EE360 - Signals & Systems

Oct 21st, 2020

Impulse Response

Discrete $\delta(n)$

$x(n) = \delta(n)$

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n)$$

or

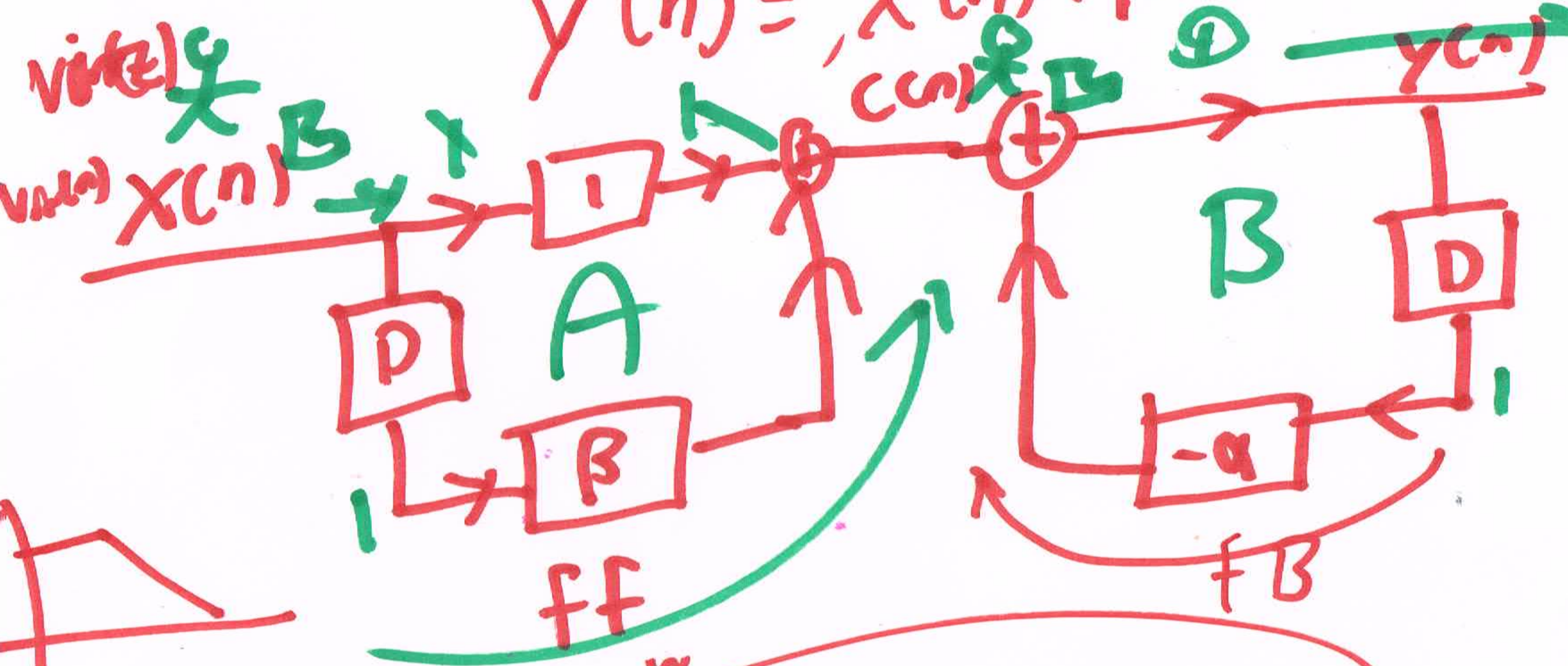
$$y(n) = x(n) * h(n)$$

$$\delta(n) * h(n)$$

$$\boxed{y(n) = h(n)}$$

LTI $y(n) + \alpha\, y(n-1) = x(n) + \beta\, x(n-1)$

$$y(n) = x(n) + \beta\, x(n-1) - \alpha\, y(n-1)$$



ff

$$\frac{A}{B_{fB}} = Q$$

f, w

$$H(z) = \frac{V_{out}(z)}{V_{in}(z)} = \boxed{\frac{Y(z)}{X(z)} = \frac{h_{zero}}{h_{poles}}}$$

$$Y(z)\, h_{poles}(z) = X(z)\, h_{zero}(z)$$

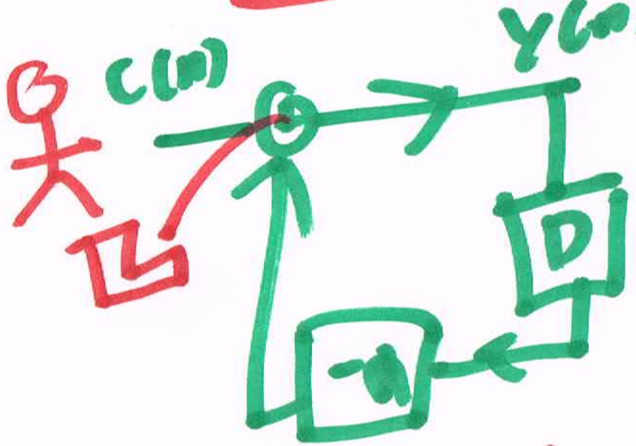
$$y(n)\left[H_p\right] = x_n\left[H_z\right]$$

(poles) $\rightarrow$ $\underbrace{y(n) + \alpha\, y(n-1)}_{c(n)} = \underbrace{x(n) + \beta\, x(n-1)}_{h_{zeros}(n)}$

$$h_{zero}(n) = c(n)$$

$$x(n) = \delta(n)$$

$$c(n) = \boxed{h_{zero}(n) = \delta(n) + \beta\,\delta(n-1)}$$



$$(1 + \alpha\lambda^{-1} = 0)\,\lambda$$

$$\lambda + \alpha = 0 \quad \text{aux eqn}$$

$$\boxed{\lambda = -\alpha}$$

$$h_{pole}(n) = y(n) = C_1 \lambda^n u(n) = C_1(-\alpha)^n u(n)$$

$$h(0) = \boxed{1 = C_1}(1)$$

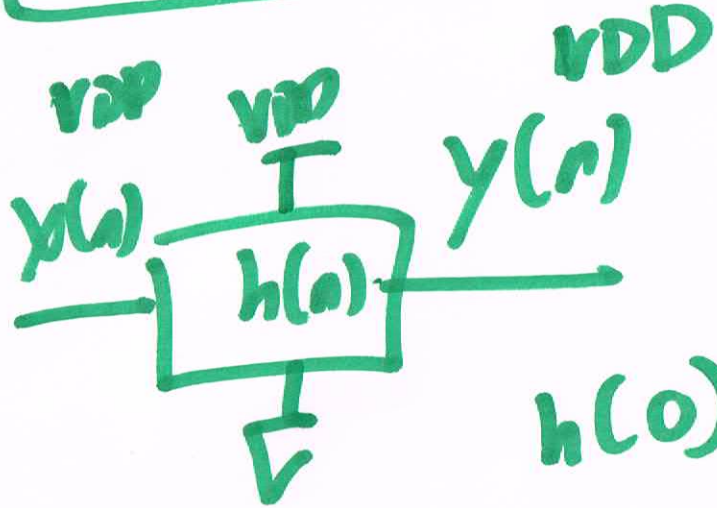
$$\boxed{h_{pole}(n) = (-\alpha)^n u(n)}$$

$$h(n) = h_{zeros}(n) * h_{poles}(n) \qquad h_{zeros}(n) = \delta(n) + \beta\delta(n-1)$$

$$= [\delta(n) + \beta\delta(n-1)] * [(-\alpha)^n u(n)]$$

$$\boxed{h(n) = (-\alpha)^n u(n) + \beta(-\alpha)^{n-1} u(n-1)}$$

$$\frac{\beta}{-\alpha}(-\alpha)^n u(n-1)$$



$h(0)$

$$\underline{h(n)} = \underline{\delta(n)} + \beta\,\underline{\delta(n-1)} - \alpha\,\underline{h(n-1)}$$

$$\boxed{h(0) = \Delta\,\delta(0)} + \beta\,\delta(\cancel{-1}) - \alpha\,h(\cancel{-1})$$

$$\Delta = 1.01$$

$$\boxed{\Delta = 1}$$

$$h(1) = \delta(\cancel{1}) + \beta\,\delta(0) - \alpha\,h(0)$$

$$h(1) = \beta + (-\alpha)[\Delta]$$

$$y(n) = \left|\sum_{-\infty}^{\infty} h(n)\,\cancel{x}(k-n)\right|$$

$$= \left|\sum_{-\infty}^{\infty} h(n)\right|\,|\cancel{x}|$$