The Z-Transform

Nov. 21st, 2020

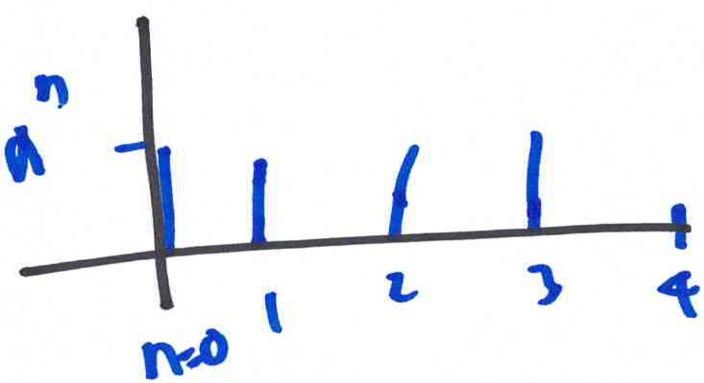
$$Z(x(n)) = \sum_{n=-\infty}^{n=\infty} x(n) z^{-n}$$

Elemts

$N = 4$ elemts

$$x(n) = a^n\left[u(n) - u(n-4)\right]$$

$$Z(x(n)) = \sum_{n=-\infty}^{n=+\infty} x(n) z^{-n}$$



$$\sum_{n=-\infty}^{+\infty} \alpha^n [u(n) - u(n-4)] z^{-n}$$

$N \to 100$!!

$n=-1$ $u(n)=0$ $u(n-4)=0$

$n=4$ $u(n)=1$ $u(n-4)=1$

$$\sum_{n=0}^{3 \leftarrow N-1} \alpha^n z^{-n} = \alpha^0 z^{-0} + \alpha z^{-1} + \cdots \alpha^{N-1} z^{-(N-1)}$$

$n=0$, $n=1$, $n=N-1$

$$\alpha z^{-1} \sum_{n=0}^{N-1} \alpha^n z^{-n} = \alpha z^{-1} + \alpha^2 z^{-2} + \cdots + \alpha^N z^{-N} + \alpha z^{-1}$$

$$\sum_{n=0}^{N-1} \alpha^n z^{-n} (1 - \alpha z^{-1}) = 1 + \cancel{\alpha z^{-1}} + \cancel{\alpha^2 z^{-2}} + \cdots + \cancel{\alpha^{N-1} z^{-(N-1)}} + \cancel{-\alpha z^{-1}} - \cancel{\alpha^2 z^{-2}} + \cdots - \cancel{\alpha^{N-1} z^{-(N-1)}} - \alpha^N z^{-N}$$

$$(1-\alpha z^{-1})\sum_{n=0}^{N-1}\alpha^n z^{-n} = 1-\alpha^N z^{-N}$$

For finite symbols

$$\sum_{n=0}^{N-1}\alpha^n z^{-n} = \frac{1-\alpha^N z^{-N}}{1-\alpha z^{-1}}$$

$N\to\infty$ Assumption: $|\alpha| \leq 1$

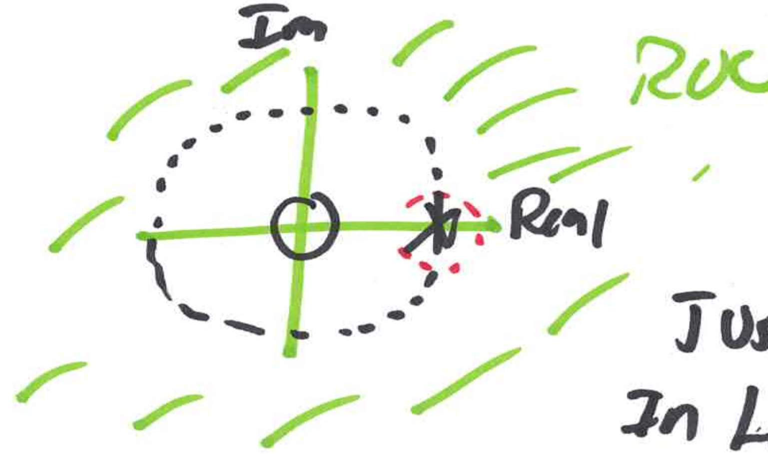
Z-Transform

$$\sum_{n=0}^{n=\infty}\alpha^n z^{-n} = \frac{1}{1-\alpha z^{-1}} = Z(x(n))$$

$\alpha = 1$

$$Z(u(n)) = \frac{1}{1-z^{-1}}$$

$$\frac{1}{1-z^{-1}}\,\frac{(z)}{(z)} = \frac{z}{z-1}$$



Just like In Laplace S

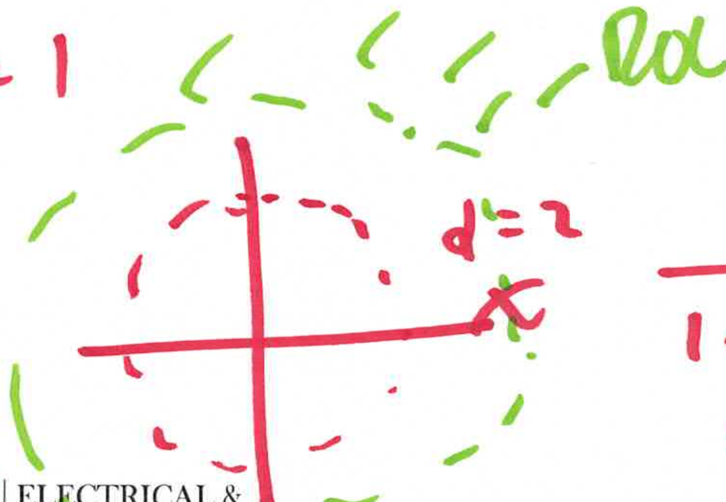
$$Z(x(n)) \rightarrow X(z) =$$

$$\frac{C}{1-\alpha z^{-1}} \xrightarrow[\text{Z-T}]{\text{inv.}} x(n) = \overset{\text{Res}}{\downarrow}{C}\,(\text{pole})^n \; u(n)$$

$$+ \dots c_2(\text{pole}_2)^n \dots$$

$z = \mathrm{Re} + j\,\mathrm{Im}$

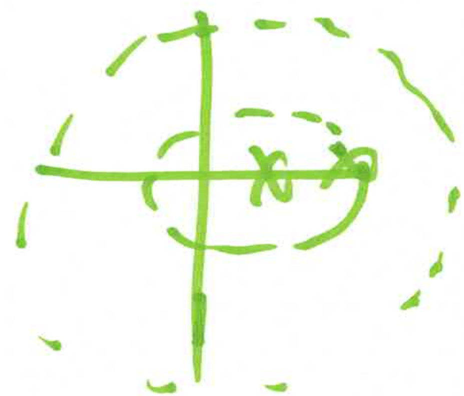
$\alpha = 1$

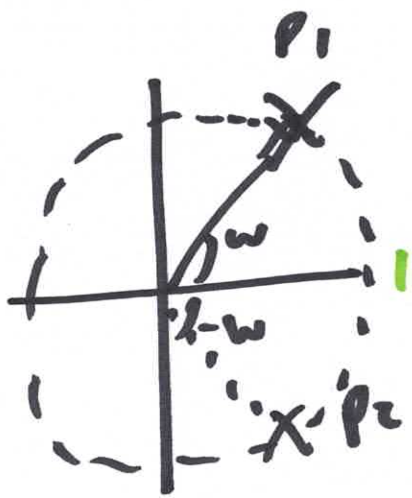


$$\frac{1}{1-2z^{-1}}$$

$$\rightarrow h(n) = 2^n \, u(n)$$

$n \rightarrow \infty \quad h(n) \rightarrow \infty$





$$\frac{R_{e1}}{1-p_1 z^{-1}} + \frac{R_{e2}}{1-p_2 z^{-2}} = H(z)$$

$\to h(n)$

$$= \left(R_{e1}(p_1)^n + R_{e2}(p_2)^n\right) u(n)$$

$$H(z) = \frac{1}{(1-p_1 z^{-1})(1-p_2 z^{-1})}$$

$p_1 = r e^{jw}$

$p_2 = r e^{-jw}$

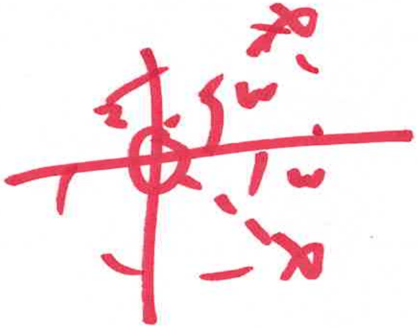
$$\underline{1 - r e^{-jw} z^{-1} - r e^{+jw} z^{-1}} + r^2 z^{-2}$$

$$1 - 2r\cos w \cdot z^{-1} + r^2 z^{-2}$$

$$\cos w = \frac{e^{jw} + e^{-jw}}{2}$$

$$-\frac{(e^{jw} + e^{-jw}) \cdot 2}{2} \to -2\cos w\, z^{-1}$$

$$\frac{1}{1-2r\cos z^{-1}+r^2z^{-2}}\frac{(z^2)}{(z^2)} = \frac{z^2}{z^2-2r\cos z+r^2}$$

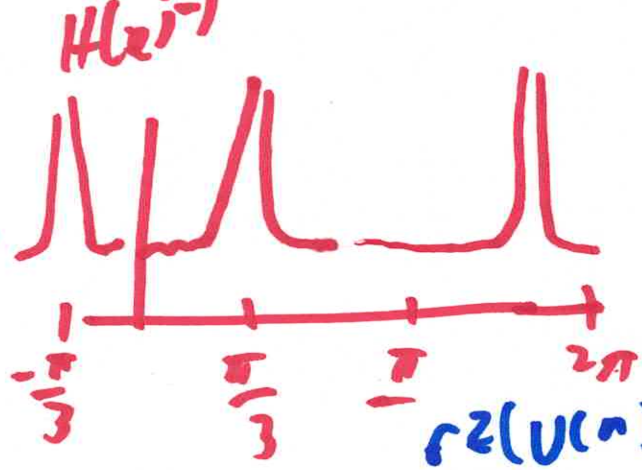


$H(j\omega)$

$j\omega$

$z = re$

$r=1$



$r\,Z(u(n))$

$$\frac{1}{1-2r\cos z^{-1}+r^2z^{-2}} \quad \frac{1}{(1-z^{-1})}$$

$$\frac{r}{1-e^{j\omega}z^{-1}} \quad j\omega n$$

$\to r e^{j\omega n} = h(n)$

$$y(n) - \alpha y(n-1) = x(n) - \beta x(n-1)$$

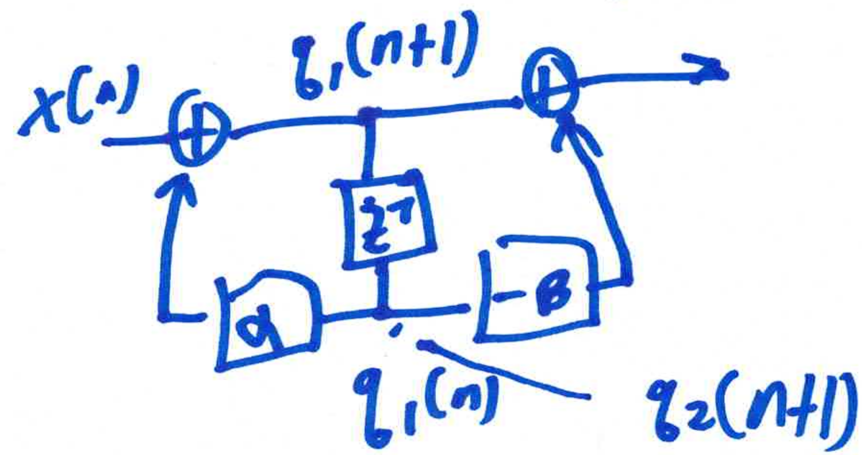
$$Y(z) - \alpha\left[z^{-1}Y(z) - y(n=-1)\right] = X(z) - \beta\left(z^{-1}X(z) - x(-1)\right)$$

$$y(n=-1) = 0 = x(-1)$$

$$\frac{Y(z)}{X(z)}\,\frac{\left[1-\alpha z^{-1}\right]}{\left(1-\alpha z^{-1}\right)} = \frac{X(z)}{X(z)}\left[1-\beta z^{-1}\right]$$

$$y(n) = x(n) - \beta x(n-1) + \alpha y(n-1)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1-\beta z^{-1}}{1-\alpha z^{-1}}$$



SS:

$$q_1(n+1) = x(n) + \alpha q_1(n)$$

$$y(n) = (-\beta+\alpha)q_1 + x(n)$$

$$\overbrace{\begin{bmatrix} q_1(n+1) \\ q_2(n+1) \end{bmatrix}}^{q(n+1)} = A \overbrace{\begin{bmatrix} q_1(n) \\ q_2(n) \end{bmatrix}}^{q} + B \overbrace{x(n)}^{x}$$

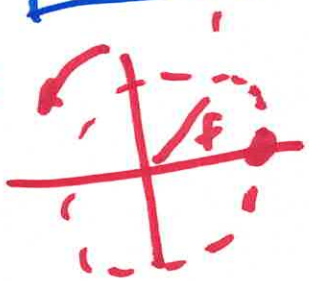
$$y(n) = C \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + D\,x(n)$$

$$q_k(n+1) \rightarrow z^{-1} Q_k$$

$$\boxed{Y(z) = \left[C(zI-A)^{-1}B + D\right]X(z)}$$

$z = e^{j\theta}$

$z = 1$ F

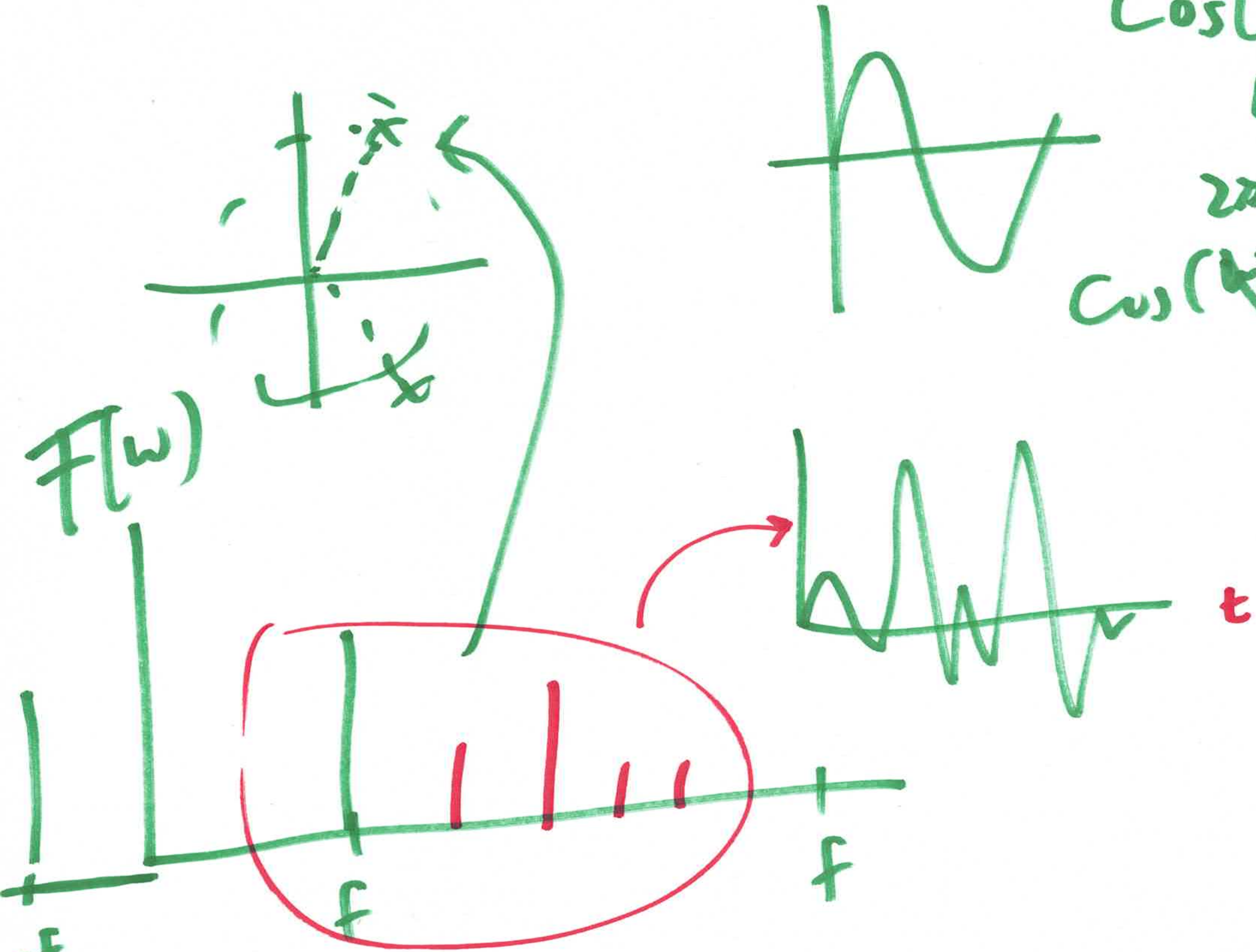


DC

$B = .2$

$$H(z) = \frac{1-Bz^{-1}}{1-2z^{-1}-z^{-2}} \overset{z=1}{=} \frac{1-B}{1-2-1} = \underline{DC}$$

$$U(n) = \frac{1}{1-z^{-1}} = \frac{z-0}{z-1} \quad \begin{matrix} [1 \;\; 0] \\ [1 \;\; -1] \end{matrix}$$



$\cos(\cancel{\omega_0 t}\ \omega_0 T_s)$

$2\pi f$

$\cos(\omega_0 t)$