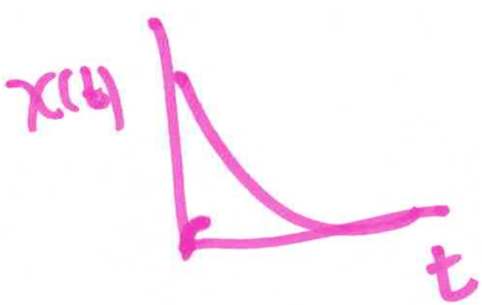
EE 360:

The Laplace Transform

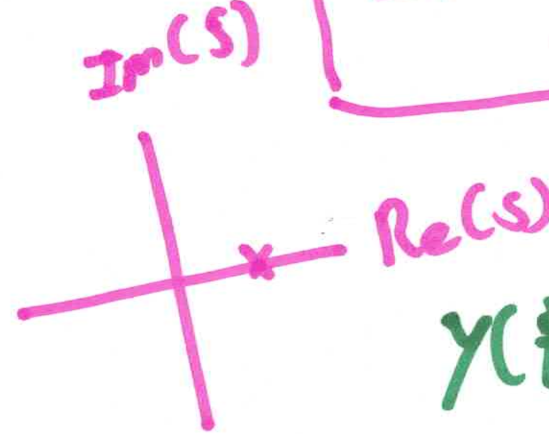
Nov. 1st, 2020

$$\boxed{\mathcal{L}(x(t)) = \int_{t=-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

$$x(t) = e^{at}$$



$\rightarrow$



$$y(t) = x(t) * h(t)$$

$$\Downarrow$$

$$Y(s) = X(s)\, H(s)$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)}}$$

$$\frac{d^2y(t)}{dt^2} + 7\frac{dy(t)}{dt} + 12\,y(t) = \frac{d^2x}{dt^2} + 3\frac{dx}{dt} + 2\,x(t)$$

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$t = 0$

$\downarrow$

$H(s)$

$$\frac{d^2y(t)}{dt^2} \Rightarrow Y(s)\left[s^2 - \cancel{s\,y(0) - \dot{y}(0)}\right]$$

$y(t=0)$ $\quad \frac{dy(t)}{dt}$

I.C. $= 0$

For Unilateral LT

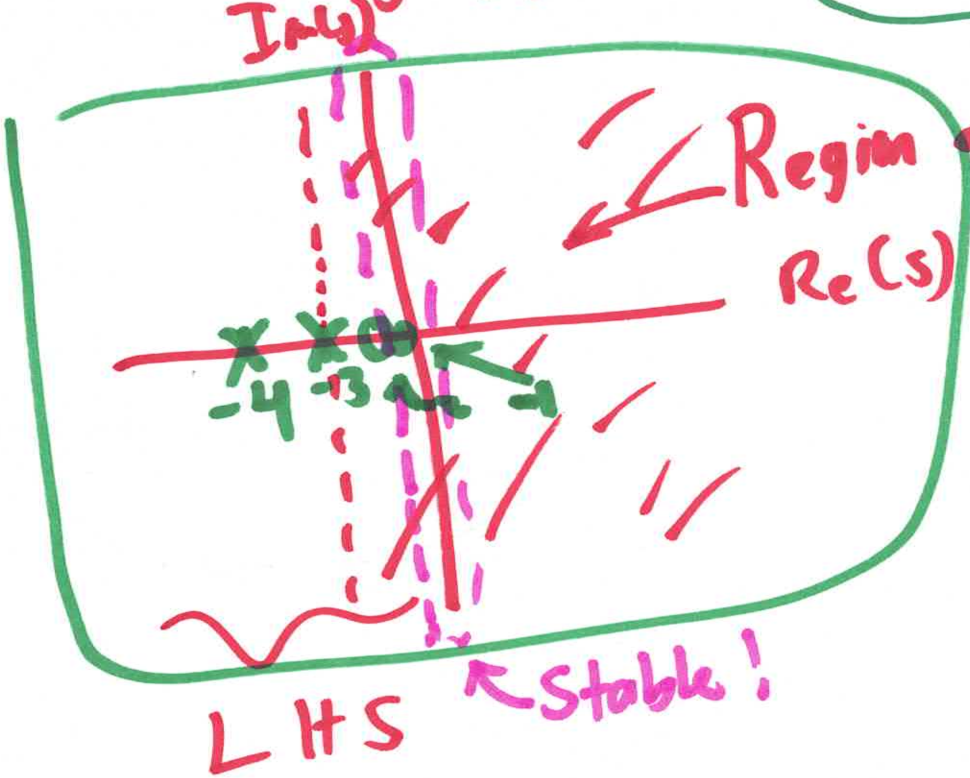
$$s^2Y(s) - 7sY(s) - 12Y(s) = s^2X(s) + 3sX(s) + 2X(s)$$

$$Y(s)[s^2 + 7s + 12] = X(s)[s^2 + 3s + 2]$$

$$\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{[s^2 + 3s + 2]}{[s^2 + 7s + 12]}}$$

$$H(s) = \frac{s^2+3s+2}{s^2+7s+12} = \frac{(s+2)(s+1)}{(s+4)(s+3)}$$

$s = Re + j\,Im$



$$y(t) = H(s=s_0)\,e^{-st}$$

Real s: $-10 \to 10$

$s = -10, -9 \ldots 10$

Im s: $-j10 \to +j10$

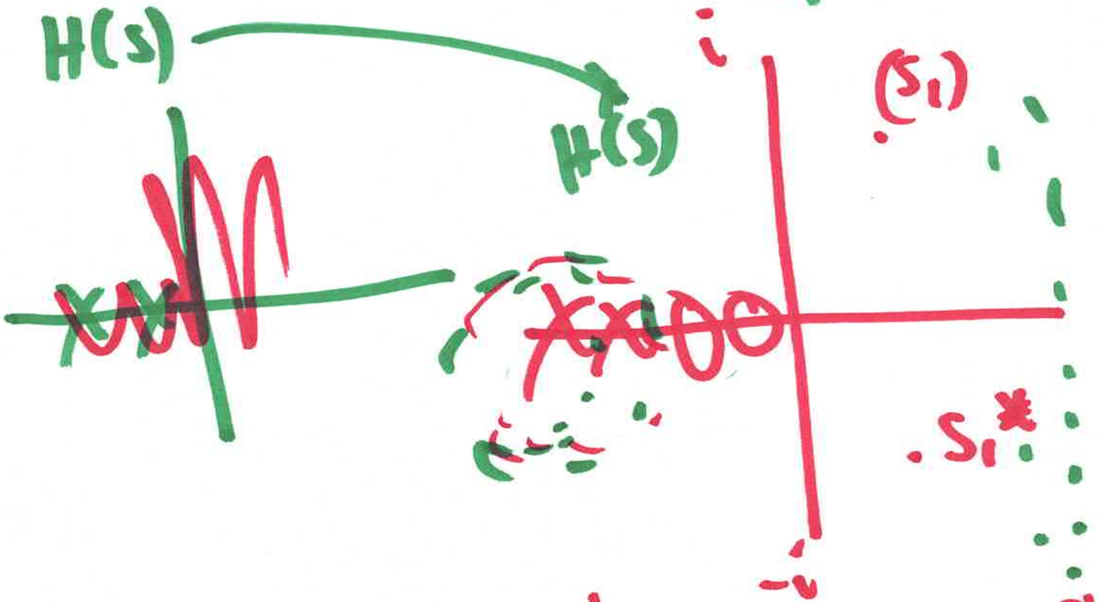
$s = -j10, -j9 \ldots \underline{+j10}$

$s = -10 - j10$

$$\overbrace{s^3 + s^2 + s^1 + 2s^0}^{H_N} \Big/ \overbrace{s^4 + s^2 + 3}^{H_N}$$

$$[z, p, k] = tf2zp(H_N, H_D)$$

$H_N \rightarrow [1 \ 1 \ 1 \ 2]$ $\quad H_D \rightarrow [1 \ 0 \ 1 \ 0 \ 3]$



$$y(t) = H(s_1)e^{+s_1 t} + H(s_1^*)e^{+s_1^* t}$$

$$y(t) = \frac{1}{2\pi i}\int_{s=a-\infty}^{a+\infty} H(s)e^{st}\,ds$$

Cauchy Integral theorem $\rightarrow$ $\oint_C H(s) = 2\pi i\,[\text{Residue}]$

$$H(s) = \frac{-6}{s+4} + \frac{2}{s+3} + 1$$

$$H(t) = -6e^{-4t} + 2e^{-2t}$$

$$x(t) = u(t)$$

$$X(s) = \frac{1}{s}$$

conv(Hd, [1, 0])

$$= \cancel{[1 \; 7 \; 12 \; 1 \; 0]}$$

$$[1 \; 7 \; 12 \; 0]$$

$$Y(s) = H(s)X(s)$$

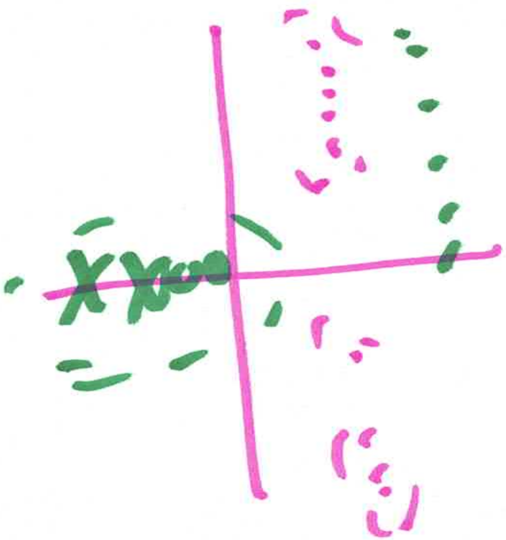
$$= \left[\frac{s^2+3s+2}{s^2+7s+12}\right]\frac{1}{s}$$

$$= \frac{s^2+3s+2}{s^3+7s^2+12s}$$

~~Y(t) = 1.5~~

$$\boxed{Y(s) = \frac{1.5}{s+4} + \frac{-.666}{s+3} + \frac{0.1666}{s}}$$

$$y(t) = 1.5e^{-4t} + (-.666)e^{-3t} + 0.166e$$



$$\frac{1}{2\pi i}\oint_C H(s)\,e^{st}\,ds$$

$$\int \left[\frac{\partial^2 y}{\partial t^2} + \frac{\partial y}{\partial t} + y = \frac{\partial x}{\partial t} + x \right]$$

$$\int \left[\frac{\partial y}{\partial t} + y(t) + \int y = x + \int x \right]$$

$$y(t) + \underbrace{\int y}_{\text{Integrator}} + \int\int y = \int x + \int\int x$$

$$\boxed{y(t) = \int x + \int\int x - \int y - \int\int y}$$

The Integral form!