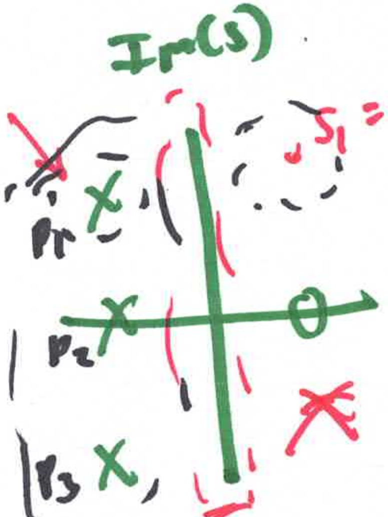
The Laplace Transform (Pt. 2)

$P_1 = -Re + jIm$

Nov 7^{th}, 2020



$$y(t) = H(s)\, e^{st}$$

$$y_1(t) = H(s_1)\, e^{s_1 t}$$

$$y_{p_1}(t) = H(s_{p_1})\, e^{(-Re + jIm)t}$$

$$\oint_S H(s)\, ds = \underline{2\pi i}\left(\text{Residue}_1 + R_2 + R_3\right)$$

$$\int\left[\frac{d^2}{dt^2}y(t) + 7\frac{dy(t)}{dt} + 12y(t) = \frac{d^2x(t)}{dt^2} + \frac{3\,dx(t)}{dt} + 2x(t)\right]$$

LTI diff eqns

$$q_1(n) \longrightarrow \boxed{D} \longrightarrow q_1(n-1)$$

$q_1(n+1) =$ current inpts $+$ current states

$y(n) =$ " $+$ "

LTI ss eqns

$$q_1(n+1) \longleftarrow \boxed{D} \longleftarrow q_1(n)$$

differential eqns

$$x(t) \longrightarrow \boxed{d/dt} \longrightarrow \frac{dx(t)}{dt}$$

$$\int\left[\frac{dy(t)}{dt} + 7y(t) + 12\int y(t) = \frac{dx(t)}{dt} + 3x(t) + 2\int x(t)\right]$$

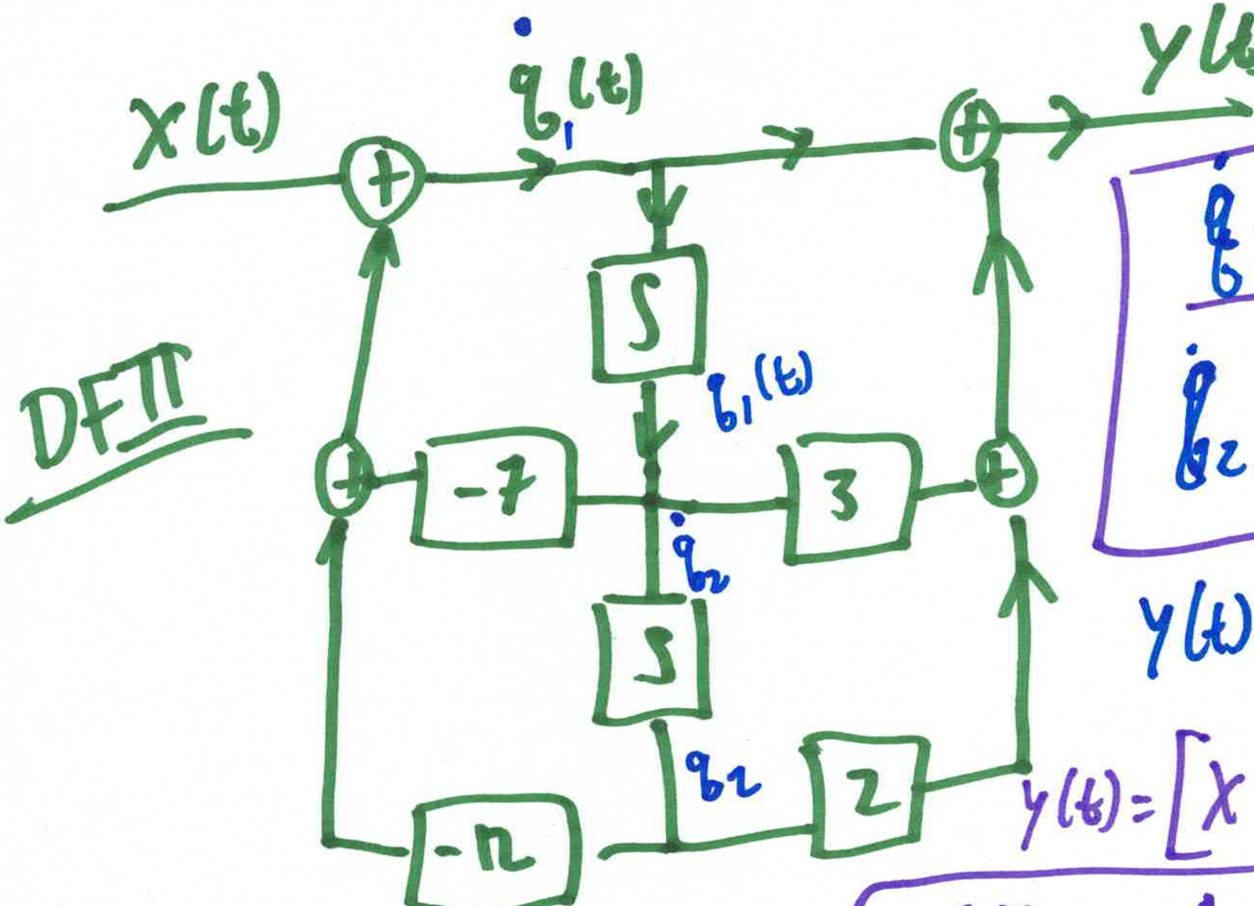
ss

$$\frac{dx(t)}{dt} = \dot{x} \longrightarrow \boxed{\int} \longrightarrow x(t)$$

$$y(t) + 7\int y(t) + 12\iint y(t) = x(t) + 3\int x(t) + 2\iint x(t)$$

$$y(t) + 7\int y + 12\iint y = x(t) + 3\int x + 2\iint x$$

$$y(t) = x(t) + 3\int x + 2\iint x - 7\int y - 12\iint y$$



$$\boxed{\begin{aligned} \dot{q}_1 &= x(t) - 7q_1(t) - 12q_2(t) \\ \dot{q}_2 &= q_1(t) \end{aligned}}$$

$$y(t) = \dot{q}_1 + 3q_1(t) + 2q_2(t)$$

$$y(t) = [x - 7q_1 - 12q_2] + 3q_1 + 2q_2$$

$$\boxed{y(t) = -4q_1 - 10q_2 + x}$$

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$$\dot{q}_1 = x(t) - 7q_1 - 12q_2$$

$$\dot{q}_2 = q_1$$

$$y = -4q_1 - 10q_2 + x$$

$$\dot{q}(t) \rightarrow sQ(s)$$

$$s\dot{I}$$

$$sQ = AQ + Bx$$

$$Q[sI - A] = Bx$$

$$\boxed{Q(s) = [sI - A]^{-1} Bx}$$

$$\underbrace{\begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{bmatrix}}_{\dot{q}} = \underbrace{\begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} q_1(t) \\ q_2(t) \end{bmatrix}}_{q} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} \underbrace{x(t)}_{x}$$

$$\underbrace{y(t)}_{y} = \underbrace{\begin{bmatrix} -4 & -10 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} q_1 \\ q_2 \end{bmatrix}}_{q} + \underbrace{\begin{bmatrix} 1 \end{bmatrix}}_{D} \underbrace{x(t)}_{x}$$

SS eqns / model

$$Y(s) = C\,Q(s) + D\,X(s)$$

$$Y(s) = C\left[[sI-A]^{-1} Bx(s)\right] + D\,x(s)$$

$$\frac{Y(s)}{X(s)} = H(s) = \boxed{\cancel{Y(s)} = \left\{ C[sI-A]^{-1}B + D \right\} \cancel{X(s)}}$$

$$A = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \cancel{y(s) =}$$

$$C = \begin{bmatrix} -4 & -10 \end{bmatrix} \quad D = 1$$

$$y(s) = \left\{ \begin{bmatrix} -4 & -10 \end{bmatrix} \left[s\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix} \right]^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + D\begin{bmatrix} 1 \end{bmatrix} \right\} x(s)$$

$$= \left\{ \begin{bmatrix} -4 & -10 \end{bmatrix} \begin{bmatrix} \overset{a}{s+7} & \overset{b}{12} \\ \overset{c}{-1} & \overset{d}{s} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 1 \right\} x$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\left[\frac{1}{(s+7)s - 12(-1)} \begin{bmatrix} s & -12 \\ +1 & s+7 \end{bmatrix} \right.$$

$$\underbrace{\left(\frac{1}{s^2 + 7s + 12} \right)}_{H_{poles}(s)}$$

$$y(s) = \Bigg\{ \overset{1\times 2}{[-4 \quad -10]} \; \underset{\text{constant}}{\underset{\&}{H_{pols}(s)}} \; \overset{2\times 2}{\begin{bmatrix} s & -12 \\ 1 & s+7 \end{bmatrix}} \overset{2\times 1}{\begin{bmatrix} 1 \\ 0 \end{bmatrix}} + 1 \Bigg\} x$$

$$= \Bigg\{ \cancel{[-4 \quad -10]} \; H_{poles}(s) \; \overset{1\times 2}{[-4 \quad -10]} \overset{2\times 1}{\begin{bmatrix} s \\ 1 \end{bmatrix}} + 1 \Bigg\} x$$

$$\frac{y(s)}{x(s)} = \Big\{ H_{poles}(s)\left(-4s - 10\right) + 1 \Big\} \frac{x(s)}{x(s)}$$

$$H(s) = H_{poles}(s)\left(-4s - 10\right) + 1$$

$$\boxed{H(s) = \frac{-4s - 10}{s^2 + 7s + 12} + 1}$$

$$\frac{-4s-10}{s^2+7s+12} = \frac{A}{(s+4)} + \frac{B}{(s+3)}$$

$(s+4)(s+3)$

$$-4s-10 = A(s+3) + B(s+4)$$

$s: \; -4 = A + B$

$A = -6$

$s = -3$

$+12 - 10 = 0 + B$

$$\boxed{B = 2}$$

$s = 0$ DC

$$\cancel{H(0) = \frac{-6}{4} + \frac{2}{3}} + 1$$

$$\cancel{= \frac{-18+8}{12}} + 1$$

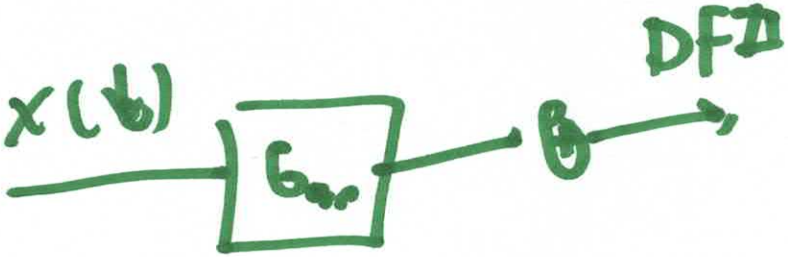
$$H(s) = \frac{-6}{s+4} + \frac{2}{s+3} + 1$$

$$\boxed{H(t) = \left[-6e^{-4t} + 2e^{-3t} + \delta(t)\right] u(t)}$$

$$H(s) = \frac{s^2 + 3s + 2}{s^2 + 7s + 12} \qquad s \to 0$$

Gain

$$= \frac{2}{12} = \frac{1}{6} = \boxed{0.1666}$$



$$h(t=0) = -6 + 2 + \infty$$
$$= -4$$

$$= \int x + \iint x$$