SIGNALS AND SYSTEMS I

Computer Assignment 5

Analog systems can be designed, analyzed and simulated in MATLAB using the signal processing and controls toolboxes.

Signal Processing Toolbox

In MATLAB's signal processing toolbox, the transfer function is the principal system representation. For single input, single output (SISO) systems, MATLAB assumes that the system's transfer function or system function, $H(s)$, has the form

$$H(s) = \frac{b(1)s^M + b(2)s^{M-1} + \cdots + b(M)s + b(M+1)}{a(1)s^N + a(2)s^{N-1} + \cdots + a(N)s + b(N+1)}$$

In MATLAB, the row vector, **b**, stores the numerator coefficients, $b(1), b(2), \ldots, b(M), b(M+1)$, and the row vector, **a**, stores the denominator coefficients, $a(1), a(2), \ldots, a(N), a(N+1)$. Thus, a system function is defined by two row vectors, one row vector for the numerator and one for the denominator.

The system function can be evaluated for particular values of s using the *polyval* function. For example,

```
H = polyval(b,s) ./ polyval(a,s)
```

evaluates the system function defined by the vectors, **b** and **a**, at the values in the matrix, **s**. The system function, H(s) can be evaluated along the imaginary axis using the *freqs* function. For example,

```
H = freqs(b,a,w)
```

evaluates the system function defined by the vectors, **b** and **a**, at the imaginary values in the vector, **w**. By convention, the values in **w** are real; however, the function *freqs* evaluates the system function at the values j**w**. Typically the system function, $H(s)$, is complex valued. The *abs* and *angle* functions can be used to generate the magnitude and phase of $H(s)$, respectively.

A system function that can be written in the form of (1) can also be written in the factored or zero pole gain form

$$H(s) = k\,\frac{[s - z(1)][s - z(2)]\cdots[s - z(M)]}{[s - p(1)][s - p(2)]\cdots[s - p(N)]}$$

where k is the system's gain, $z(1), z(2), \ldots, z(M)$ are the system's zeros and $p(1), p(2), \ldots, p(N)$ are the system's poles.

By convention, MATLAB stores polynomial coefficients in row vectors and polynomial roots in column vectors. Therefore, functions, such as *zplane*, generate different results depending on whether the function's input is a row or column vector.

For example,

```
b = [1 3 2];
a = [1 7 12];
zplane(b,a)
```

generates a pole-zero plot of the system function,

$$H(s) = \frac{s^2 + 3s + 2}{s^2 + 7s + 12}$$

generates an identical plot.

MATLAB's *poly* and *roots* functions can be used to convert between polynomial and root representations. For example,

```
roots([1 3 2])
```

generates the column vector $[-2\ -1]^T$, and,

```
poly([-2;-1])
```

generates the row vector, $[-2\ -1]$, that represents the polynomial, $s^2 + 3s + 2$.

A system function that can be written in the form of (1) also has a corresponding partial fraction expansion or residue representation of the form

$$H(s) = \frac{r(1)}{s - p(1)} + \cdots + r(N) + k(1) + k(2)s + \cdots + k(M - N + 1)s^{M-N+1}$$

if multiple roots do not exist. MATLAB's *residue* function can be used to determine a system function's residue representation. From this representation, the system's impulse response can be calculated. For example,

```
b = [1 3];
a = [1 7 12];
[R,P,K] = residue(b,a)
t = 0:0.025:5;
h = R.' * exp(kron(P,t));
```

generates the impulse response of the system function,

$$H(s) = \frac{s + 3}{s^2 + 7s + 12}$$

for $0 \le t \le 5$.

For linear time invariant systems,

$$Y(s) = H(s)X(s)$$

where $Y(s)$ is the Laplace transform of the system's output, $H(s)$ is the system's transfer function and $X(s)$ is the Laplace transform of system's input. To generate $Y(s)$, the numerator polynomials of $H(s)$ and $X(s)$ must be multiplied together and the denominator polynomials of $H(s)$ and $X(s)$ must be multiplied together. In Matlab, the *conv* function can be used to multiply polynomials.

Exercises

For Exercises 1 - 8, use the analog system described by the differential equation,

$$\frac{d^2y(t)}{dt} + 2a\frac{dy(t)}{dt} + (a^2 + \omega_0^2)y(t) = \frac{dx(t)}{dt} + ax(t)$$

where $x(t)$ is the system's input, $y(t)$ is the system's output and

$$a = 1.5 \text{ and } \omega_0 = 7 \text{ rad/sec.}$$

1. Via hand calculations, determine the system's transfer function, $H(s)$. Show all work for credit

2. Recall that $s = Re\{s\} + jIm\{s\}$. Using MATLAB, calculate $H(s)$ for $-10 \le Re\{s\} \le 10$ and $-10 \le Im\{s\} \le 10$. Plot $|H(s)|$ in decibels (dB): Power = $20 * \log_{10}|H(s)|$. Use the functions *mesch*, *title*, *xlabel*, *ylabel*, *zlabel* and ***log10***.

3. Using the *freqs* function, calculate the frequency response, $H(j\omega)$, for $0 \le \omega \le 100$. Plot $|H(j\omega)|$ in dB and the phase of $H(j\omega)$ in degrees using the *semilogx*, *title*, *xlabel*, *ylabel*, ***log10*** and *subplot* functions. (You should generate 2 plots on 1 page.) Note that this plot should be a slice of example 1, where we analyze when $s = Re\{s\} + jIm\{s\}, Re\{s\} = 0$.

4. Using the *roots* function, determine the system function's poles and zeros. Using these poles and zeros, use the *poly* function to generate the system function. Look at the plot for Exercise 1. Do your zeros and poles from Exercise 4 agree with Exercise 1? (you may want to play around with your figure).

5. Using the *tf2zp* function, generate the factored form of the system function. Using the *zp2tf* function, convert the factored form of the system function back into its original form.

6. Using the *zplane* function, generate a pole zero plot of $H(s)$.

7. Using the *residue* function, generate the partial fraction expansion representation of $H(s)$. Using this representation, generate the system's impulse response, $h(t)$, for $0 \le t \le 5$. Plot $h(t)$ for $0 \le t \le 5$ (using 0.01 precision) using the *plot*, *title*, *xlabel*, and *ylabel* functions.

8. Using the *conv* function, generate the transfer function, $Y(s)$, when $x(t) = u(t)$. Using the *residue* function, generate the partial fraction expansion representation of $Y(s)$. Using this representation, generate the system's step response, $y(t)$, for $0 \le t \le 5$. Plot $y(t)$ for $0 \le t \le 5$ using the *plot*, *title*, *xlabel*, and *ylabel* functions.

Controls Toolbox

Although MATLAB's controls toolbox allows systems to be modeled as transfer functions in rational and factored representations, the state space representation is the principal system representation in MATLAB's controls toolbox. For example, many of Matlab's functions which accept rational, factored and partial fraction expansion system representations as inputs use a state space representation of the system internally. However, the Control System Toolbox has a comprehensive library of state space tools. Regardless of the system representation, MATLAB's controls toolbox requires that the model be defined. For example, rational and factored systems can be defined as

```
sys_tf = tf(num,den)

sys_zpk = zpk(z,p,k)
```

respectively. A state space model of the form,

$$\mathbf{q}(n+1) = \mathbf{A}\mathbf{q}(n) + \mathbf{B}\mathbf{x}(n)$$
$$\mathbf{y}(n) = \mathbf{C}\mathbf{q}(n) + \mathbf{D}\mathbf{x}(n)$$

can be defined as

```
sys = ss(A,B,C,D)
```

After the system has been defined, the *freqresp*, *pole*, *zero*, *dcgain*, *impulse*, and *step* functions can be used to determine the system's frequency response, poles, zeros, gain, impulse response and step response, respectively. Systems can also be combined using addition and multiplication. Addition performs a parallel interconnection, and multiplication performs a series interconnection. For example,

```
sys1  =  tf([1],[1   2]);
sys2  =  tf([1],[1   3]);
sys3 = sys1*sys2
```

generates the system function,

$$H_3(s) = H_1(s)H_2(s) = \frac{1}{s+2} \cdot \frac{1}{s+3} = \frac{1}{s^2+5s+6}$$

Exercises

For Exercises 9 - 16, again use the analog system described by the differential equation,

$$\frac{d^2y(t)}{dt} + 2a\frac{dy(t)}{dt} + (a^2 + \omega_0^2)y(t) = \frac{dx(t)}{dt} + ax(t)$$

where $x(t)$ is the system's input, $y(t)$ is the system's output and

a=1.5 and ω_0= 7 rad/sec.

9. Draw (and label) a Direct Form II block diagram of the system, and using this diagram, write out the state space equations of the system **in matrix form** (Hint, find $\frac{dq_1(t)}{dt} = \dot{q}_1(t)$, $\frac{dq_2(t)}{dt} = \dot{q}_2(t)$, etc..). Note you should use integrators for this problem.
10. Using the *tf2ss* function, generate a state space model from the transfer function model that you developed in Exercise 1. Compare the result to the hand calculated state space equations that you found in Exercise 9. What assumptions does MATLAB make? (Note, the results should be the same. If not, check your work in Exercise 9.)
11. Using the *ss, freqresp*, and *squeeze* functions, calculate the frequency response, $H(j\omega)$, for $0 \le \omega \le 100$.

    Plot $|H(j\omega)|$ in dB and the phase of $H(j\omega)$ in degrees using the *semilogx, title, xlabel, ylabel, **log10*** and *subplot* functions. (You should generate 2 plots on 1 page.). Compare this figure with Exercise 3. Note: Where is the resonating peak at? (hint, ω_0)
12. Using the *zero, pole* and *dcgain* functions, determine the system's poles, zeros and gain. Take a look at the gain factor. What value appears?

    Explain why this gain factor appears via hand calculations. <u>Show all work for credit</u>. (Hint: Analyze the transfer function at DC. What assumption should be made to make our system practical?)
13. Using the *pzmap* function, generate a pole zero plot of $H(s)$.
14. Using the *impulse* function, generate the system's impulse response, $h(t)$ for $0 \le t \le 5$. Plot $h(t)$ for $0 \le t \le 5$ using the *plot, title, xlabel,* and *ylabel* functions. Compare this to your result in Exercise 7. Are they the same? If not, check your work. Hint: The Impulse Response should naturally decay to 0.
15. Using the *step* function, generate the system's step response, $s(t)$ for $0 \le t \le 5$. Plot $s(t)$ for $0 \le t \le 5$ using the *plot, title, xlabel,* and *ylabel* functions. Compare this to your result in Exercise 8. Are they the same? What value does this plot settle to with a input signal of DC? Does this gain match what you have found in Exercise 12?
16. Generate a system model (transfer function using *tf*, state space syetem model using *ss*) for the step function and multiply it with your system model from Exercise 11. Using the resulting model and the *impulse* function, generate the system's step response, $s(t)$ for $0 \le t \le 5$. Plot $s(t)$ for $0 \le t \le 5$ using the *plot, title, xlabel,* and *ylabel* functions. Compare this to Exercise 15.