:

pulse FB

$$\frac{dy(t)}{dt} + 3y(t) = 0$$

$\frac{C(n)}{\delta(t)}$



$y(t=0) = 1$

$$\lambda + 3 = 0$$

$$\lambda = -3 \rightarrow e^{\lambda t}$$

$$h_{pulse}(t) = C_1 e^{\lambda t}$$

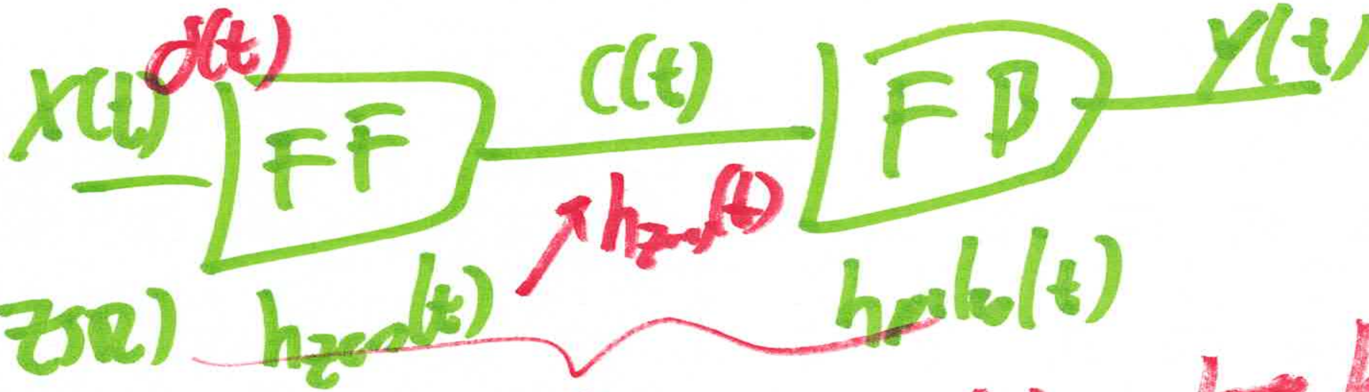
$$h_{pulse}(t=0) = 1 = C_1 e^{-3(0)}$$

$$\boxed{C_1 = 1}$$

Part/Piece! #2!

$$\boxed{h_{pulse}(t) = e^{-3t} u(t)}$$

Don't forget!



$$y(t) = h_{zeros}(t) * h_{poles}(t) = h(t)$$

$$= \left(4\delta(t) + 2\frac{d\delta(t)}{dt}\right) * e^{-3t}u(t)$$

$$\cancel{y(t)}\; y = 2\dot{x} + 4x = c(t)$$

Practically:

h_{zeros}

$$x(t) = \delta(t)$$

$$\boxed{h_{zeros}(t) = 2\frac{d\delta(t)}{dt} + 4\delta(t)}$$

$$4e^{-3t}u(t) + 2\frac{d\delta(t)}{dt} * e^{-3t}u(t)$$

$$2\int_{-\infty}^{\infty} e^{-3(t-\tau)}\, u(t-\tau) \cdot \frac{d\delta(\tau)}{d\tau}$$

~~Direct~~ Derivative Sifting Property

Ch. 1 Pg. 82 Ex 1.56

$$\ominus 2\,\frac{d}{d\tau}\left[e^{-3(t-\tau)}\,u(t-\tau)\right]_{\tau=0}$$

Don't forget!!

$$= -2\left[-3e^{-3(t-\cancel{\tau})}\,u(t-\cancel{\tau}) + e^{-3(t-\cancel{\tau})}\,\delta(t-\cancel{\tau})\right]_{\tau=0}$$

$$2\frac{d\delta(t)}{dt} * e^{-3t}u(t) = +6e^{-3t}u(t) + e^{-3t}\delta(t)$$

$$y(t) = h(t) = 4e^{-3t}u(t) + 6e^{-3t}u(t) + e^{-3t}\delta(t)$$

Impulse Response (ZSR)

$$\boxed{= \underline{10e^{-3t}u(t) + e^{-3t}\delta(t)}}$$

Total Response

$$= y_{ZIR} + y_{ZSR} = \underbrace{-e^{-3t}u(t)}_{ZIR} + \underbrace{10e^{-3t}u(t)}_{ZSR} + e^{-3t}\delta(t)$$

$$\boxed{y_{Total}(t) = 9e^{-3t}u(t) + e^{-3t}\delta(t)}$$