temperature in the home is moving around 2C centered around 22C but, from the occupants point of view, the temperature is relatively constant. In this feedback system the input is the temperature the occupant sets the thermostat to. The feedback is the thermostat sensing the temperature of the home and determining when it's 1C above or below the set temperature of 22C (the hysteresis, see Ch. 18 Fig. 18.1, V_{SPL} = 21C and V_{SPH} = 23C) to turn the heater off or on respectively. The forward path of the feedback system is the heater driving the temperature of the house (the feedback system's output that is sensed by feedback to the thermostat). Note, again, that the output of the system is oscillating (the feedback system is unstable). In a HPS if we can minimize the amplitude of the oscillations we can make a very useful and simple power supply.

32.1 A Review of Power and Energy Basics

Instantaneous power, $p(t)$ with units of watts (joules/second), is a product of the voltage across a circuit (as shown in Fig. 32.1) and the current through a circuit or

$$p(t) = v(t) \cdot i(t) \tag{32.1}$$

where $i(t)$ flows from the + to – of how $v(t)$ is defined. The energy, E with units of joules, stored (positive) in or supplied (negative) by a circuit is calculated using

$$E = \int_{t_1}^{t_2} p(t) \cdot dt \tag{32.2}$$

The average power over a time interval T is calculated using

$$P = \frac{E}{T} = \frac{1}{T} \int_{t_0}^{t_0+T} p(t) \cdot dt = \frac{1}{T} \int_{t_0}^{t_0+T} v(t) \cdot i(t) \cdot dt \tag{32.3}$$

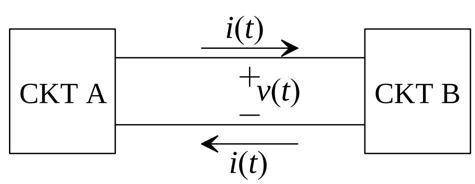

Figure 32.1 Illustrating supplying/receiving energy in an electronic system.

Example 32.1
Estimate the power dissipated by the battery and resistor seen in Fig. 32.2. How much energy does the battery supply to the resistor in 10 hours?

In this simple circuit $v(t)$ = 1.5V and $i(t)$ = 1.5 mA. The instantaneous power dissipated by the resistor is, Eq. (32.1), 2.25 mW and by the battery –2.25 mW,

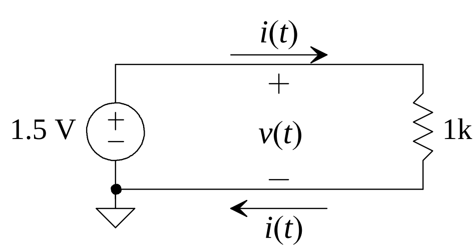

Figure 32.2 A simple circuit used in Ex. 32.1.

that is, $(-1.5mA)\cdot(1.5V)$. The minus sign indicates that the battery is supplying energy (a source of power) because $i(t)$ flows from the − terminal to + terminal. Of course the power supplied by the voltage source and dissipated by the resistor are equal and opposite.

The total amount of energy supplied to the resistor by the battery in 10 hours is (2.25mW· 3,600s/hr·10hr) or 81 joules. Note that we could've used Eq. (32.2) but since the voltage and current signals don't change with time the integration results in simple multiplication. The average power, looking at Eq. (32.3), dissipated by the resistor is equal to the instantaneous power of 2.25 mW.

Note that the resistor, since it's dissipating power, will heat up. Does the battery, since it's supplying energy, then correspondingly cool down? Answer: no. Storing and releasing energy doesn't result in using the stored energy (heat) and thus power dissipation (and no cooling, or removing heat from the system, since that would require energy!). This assumes that the internal resistance of the battery (that ultimately limits how much current the battery can supply) is small. This internal battery resistance will result in the battery heating-up when it's being charged or discharged. ■

We covered related material on power and energy back in the beginning of Ch. 8. Note, as discussed there, energy stored in a battery is often incompletely characterized using mA·hrs (milliamp·hours). To get the amount of energy stored in a (full-capacity or fully-charged) battery we also need to know the battery's voltage. A 1.5V battery characterized as having a capacity of 1000 mAh (milliamp·hour) would have an energy storage capacity of 1.5V·1,000mA·3600s. Since mA are coulombs/s and volts· coulombs are joules, the battery's capacity is 5,400 joules (so this battery could supply power to the resistor in Ex. 32.1 for 5,400/81 or 66.7-10 hour time periods [667 hours]). To further complicate matters, instead of using joules battery suppliers often use Wh (watt·hour). Using this notation the battery has a capacity of 1.5 Wh (= 5,400 joules) of energy.

An Analogy

Finally, to ensure that the concepts of energy and power are understood by the reader let's provide an analogy. Energy is like the gas in the gas tank of a car. Power is the amount of gas used over time. If a battery is dead it's like having no gas in the tank of a car. A car gas tank, or battery, can be fully filled or charged but until gas is used (energy is supplied by the battery) no work is done. The car doesn't start/move or no power is dissipated in a circuit. In our analogy we would characterize power as the amount of gas (= energy) used over time. If we accelerate quickly we use more gas (the instantaneous power is high) while if we take our foot off the gas pedal and coast the amount of gas used goes down (the instantaneous power is low). At this point the reader should have a clear understanding of why we talk about instantaneous power. The reader should also understand then why we talk about average power, the power used over some time (for example an hour) T in Eq. (32.3).

It's interesting to note that a PMIC (in a cell phone for example) will measure the energy capacity remaining in a battery and report it as "fuel level." The circuit used on the PMIC to perform this function is often called the "fuel gauge."

32.1.1 Energy Storage in Inductors and Capacitors

Ideally, inductors and capacitor don't dissipate power. Rather they store and release energy. Real inductors, especially large-valued ones using a lot of wire, can have a significant series resistance. Real capacitors, especially at higher frequencies, can also have a significant series resistance (called an effective series resistance or ESR). This resistance can result (in real circuits) in an inductor or capacitor dissipating power but, again, an ideal inductor or capacitor doesn't dissipate power. In the following discussions we'll assume that the inductor and capacitor are ideal (no series resistance). Later in the chapter we'll discuss this further.

Energy Stored in an Inductor

Examine the inductor seen in Fig. 32.3a. The voltage across the inductor, $v_L(t)$, is related to the current through the inductor, $i(t)$, using

$$v_L(t) = L \cdot \frac{di(t)}{dt} \tag{32.4}$$

Using Eqs. (32.1) and (32.2) the energy stored in an inductor can be calculated using

$$E = \int_{t_1}^{t_2} v_L(t) i(t) \cdot dt \tag{32.5}$$

Re-writing Eq. (32.4)

$$dt = L \cdot \frac{di(t)}{v_L(t)} \tag{32.6}$$

and substituting Eq. (32.6) in Eq. (32.5) and assuming the current in the inductor is initially 0 (at t_1) and when we are looking at the energy stored in the inductor, E, the current is I (at t_2)

$$E = L \cdot \int_0^I i(t) \cdot di(t) = \frac{1}{2} \cdot L \cdot I^2 \tag{32.7}$$

Note that if a sinusoidal current flows in the inductor that half of the time the inductor is storing energy while the other half of the time the inductor is supplying energy (so E is negative in this other half).

Energy Stored in a Capacitor

Examine the capacitor seen in Fig. 32.3b. The voltage across the capacitor, $v_C(t)$, is related to the charge on the capacitor, $q(t)$, using $v_C(t) = q(t)/C$. The energy stored in a capacitor is then (assuming the capacitor is initially discharged and then is charged to a voltage V when we are interested in know the energy stored in the capacitor)

$$E = \int_0^Q v_c(t) \cdot dq = \int_0^Q \frac{q(t)}{C} \cdot dq = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} C V^2 \tag{32.8}$$

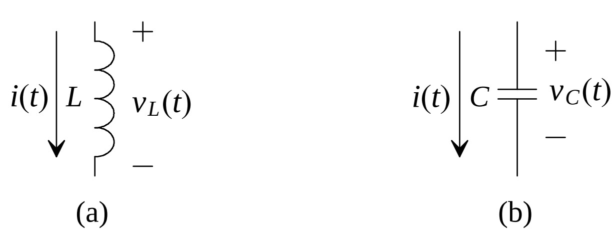

Figure 32.3 Calculating energy stored in an inductor and capacitor.

Another way to derive this equation is by writing the relationship between the voltage across the capacitor and the current through the capacitor

$$i(t) = C \cdot \frac{dv_C(t)}{dt} \tag{32.9}$$

and then writing

$$E = \int_{t_1}^{t_2} v_C(t) i(t) \cdot dt \text{ and } dt = C \cdot \frac{dv_C(t)}{i(t)} \tag{32.10}$$

Assuming the voltage on the capacitor is initially 0 (at t_1) the energy stored on the capacitor, E, charged to a voltage V (at t_2) is

$$E = C \int_0^V v_C(t) \cdot dv_C(t) = \frac{1}{2} C V^2 \tag{32.11}$$

Example 32.2

If C1 in Fig. 32.4 is charged to 1V and C2 is charged to 2V show that the energy stored on the capacitors before the switch closes does not equal to the energy stored on the capacitors after the switch closes. Knowing that a capacitor doesn't dissipate power (convert energy into heat) where does the energy go? Verify your answer with simulations. Note that these two capacitors are *charge sharing*.

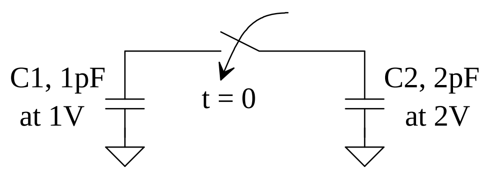

Figure 32.4 Where does the energy go when sharing charge on capacitors?

If we call the initial voltage on C1, V_1 (= 1V in this example) and the initial voltage on C2 V_2 (= 2V in this example) then the charged stored on both capacitors before the switch closes is

$$Q = C1 \cdot V_1 + C2 \cdot V_2 \tag{32.12}$$

After the switch closes the capacitors are in parallel so their values add. If we call the voltage across the capacitors V_f after the switch closes then the charge on the capacitors, because charge is conserved (no charge enters or exits the circuit) can be written as

$$Q = (C1 + C2) \cdot V_f \tag{32.13}$$

The final voltage across the capacitors after the switch closes is

$$V_f = \frac{C1 \cdot V_1 + C2 \cdot V_2}{C1 + C2} \tag{32.14}$$

or, using the numbers from Fig. 32.4, V_f = 1.667 V, see Fig. 32.5a. The energy stored in the capacitors before the switch closes is

$$E_{before} = \frac{1}{2} \cdot 1pF \cdot 1^2 + \frac{1}{2} \cdot 2pF \cdot 2^2 = 4.5\ pJ \tag{32.15}$$

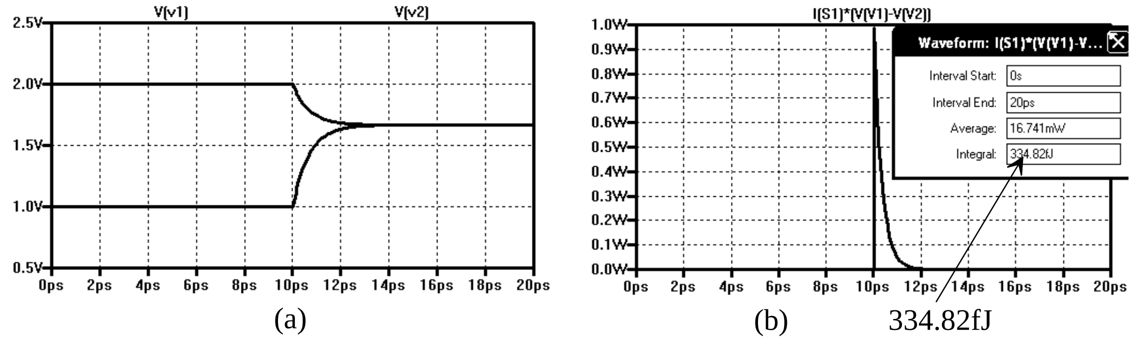

Figure 32.5 Simulating charge sharing between two capacitors.

After the switch closes the energy stored in the capacitors is

$$E_{after} = \frac{1}{2} \cdot 3pF \cdot (1.667V)^2 = 4.168\ pJ \tag{32.16}$$

The difference is 332 fJ. So, where does this energy go? Answer, the energy is converted to heat through the resistance of the interconnecting wires and switch resistance. In the simulation the wires don't have resistance but the switch has a finite resistance (in the simulation here it's 1 Ω). Figure 32.5b plots the power dissipated by the switch. Integrating this power, see Eq. (32.2), we get the energy converted to heat and thus lost in the circuit (to integrate in LTspice press CTRL and left click on the trace label at the top of the plot). Note that whenever charge is moved some work is done. Therefore, we shouldn't expect it to be free (the movement of charge takes energy unless it's through a superconductor, that is, a conductor having zero resistance). ■

32.1.2 Energy Use in Transmitting Data

The energy used when transmitting data is characterized by the power used divided by the data-rate or

$$\frac{\text{Power, watts}}{\text{Data rate, } bits/s} = \frac{joules}{bit} \tag{32.17}$$

Reviewing Fig. 11.13 back in Ch. 11, if the input to the inverter is data that is an alternating sequence of 1s and 0s the largest amount of energy is used. This is because data that is repeating, such as 1111 or 000, doesn't charge or discharge the load capacitance as often. For an alternating sequence of 1s and 0s the data rate is $2f_{clk}$. That is, two bits are transmitted every T clock period. Using Eq. (11.15) and (32.17) we can write

$$\frac{C_{tot} \cdot VDD^2 \cdot f_{clk}}{2f_{clk}} = \frac{C_{tot} \cdot VDD^2}{2},\ joules/bit \tag{32.18}$$

Typical numbers for transmitting signals off-chip range from 1 pJ/bit to greater than 50 pJ/bit (or considerably higher if transmission lines are used terminated with resistors). Low power transmission between, for example, two chips packaged in the same package, may be in the 10-100 fJ/bit range. Using this (J/bit) figure-of-merit is useful to compare the efficiency of different methods of transmitting information.

One *important take-away* from this discussion is that the data rate doesn't determine the energy used and thus the drain on the battery. For example, if it takes 1-pJ/bit of energy to read a bit from a communication channel then downloading a 1GB video at 10 Gbits/s results in the same drain on the battery (energy usage) as downloading the 1GB video at 100 Mbits/s. Therefore, operating faster doesn't reduce battery life. This may be confusing since the power used while downloading at 10 Gbits/s is 100 times higher than the power used when downloading at 100 Mbits/s. However, the video is downloaded at 1/100th the time when downloading at the faster rate. This same discussion can be applied to reading and writing to memory in a computing system. Given a fixed amount of information to be written, or read, operating the memory at faster rates doesn't change the power that's used (assuming the energy/bit doesn't change between the faster and slower read/write rates).

The observant reader may have wondered if the material in this section is contradictory to the comment at the beginning of the chapter that clocking at lower frequencies reduces power (it does reduce the instantaneous power but read on). Performing the same number of operations with the same power supply voltage, *VDD*, results in no overall power (energy) savings when running at slower clock frequencies. Why? While the instantaneous power consumed is lower the fact that it takes longer to perform the operations results in no change in the energy used. Rather, to reduce the energy supplied by the battery there would need to be an associated reduction in power supply voltage when reducing the operating frequency. Being able to dynamically adjust the output voltage of a power supply (powered by a battery for example), while maintaining high-efficiency (energy supplied to a load is nearly the same as the energy drained from a battery), is an important attribute of a switching power supply. Note that there are many situations where the number of operations is not fixed, such as waiting or checking for something to occur, and so lowering the clock frequency will reduce power as discussed back in Ch. 11.

32.1.3 Selection and use of Switches

The last topic we'll discuss in this section is the selection of switches (MOSFETs), that is, sizes, type (NMOS or PMOS), and design/layout for higher voltages. Before we get started let's take a look at the circuit in Fig. 32.6. The input to this circuit is a pulse waveform swinging between ground (0V) and V_S. A lowpass filter, here an *LC* circuit, is used to smooth out the pulse to generate a DC voltage for supplying energy to a load (cell phone, laptop, rechargeable battery, etc.). Here we are obviously modeling the load as resistor (but the load is not part of the power supply, it's what the power supply supplies energy to). Note that the only item that dissipates power in this circuit is the load. All of the power supplied to the input of the low pass filter is transferred to the load. This is why switching power supplies are so useful in practical electronics. Unlike a linear supply (voltage regulator), see Fig. 24.54, which dissipates significant power to regulate an output voltage, the switching regulator, as we'll see in a moment, averages an input pulse waveform to, ideally, ensure most of the power is transferred to the load.

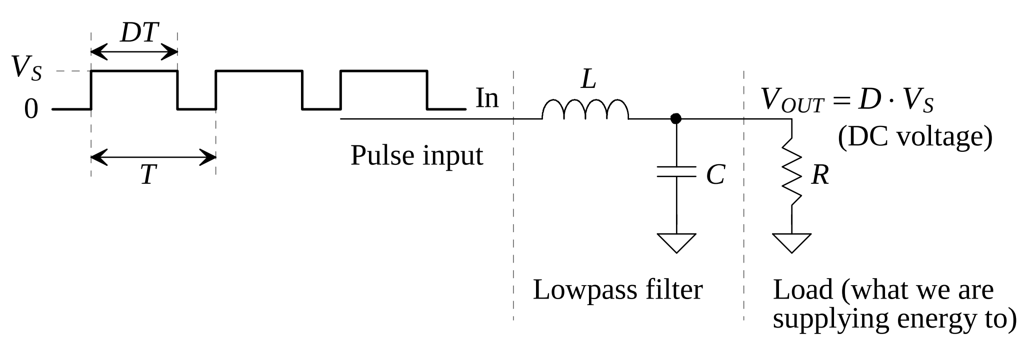

Figure 32.6 Basic idea behind a switching regulator.