

34.1) We can approach this problem in one of two ways, either by utilizing the equations given in the book or using circuit analysis, we'll do both.

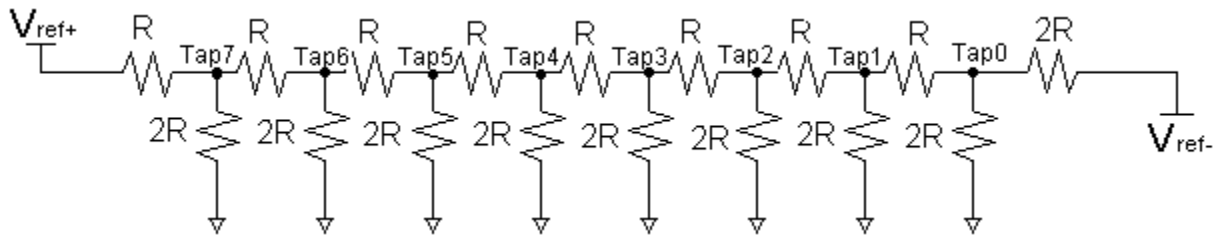
$$V_{TAPX} = \frac{2^X}{2^N} \cdot (V_{REF+} - V_{REF-}) + V_{REF-} \quad \text{EQ(34.1)}$$

8-Bit R-2R DAC

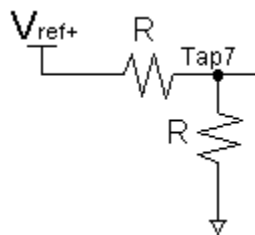
N	X	V _{TAPX}
8	0	.00586
8	1	.01172
8	2	.02343
8	3	.04687
8	4	.09375
8	5	.1875
8	6	.375
8	7	.75

Using circuit analysis will require a little more work but will give more of an understanding how the circuit is working.

Schematic of the circuit is shown with just the resistors, V_{ref+}=1.5V and V_{ref-}=0, in the circuit.



Starting from the left we have the 2R resistors in parallel which gives us an R in series with the R between the two Tap voltages and continuing all the way we are left with a simple voltage divider.



$$\begin{aligned} V_{tap7} &= 1.5/2 = .75V \\ V_{tap6} &= .75/2 = .375V \\ V_{tap5} &= .375/2 = .1875V \\ V_{tap4} &= .1875/2 = .09375V \\ V_{tap3} &= .09375/2 = .04687V \\ V_{tap2} &= .04687/2 = .02343V \\ V_{tap1} &= .02343/2 = .01172V \\ V_{tap0} &= .01172/2 = \mathbf{.00586V} \end{aligned}$$

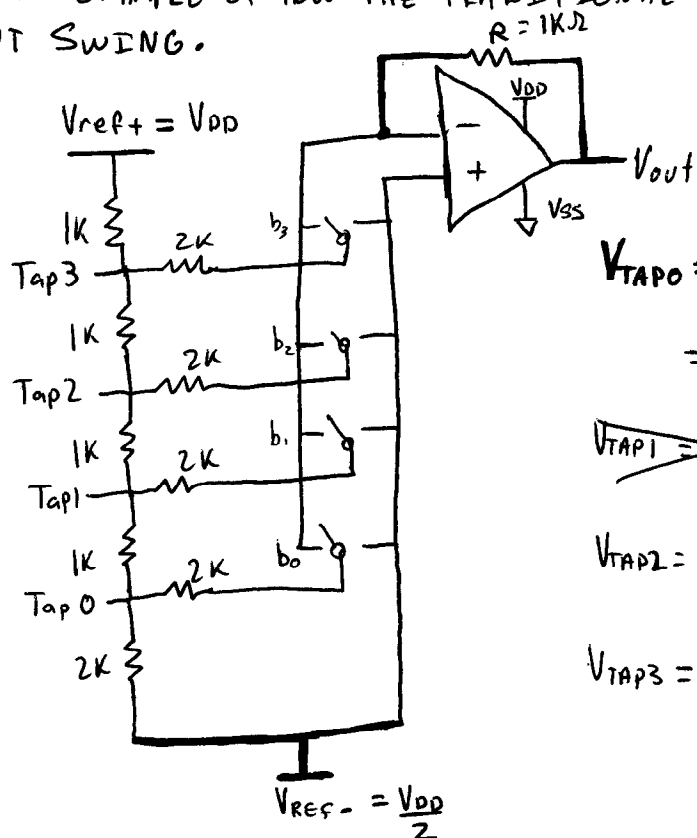
This result agrees with the value for the LSB of the circuit: 1LSB=1.5/2⁸=.00586V

34.2

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①

GIVE AN EXAMPLE OF HOW THE TRADITIONAL CURRENT MODE DAC WILL HAVE LIMITED OUTPUT SWING.



$$V_{DD} = 5V$$

$$(34.1) V_{TAPx} = \frac{2^x}{2^N} (V_{REF+} - V_{REF-}) + V_{REF-}$$

$$V_{TAPO} = \frac{2^0}{2^4} \left(\frac{V_{DD}}{2} \right) + \frac{V_{DD}}{2}$$

$$= \frac{V_{DD}}{32} + \frac{V_{DD}}{2} = 2.65625V$$

$$V_{TAP1} = \frac{V_{DD}}{2} + \frac{V_{DD}}{16} = 2.8125V$$

$$V_{TAP2} = \frac{V_{DD}}{2} + \frac{V_{DD}}{8} = 3.125V$$

$$V_{TAP3} = \frac{V_{DD}}{2} + \frac{V_{DD}}{4} = 3.75V$$

The maximum current draw is when each switch is connected to the inverting input.

This current is

$$I_{MAX} = \frac{V_{TAPO} + V_{TAP1} + V_{TAP2} + V_{TAP3} - 4V_{REF-}}{2K\Omega}$$

$$= \frac{2.65625 + 2.8125 + 3.125 + 3.75 - 4 \cdot 5/2}{2K\Omega} = 1.1718mA$$

$I_{min} = 0$, when all the switches are connected to the non inverting input.

$$V_{REF-} - V_{OUT_{min}} = I_{max}$$

$$V_{OUT_{min}} = V_{REF-} - I_{max}(1K\Omega) = \frac{V_{DD}}{2} - (1.1718)(1) = 1.328$$

problem 34.2 continued...

②

$$\begin{aligned} V_{OUT_{max}} &= V_{REF} - I_{min}(R) \\ &= 5/2 - 0 = \frac{V_{DD}}{2} \end{aligned}$$

So for this standard current mode DAC

V_{out} ranges from (1.328 - 2.5 Volts)

If the DAC has more bits it only causes the minimum V_{out} to decrease, but it doesn't change $V_{out_{max}} = V_{REF}$.

The op-amp can't have its output go below the power supply $V_{SS} = 0V$.

Solution 34.3:

By applying superposition theorem, for fig. 3.2

Voltage at R-2R tap we can write as,

$$V_{(R-2R)_{n-1}} = \frac{b_{(N-1)}V_{REF}^+ + \overline{b_{(N-1)}}V_{REF}^-}{2^1} + \frac{b_{(N-2)}V_{REF}^+ + \overline{b_{(N-2)}}V_{REF}^-}{2^2} + \dots + V_{REF}^-$$

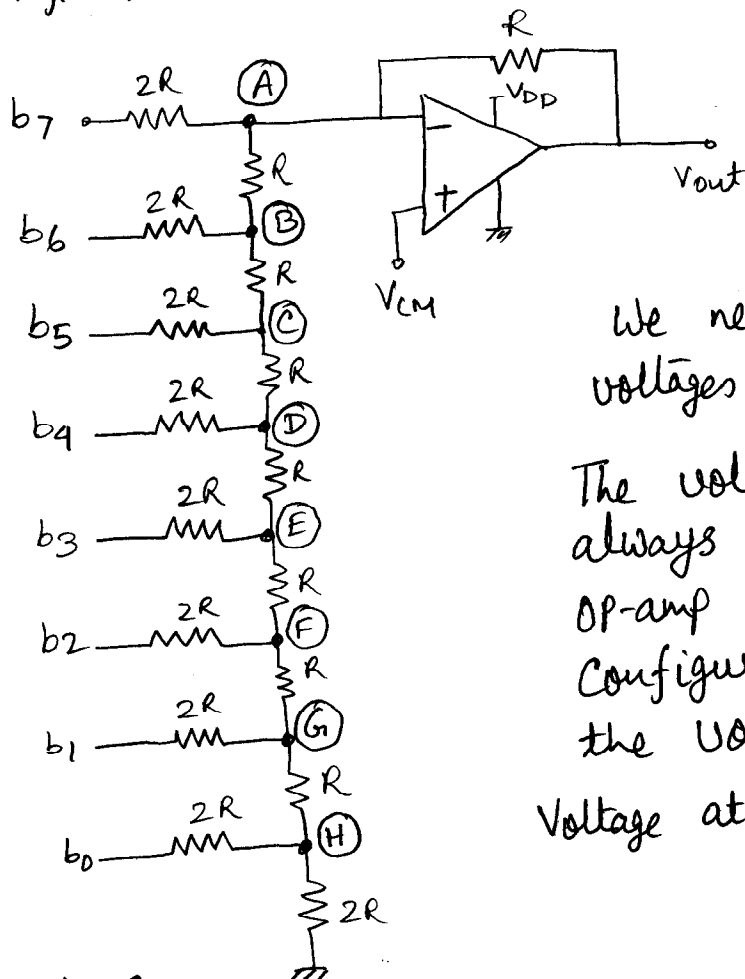
For 8 bit DAC, N=8, and $V_{REF}^+ = 1.5V$, $V_{REF}^- = 0V$,

So voltage at $V_{(R-2R)}$ tap would be:

tap no.	$V_{(R-2R)}$ tap	$V_{(R-2R)}$ tap, if $b_0, \dots, b_7=1$
0	$\frac{3}{2}(\frac{b_0}{2^8})$	5.86mV
1	$\frac{3}{2}(\frac{b_0}{2^8} + \frac{b_1}{2^7})$	17.58mV
2.	$\frac{3}{2}(\frac{b_0}{2^8} + \dots + \frac{b_2}{2^6})$	41mV
3	$\frac{3}{2}(\frac{b_0}{2^8} + \dots + \frac{b_3}{2^5})$	87.88mV
4	181.63mV
5	369.13mV
6	750mV
7	$\frac{3}{2}(\frac{b_0}{2^8} + \dots + \frac{b_7}{2})$	1.5V

Q: For the wide-swing current mode DAC shown in Fig. 34.3 what are the voltages at the taps along the R-2R string assuming 8-bits, $V_{REF+} = 1.5V$, $V_{REF-} = 0$, and a digital input code of 0000 0000?

Ans: Fig. 34.3 is redrawn as shown for 8 bit case



We need to calculate the voltages at nodes A to H.

The voltage at node A is always held at V_{CM} by the op-amp working in feedback configuration irrespective of the voltage values at $b_7 - b_0$.

Voltage at $b_i = V_{REF+}$ if bit $i = 0$
 $= V_{REF-}$ if bit $i = 1$
 $i = 0, 1, 2, \dots, 7$

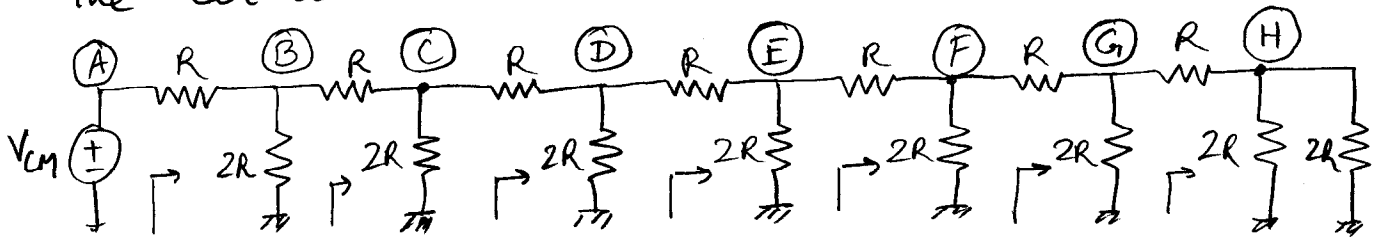
Technique

To solve this problem, we use the superposition theorem. $b_7 - b_0$ are 8 independent sources along with node A which is always held at V_{CM} by the op-amp. At each node from B to H, we superpose the effect of these 9 independent sources individually and then sum the individual effects to determine the expression for the voltage at each node. We then calculate the voltage value at each node for the given digital input code "0000 0000".

Independent source V_{CM}

We ground all other independent sources (if $b_0 - b_7$).

The circuit becomes:



We find that the resistance looking to the right of A, B, ..., G is $2R$ to ground.

Voltage at A = V_{CM}

Voltage at B = $\frac{V_{CM}}{2}$ →

" " C = $\frac{V_{CM}}{4}$ →

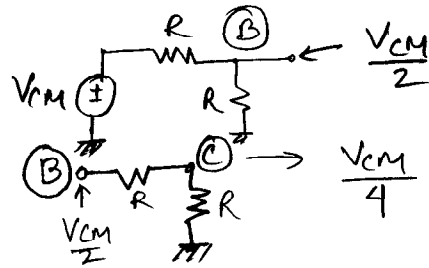
" " D = $\frac{V_{CM}}{8}$

" " E = $\frac{V_{CM}}{16}$

" " F = $\frac{V_{CM}}{32}$

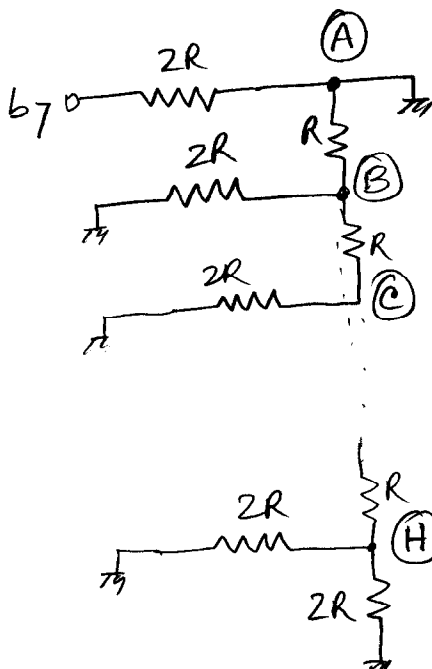
" " G = $\frac{V_{CM}}{64}$

" " H = $\frac{V_{CM}}{128}$



Independent source b_7

We ground all other independent sources (V_{CM} , $b_0 - b_6$)



We find that the contribution of b_7 at nodes A-H is zero.

Independent source b_6

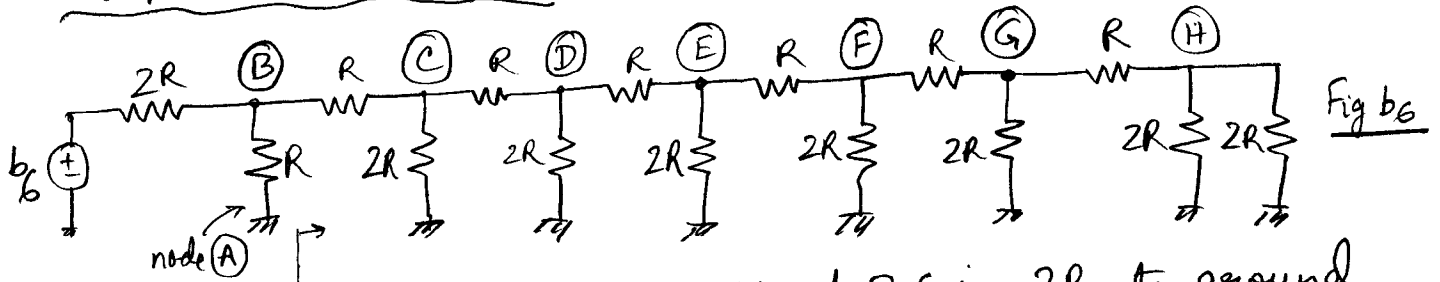
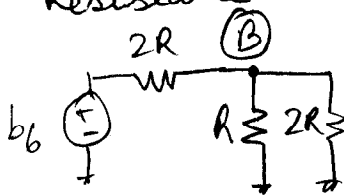


Fig b_6

Resistance looking to the right of B-G is $2R$ to ground.

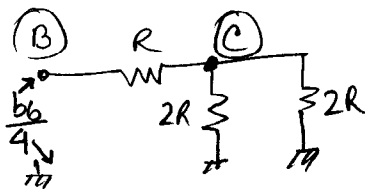


$$\text{Voltage at B} = b_6 \cdot \frac{R // 2R}{2R + (R // 2R)}$$

$$= b_6 \cdot \frac{\frac{2R}{3}}{2R + \frac{2R}{3}}$$

$$= b_6 \cdot \frac{\frac{2R}{3}}{\frac{8R}{3}}$$

$$= \frac{b_6}{4}$$



$$\text{Voltage at C} = \frac{b_6}{4} \cdot \frac{2R // 2R}{R + (2R // 2R)}$$

$$= \frac{b_6}{4} \cdot \frac{R}{R + R}$$

$$= \frac{b_6}{8}$$

Similarly, Voltage at D = $\frac{b_6}{16}$

$$E = \frac{b_6}{32}$$

$$F = \frac{b_6}{64}$$

$$G = \frac{b_6}{128}$$

$$H = \frac{b_6}{256}$$

Independent source b_5

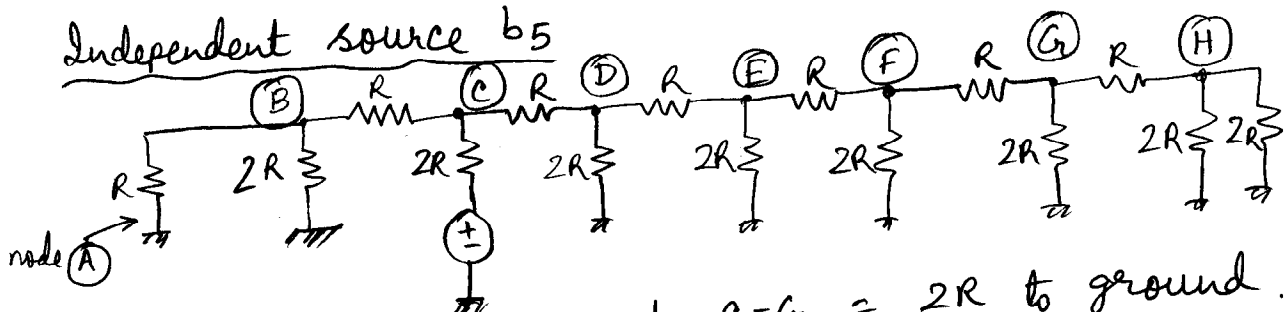
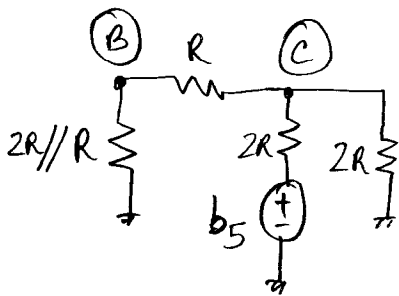


Fig b_5

Resistance looking right of C-G = $2R$ to ground.



Resistance looking left of C = $\frac{2R}{2} + R$ to ground
 $= \frac{2R}{3} + R = \frac{5R}{3}$

$$\text{Voltage at } (C) = b_5 \cdot \frac{2R // \frac{5R}{3}}{2R + 2R // \frac{5R}{3}}$$

Now $2R // \frac{5R}{3}$

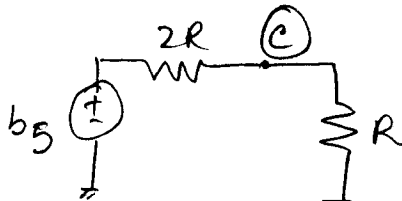
$$= \frac{10R}{11}$$

$$\approx R$$

$$= b_5 \cdot \frac{\frac{10R}{3} / (\frac{11}{3})}{2R + \frac{10R}{2}}$$

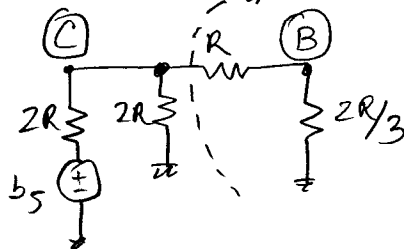
$$= b_5 \cdot \frac{10^5}{3216}$$

If we make this approximation, then the voltage at (C) becomes



$$b_5 \cdot \frac{R}{R+2R} = \frac{b_5}{3}$$

Voltage at B



Therenzizing the circuit to the right of B, we get

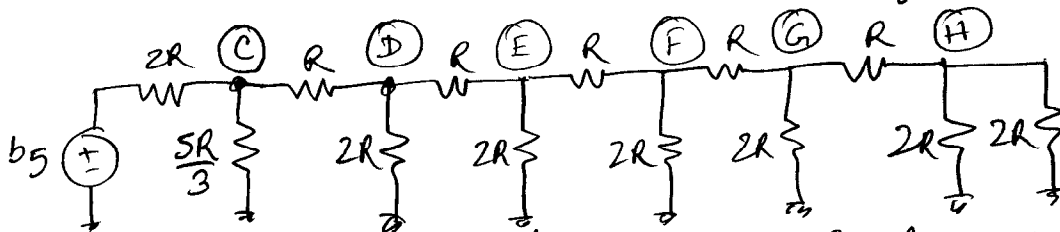
$$V_{Th} = b_5 \cdot \frac{2R}{2R+2R} = \frac{b_5}{2}$$

$$R_{Th} = 2R // 2R = R$$

$$\text{Voltage at B} = \frac{b_5}{2} \cdot \frac{\frac{2R}{3}}{2R + \frac{2R}{3}}$$

$$= \frac{b_5}{2} \cdot \frac{\frac{2R}{3}}{\frac{48R}{3}}$$

$$\text{Voltage at B} = \frac{b_5}{8}$$



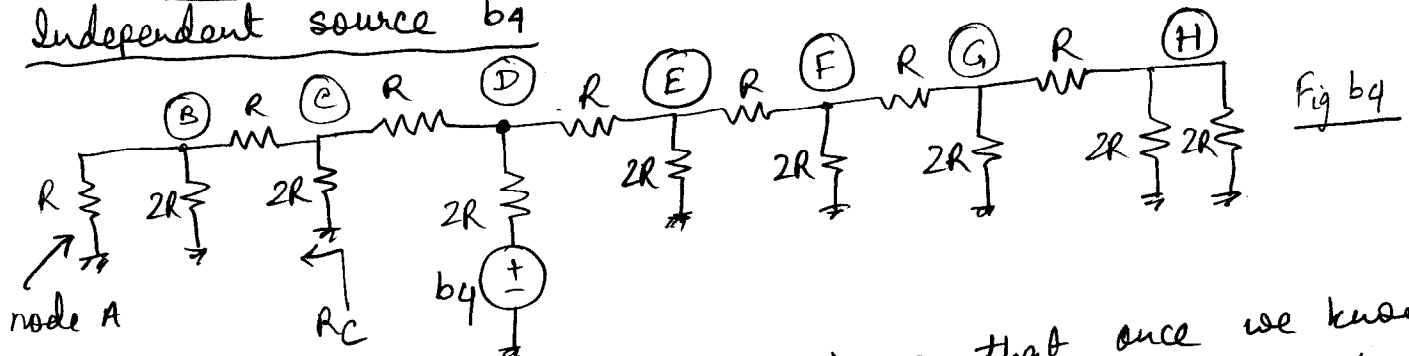
Resistance looking right of C-G = $2R$ to ground.
 Looking at the previous analysis for b_6 , we know that the voltage at D-H will be $\frac{V_C}{2}, \frac{V_C}{4}, \frac{V_C}{8}, \frac{V_C}{16}, \frac{V_C}{32}$
 where V_C = Voltage at C.

The exact expression for the voltage at C = $\frac{5}{16} b_5$

The approximate expression is $\frac{b_5}{3}$

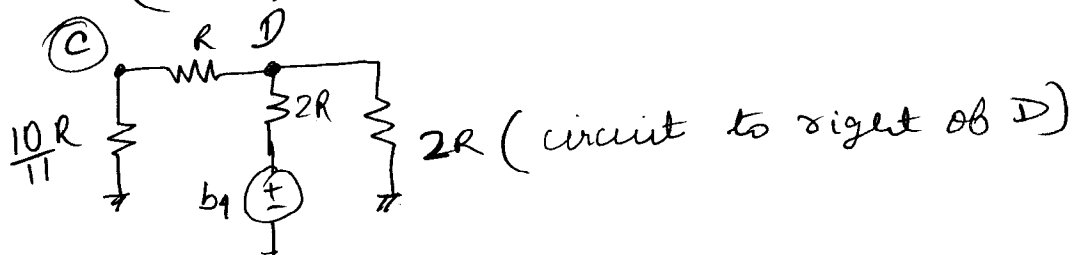
	Exact	Approx.
Voltage at D	$\frac{5b_5}{32}$	$\frac{b_5}{6}$
E	$\frac{5}{24} b_5$	$\frac{b_5}{12}$
F	$\frac{5}{128} b_5$	$\frac{b_5}{24}$
G	$\frac{5}{256} b_5$	$\frac{b_5}{48}$
H	$\frac{5}{512} b_5$	$\frac{b_5}{96}$

Independent source b_4



From previous analysis, we know that once we know the voltage at D, (E, F, G, H) will have $\frac{V_D}{2}$, $\frac{V_D}{4}$, $\frac{V_D}{8}$, $\frac{V_D}{16}$ resp. Also, resistance looking left of C = $[(R//2R) + R] // 2R$ to ground

$$R_C = \left(\frac{2R}{3} + R \right) // 2R = \frac{5R}{3} // 2R = \frac{10}{11} R \approx R$$



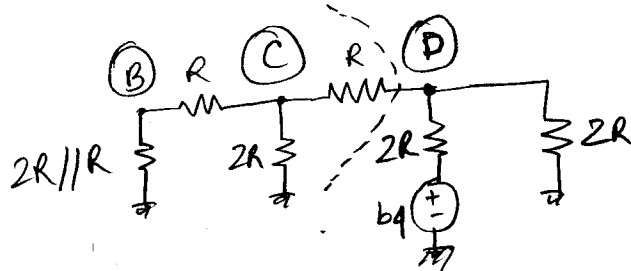
$$V_D = b_4 \cdot \frac{(R + \frac{10R}{11}) // 2R}{2R + [(R + \frac{10R}{11}) // 2R]} \approx b_4 \cdot \frac{(R + R) // 2R}{2R + [(R + R) // 2R]} = b_4 \cdot \frac{R}{2R + R}$$

$$V_D = \frac{b_4}{3}$$

Exact expression for $V_D = \frac{21}{64} b_4$

	Exact	Approx.
$V_E =$	$\frac{21}{128} b_4$	$\frac{b_4}{6}$
$V_F =$	$\frac{21}{256} b_4$	$\frac{b_4}{12}$
$V_G =$	$\frac{21}{512} b_4$	$\frac{b_4}{24}$
$V_H =$	$\frac{21}{1024} b_4$	$\frac{b_4}{48}$

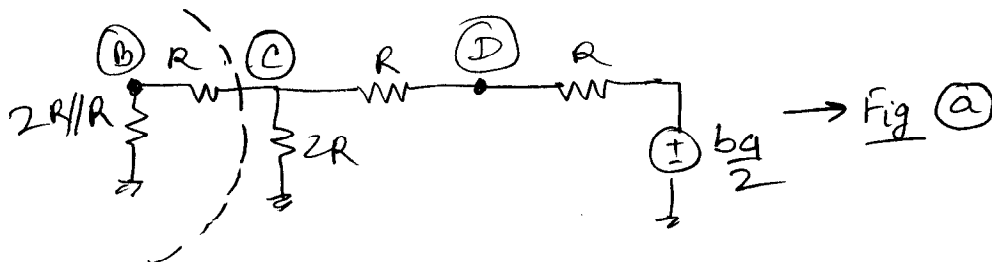
To find V_B



Thevenizing the elements at node D we get.

$$V_{TH} = b_4 \cdot \frac{2R}{2R+2R} = \frac{b_4}{2}$$

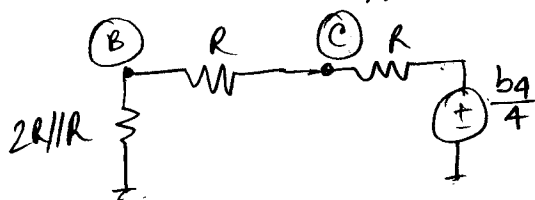
$$R_{TH} = 2R//2R = R.$$



Thevenizing elements at node C (and right of it), we get

$$V_{TH} = \frac{b_4}{2} \cdot \frac{2R}{2R+2R} = \frac{b_4}{4}$$

$$R_{TH} = 2R//2R = R.$$



$$V_B = \frac{b_4}{4} \cdot \frac{2R//R}{2R+(2R//R)}$$

$$= \frac{b_4}{4} \cdot \frac{\frac{2R}{3}}{2R+\frac{2R}{3}} = \frac{b_4}{4} \cdot \frac{1}{4}$$

$$V_B = \frac{b_4}{16}$$

To find V_C

Referring to Fig (a) above,

$$V_C = \frac{b_4}{2} \cdot \frac{2R//(R+2R//R)}{2R+[2R//(R+2R//R)]} = \frac{b_4}{2} \cdot \frac{2R//\frac{5R}{3}}{2R+(2R//\frac{5R}{3})}$$

$$= \frac{b_4}{2} \cdot \frac{\frac{10}{11}R}{2R+\frac{10}{11}R} \leftarrow R_C \quad R_C = \frac{10}{11}R \approx R.$$

$$V_C \text{ (exact)} = \frac{b_4}{2} \cdot \frac{10}{32} = \frac{5b_4}{32}$$

$$V_C \text{ (approx)} = \frac{b_4}{2} \cdot \frac{R}{2R+R} = \frac{b_4}{2} \cdot \frac{1}{3} = \frac{b_4}{6}$$

Independent Source b_3

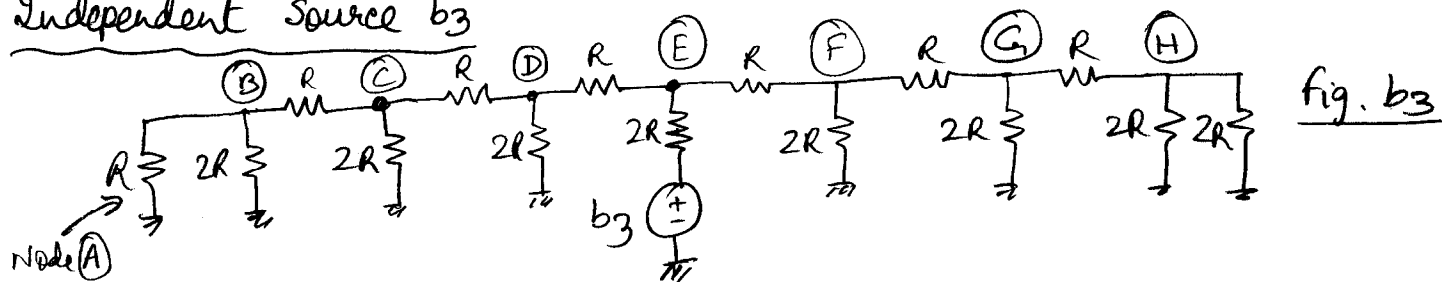
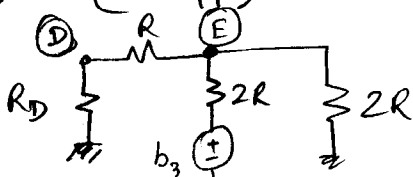


fig. b_3

We know $R_c = \frac{10R}{11} \approx R$

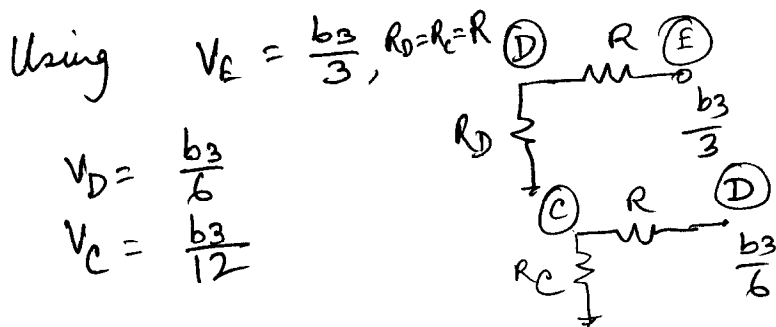
$$R_D = \left(R + \frac{10R}{11}\right) // 2R = \frac{42}{43} R \approx R$$



$$\text{Using } R_D = \frac{42}{43} R, \quad V_E = b_3 \cdot \frac{2R // \left(R + \frac{42R}{43}\right)}{2R + \left[2R // \left(R + \frac{42R}{43}\right)\right]} = b_3 \cdot \frac{\frac{170}{171} R}{2R + \frac{170}{171} R}$$

$$= b_3 \cdot \frac{170}{512} = b_3 \cdot \frac{85}{256}$$

$$\text{Using } R_D = R, \quad V_E = b_3 \cdot \frac{2R // 2R}{2R + (2R // 2R)} = \frac{b_3}{3}$$

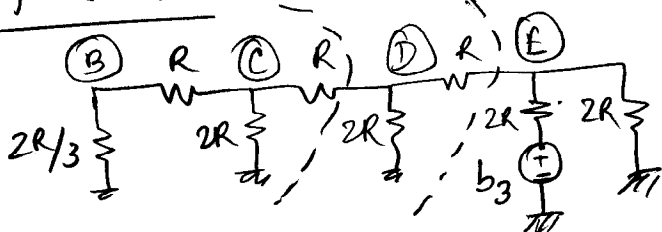


Using $V_E = \frac{b_3}{3}, R_D = R_C = R$

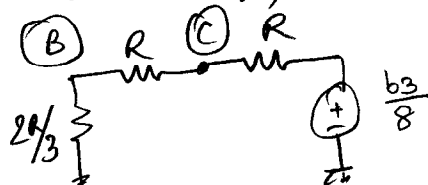
$$V_D = \frac{b_3}{6}$$

$$V_C = \frac{b_3}{12}$$

To find V_B



Thervenizing successively at \textcircled{E} and \textcircled{D} , we get.



$$V_B = \frac{b_3}{8} \cdot \frac{2R/3}{2R + 2R/3} = \frac{b_3}{8} \cdot \frac{1}{4}$$

$$\boxed{V_B = \frac{b_3}{32}}$$

Looking at the symmetry of the circuit referring to fig b3/b4/b5, we can write (extrapolate) the results for independent sources b_2, b_1, b_0 using the approximation $R_c = R_D = R_E = R_F = R_G \approx R$. The exact expression for the voltages is also given for comparison purpose.

Exact values: $R_c = \frac{10R}{11}$, $R_D = \frac{42R}{43}$, $R_E = \frac{170R}{171}$, $R_F = \frac{682R}{683}$, $R_G = \frac{2730R}{2731}$

Node	Exact Expression	Approximate Expression
A	V_{CM}	V_{CM}
B	$\frac{V_{CM}}{2} + \frac{b_6}{4} + \frac{b_5}{8} + \frac{b_4}{16} + \frac{b_3}{32} + \frac{b_2}{64} + \frac{b_1}{128} + \frac{b_0}{256}$	$\frac{V_{CM}}{2} + \frac{b_6}{4} + \frac{b_5}{8} + \frac{b_4}{16} + \frac{b_3}{32} + \frac{b_2}{64} + \frac{b_1}{128} + \frac{b_0}{256}$
C	$\frac{V_{CM}}{4} + \frac{b_6}{8} + \frac{5b_5}{16} + \frac{5b_4}{32} + \frac{5b_3}{64} + \frac{5b_2}{128} + \frac{5b_1}{256} + \frac{5b_0}{512}$	$\frac{V_{CM}}{4} + \frac{b_6}{8} + \frac{b_5}{3} + \frac{b_4}{6} + \frac{b_3}{12} + \frac{b_2}{24} + \frac{b_1}{48} + \frac{b_0}{96}$
D	$\frac{V_{CM}}{8} + \frac{b_6}{16} + \frac{5b_5}{32} + \frac{21b_4}{64} + \frac{21b_3}{128} + \frac{21b_2}{256} + \frac{21b_1}{512} + \frac{21b_0}{1024}$	$\frac{V_{CM}}{8} + \frac{b_6}{16} + \frac{b_5}{6} + \frac{b_4}{3} + \frac{b_3}{6} + \frac{b_2}{12} + \frac{b_1}{24} + \frac{b_0}{48}$
E	$\frac{V_{CM}}{16} + \frac{b_6}{32} + \frac{5b_5}{64} + \frac{21b_4}{128} + \frac{85b_3}{256} + \frac{85b_2}{512} + \frac{85b_1}{1024} + \frac{85b_0}{2048}$	$\frac{V_{CM}}{16} + \frac{b_6}{32} + \frac{b_5}{12} + \frac{b_4}{6} + \frac{b_3}{3} + \frac{b_2}{6} + \frac{b_1}{12} + \frac{b_0}{24}$
F	$\frac{V_{CM}}{32} + \frac{b_6}{64} + \frac{5b_5}{128} + \frac{21b_4}{256} + \frac{85b_3}{512} + \frac{341b_2}{1024} + \frac{341b_1}{2048} + \frac{341b_0}{4096}$	$\frac{V_{CM}}{32} + \frac{b_6}{64} + \frac{b_5}{24} + \frac{b_4}{12} + \frac{b_3}{6} + \frac{b_2}{3} + \frac{b_1}{6} + \frac{b_0}{12}$
G	$\frac{V_{CM}}{64} + \frac{b_6}{128} + \frac{5b_5}{256} + \frac{21b_4}{512} + \frac{85b_3}{1024} + \frac{341b_2}{2048} + \frac{1365b_1}{4096} + \frac{1365b_0}{8192}$	$\frac{V_{CM}}{64} + \frac{b_6}{128} + \frac{b_5}{48} + \frac{b_4}{24} + \frac{b_3}{12} + \frac{b_2}{6} + \frac{b_1}{3} + \frac{b_0}{6}$
H	$\frac{V_{CM}}{128} + \frac{b_6}{256} + \frac{5b_5}{512} + \frac{21b_4}{1024} + \frac{85b_3}{2048} + \frac{341b_2}{4096} + \frac{1365b_1}{8192} + \frac{5461b_0}{16384}$	$\frac{V_{CM}}{128} + \frac{b_6}{256} + \frac{b_5}{96} + \frac{b_4}{48} + \frac{b_3}{24} + \frac{b_2}{12} + \frac{b_1}{6} + \frac{b_0}{3}$

For an input code of "0000 0000", $b_7 - b_0$ are $V_{REF+} = 1.5V$.

Node	Exact Voltage	Approx. Voltage	% Error
A	0.750 V	0.750 V	0
B	1.119 V	1.119 V	0
C	1.298 V	1.359 V	4.70
D	1.375 V	1.406 V	2.25
E	1.391 V	1.406 V	1.08
F	1.352 V	1.359 V	0.52
G	1.238 V	1.242 V	0.32
H	0.994 V	0.996 V	0.20

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Problem 34.5

Question: Can the op-amp shown in Fig. 34.36 be used in a fully differential implementation of the DACs shown in Figs. 34.1-34.3? Why or why not?

Answer: No, the op-amp of figure 34.36 cannot be used to implement an R2R DAC, because the amplifier does not have an output buffer, and therefore cannot drive a reasonable resistive load on an IC. This is because an op-amp with reasonably high gain has a very high output resistance (on the order of mega ohms). If we add a typical resistive load in the feedback path of the op-amp in these topologies, it will load the output of the amplifier and it will kill the gain, and thus not operate correctly. If an output buffer is added, the gain will not be reduced, but the output current drive capability will be increased, thus enabling the amplifier to drive the resistive loads used in the DAC topologies.

Problem 34.6

Show the detailed derivation of eqs.(34.12)-(34.14).

For a binary-weighted DAC, the midscale transition has the worst-case DNL condition caused by resistor mismatch. In a worst-case scenario, assume the $2R$ resistor of the MSB input has a maximum mismatch of ΔR and all other resistors have a maximum mismatch of $-\Delta R$.

The equation 34.9 shows the current flowing in the feedback resistor R_f ,

$$I_F = -\frac{V_{REF+} - V_{REF-}}{2R} + \frac{V_{REF+} - V_{REF-}}{2R} \left[1 \overline{b_{N-1}} + \frac{1}{2} \overline{b_{N-2}} + \dots + \frac{1}{2^{N-1}} \overline{b_0} \right]$$

The step error of this current caused by resistor mismatch at the midscale transition is

$$\begin{aligned} \Delta I &= (I_{N-2} + I_{N-3} + \dots + I_0) - I_{N-1} \\ &= \frac{V_{REF+} - V_{REF-}}{2(R - \Delta R)} \left(\frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^{N-1}} \right) - \frac{V_{REF+} - V_{REF-}}{2(R + \Delta R)} \\ &= \frac{V_{REF+} - V_{REF-}}{2(R - \Delta R)} \left(\frac{1}{2^1} + \frac{1}{2^2} + \dots + \frac{1}{2^{N-1}} + \frac{1}{2^{N-1}} - \frac{1}{2^{N-1}} \right) - \frac{V_{REF+} - V_{REF-}}{2(R + \Delta R)} \\ &= \frac{V_{REF+} - V_{REF-}}{2(R - \Delta R)} \left(1 - \frac{1}{2^{N-1}} \right) - \frac{V_{REF+} - V_{REF-}}{2(R + \Delta R)} \end{aligned}$$

Assuming $R_f = R$, the final output error (DNL) is approximately,

$$\begin{aligned} DNL &= \Delta I \cdot R = \left[\frac{V_{REF+} - V_{REF-}}{2(R - \Delta R)} \left(1 - \frac{1}{2^{N-1}} \right) - \frac{V_{REF+} - V_{REF-}}{2(R + \Delta R)} \right] R \\ &= (V_{REF+} - V_{REF-}) \left[\left(\frac{1}{2(R - \Delta R)} - \frac{1}{2^{N-1} \cdot 2(R - \Delta R)} \right) - \frac{1}{2(R + \Delta R)} \right] R \\ &= (V_{REF+} - V_{REF-}) \left[\frac{1}{2(R - \Delta R)} - \frac{1}{2^N (R - \Delta R)} - \frac{1}{2(R + \Delta R)} \right] R \\ &= (V_{REF+} - V_{REF-}) \left[\frac{R + \Delta R - R + \Delta R}{2(R - \Delta R)(R + \Delta R)} - \frac{1}{2^N (R - \Delta R)} \right] R \\ &= (V_{REF+} - V_{REF-}) \left[\frac{\Delta R}{(R - \Delta R)(R + \Delta R)} - \frac{1}{2^N (R - \Delta R)} \right] R \\ &\cong (V_{REF+} - V_{REF-}) \left[\frac{\Delta R}{R} - \frac{1}{2^N} \right] \end{aligned}$$

For the DNL to be less than 1 LSB, the resistor-matching requirement is

$$\begin{aligned}
 DNL \leq 1LSB &= (V_{REF+} - V_{REF-}) / 2^N \\
 (V_{REF+} - V_{REF-}) \left[\frac{\Delta R}{R} - \frac{1}{2^N} \right] &\leq (V_{REF+} - V_{REF-}) / 2^N \\
 \left[\frac{\Delta R}{R} - \frac{1}{2^N} \right] &\leq \frac{1}{2^N} \\
 \frac{\Delta R}{R} &\leq \frac{2}{2^N}
 \end{aligned}$$

$$\text{Resistor mismatch} = \left| \frac{\Delta R}{R} \right| \leq \frac{1}{2^{N-1}}$$

- Yantao Ma

Chapter 34.7

Why would we want to use both current segments & binary-weighted currents to implement a current-mode DAC? (Why use segmentation?)

[Answer]: In general, segmentation achieves better DNL with less accurate components possible required.

- Use current Mode DAC: to achieve Rail-to-Rail Output swing while keeping input nodes of Op-Amp fixed.
- Use current-segments & binary-weighted current Mode DAC: Efficient method of implement high-resolution DAC.
- To against Normal current Mode DAC major disadvantages:
 1. overall accuracy is determined largely by major MSB's.
 2. Monotonicity cannot be guaranteed
 3. Design is not practical when DAC N becomes large due to current elements (max) rises exponentially, which cause resistor matching extremely critical and challenging for manufacturing.
- Fundamental principle: By limiting & Equilizing maximum current elements (controlled by MSB/upper bits) with more fine current resolution levels (controlled by lower bits), a number of maximum and fine current elements can be switched to feedback resistor.

This method guarantees monotonic operation and improves DNL within 1LSB because segment current used. The accuracy of segmented DAC is determined largely by the matching of maximum current elements.

Precision can be improved by symmetrical layout techniques and additional calibration techniques.

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Q. 348 why do we subtract ΔA in Eq. (34.36)?
Why not add the gain variation?

Ans: Referring to chapter 29. eq page 821.

Accuracy issues related to OPAMP:

It is quite difficult to implement standard OPAMPs within high resolution data converters because of the accuracy requirements. The nonideal characteristics of the OPAMP are well known and in many cases alone limit the accuracy of the data converters.

Generalizing the concept for N-bit application with feedback theory.

$$A_{CL} = \frac{V_o}{V_i} \Rightarrow \frac{A_{OLDC}}{1 + A_{OLDC}\beta}$$

** It is well known that as A_{OL} increases, A_{CL} approaches towards $\frac{1}{\beta}$ i.e. ideal.

So it is assumed that the A_{CL} will be equal to the ideal value of $\frac{1}{\beta}$ ~~minus~~ ^{minus} some maximum deviation from ideal, ΔA then

$$A_{CL} = \frac{V_o}{V_i} = \frac{A_{OL}}{1 + A_{OL}\beta} = \frac{1}{\beta} - \Delta A \quad \text{--- (*)}$$

and we know the maximum deviation would be $\frac{1}{2}$ LSB. Hence.

By solving (*) we can get

$$\boxed{|A_{OL}| = \frac{1}{\beta} (2^{N+1} + 1) = \frac{2^{N+1}}{\beta}}$$

$$|A_{OL}| \geq \frac{2^{N+1}}{\beta}$$

$\therefore N \Rightarrow$ no of bits.

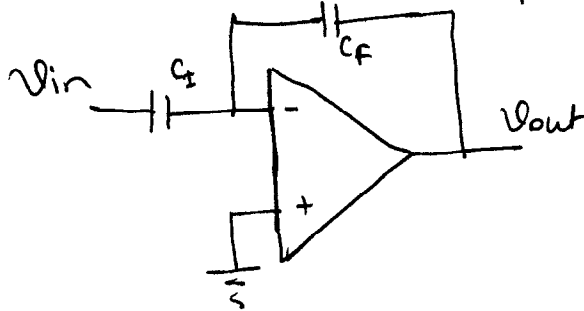
$\beta \Rightarrow$ feedback factor

$A \Rightarrow$ open loop gain

$A_{CL} \Rightarrow$ closed loop gain.

Referring to the Equation 34.36

$$|A_{CL}| = \frac{C_I}{C_F} - \Delta A \Rightarrow \frac{A_{OLDC}}{1 + A_{OLDC} \cdot \frac{C_F}{C_I + C_F}}$$



$$\Delta A = \frac{e_I}{C_F} \cdot \frac{1/2 \text{ LSB}}{\text{Full Scale}} \Rightarrow \frac{C_I}{C_F} \cdot \frac{1}{2^{N+1}}$$

Similar as previous Discussion we get

$$|A_{OLDC}| \geq \left(\frac{1}{\beta}\right) 2^{N+1}$$

So mainly the reason to minus the ΔA from ideal close loop gain is well known and i.e. as A_{OLDC} increases then and then only we can approach to $A_{CL} = 1/\beta$. But

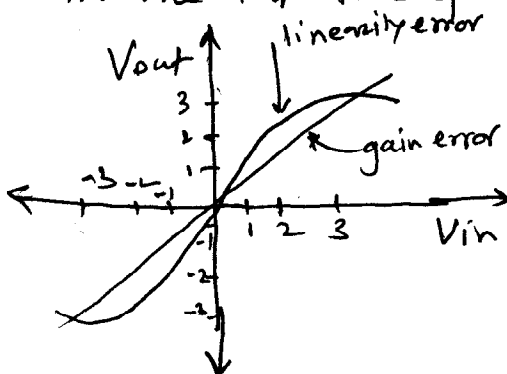
$$|A_{OLDC}| \geq \frac{1}{\beta} \cdot 2^{N+1}$$

Hence, here it can be noticeable that for every bit increase in the resolution the open loop gain requirements double.

So the ideal $A_{CL} = 1/\beta$ which is possible only when $A_{OLDC} = \infty$. But to get $A_{OLDC} = \infty$, N should be ∞ which is not practicable (from e_{in}). So always the closed loop gain of the amplifier will be equal to $1/\beta$ minus some maximum deviation from the ideal, ΔA .

* why not add the gain variation?

Ans: The amplifier must be able to linearly amplify the input over the input voltage range to within $1/2 \text{ LSB}$. Nonlinearity in devices in the amplifier go into non-saturation.



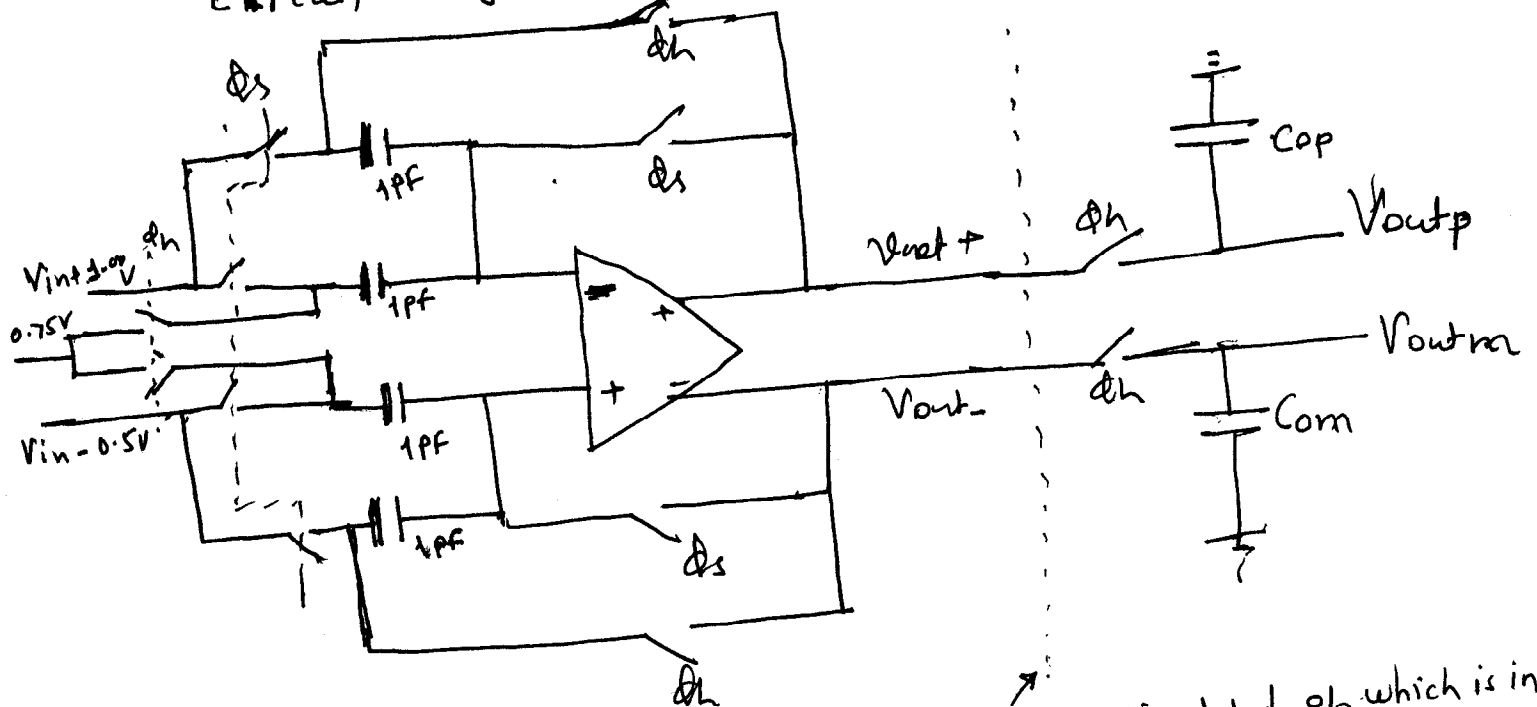
* Transfer curve shows nonlinearity introduced at both ends of $1/2 \text{ V}_{FS}$. A gain error is much less harmful to an ADC's performance than Harmonic distortion (Nonlinearity).

Q. 34.28

Repeat Ex. 34.18 if all capacitors are 1 pF (the ideal simulation) and verify that the error out of the stage is zero.

Ans:

~~circuit~~ circuit diagram of Ex. 34.18



We get the simulated o/p which is in terms of pulses. So after load capacitors we get the o/p without spikes.

The input voltage $V_{in+} - V_{in-} = 1.0 - 0.5 = 0.5V$

Ideally output should be equal to twice of input
i.e. 1V

So the output at plus = 1.25V

and output at minus = 0.25V

The simulated results are shown in fig.

Error = $V_{ideal} - V_{outactual} \Rightarrow 1V - 1V \approx 0$.

~~Input des~~
Netlist

Sample and hold circuit

```
.control
destroy all
run
let Verror=Videal-(Vop-Vom)
plot Verror
.endc
.options scale=1u ABSTOL=10u VNTOL=10m RELTOL=.01
```

***Op-amp using VCVS**

E1 Voutp Vcom Vplus Vminus 100MEG

E2 Vcom Voutm Vplus Vminus 100MEG

***Sample and hold capacitors**

Cim Vminus Vcomm 1p

Cfm Vminus Vcenm 1p

Cip Vplus Vcomp 1p

Cfp Vplus Vcenp 1p

***Switches**

S1 Vcenm Vsigg phis Vswit switmod

S2 Vcomm Vsigg phis Vswit switmod

S3 Vcomm Vcom phih Vswit switmod

S4 Vcomp Vcom phih Vswit switmod

S5 Vcomp Vsigm phis Vswit switmod

S6 Vcenp Vsigm phis Vswit switmod

S7 Voutp Vcenm phih Vswit switmod

S8 Voutp Vminus phis Vswit switmod

S9 Voutm Vplus phis Vswit switmod

S10 Voutm Vcenp phih Vswit switmod

S11 Voutp Vop phih Vswit switmod

S12 Voutm Vom phih Vswit switmod

***Load Capacitance**

Cop Vop 0 10p

Com Vom 0 10p

***Supply voltage**

VDD VDD 0 DC 1.5

***Reference voltages**

Vswit Vswit 0 DC 0.75

Vcom Vcom 0 DC 0.75

***Input Signals**

Vsigg Vsigg 0 DC 1.0

Vsigg Vsigg 0 DC 0.5

***Ideal signal**

Videal Videal 0 DC 1

***Clock Signals**

Vphis phis 0 DC 0 Pulse 0 1.5 0 200p 200p 4n 10n

Vphih phih 0 DC 0 Pulse 0 1.5 5n 200p 200p 4n 10n

Rs phis 0 1MEG

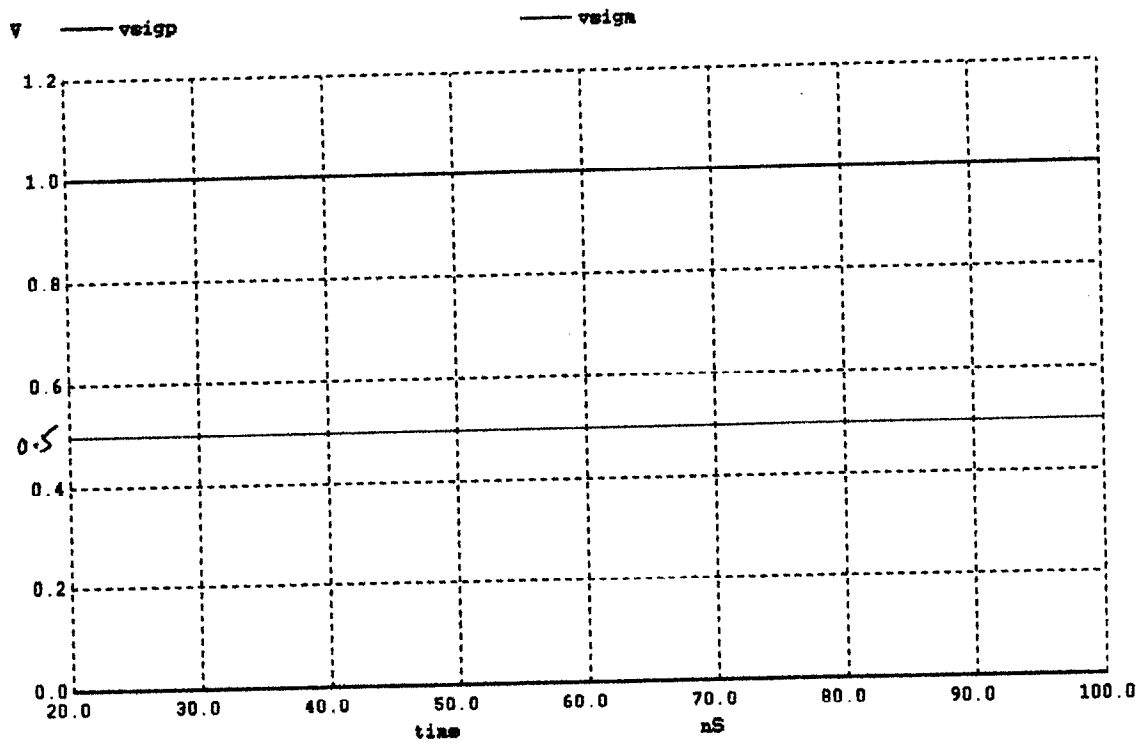
Rh phih 0 1MEG

.tran 2n 400n 0 2n UIC

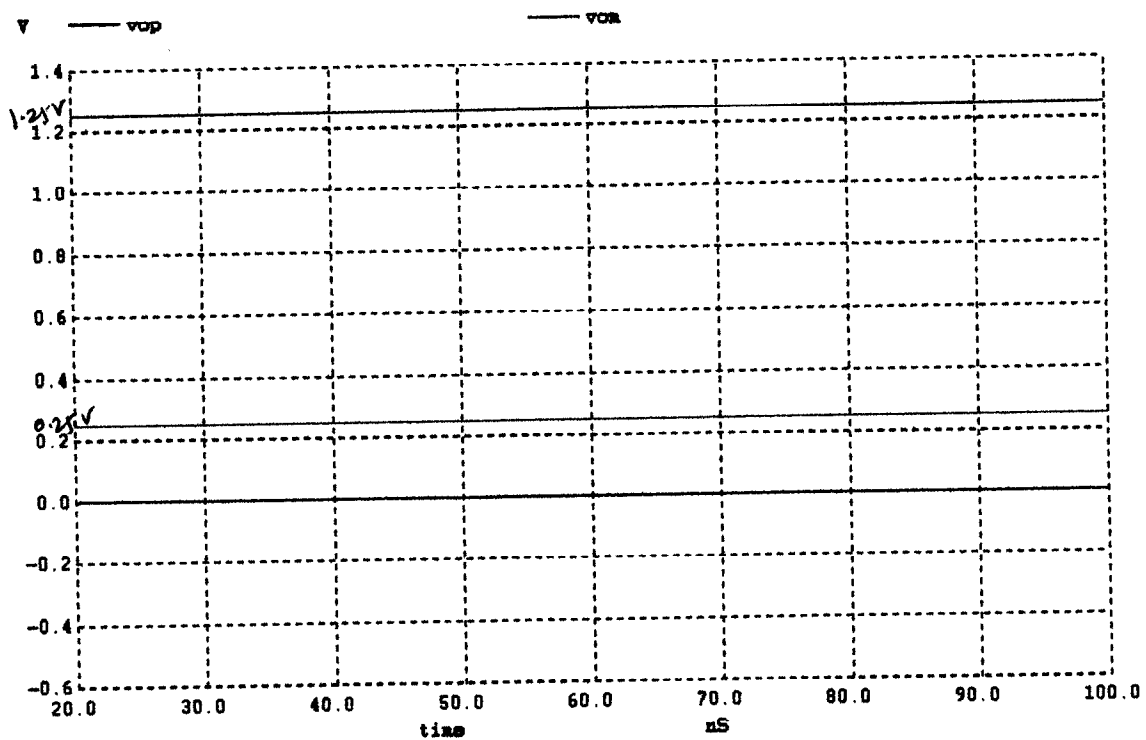
.model switmod SW RON=1

.end

Input signals

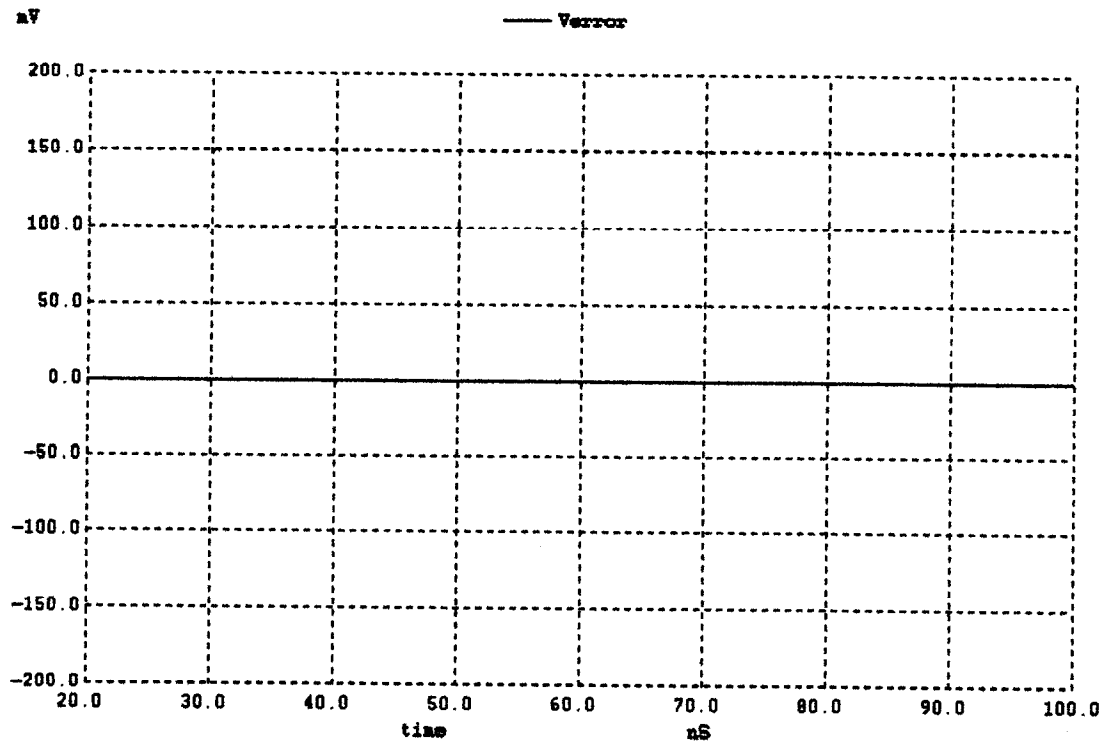


Output Signals



$$V_{op} - V_{om} = 1.25 - 0.25 = 1V$$

Error signal=Ideal signal-Actual signal



$$\begin{aligned}\text{Error} &= V_{\text{ideal}} - V_{\text{out}} \\ &= 1\text{V} - (V_{\text{op}} - V_{\text{om}}) \\ &= 1\text{V} - (1.25 - 0.25) \\ &= \underline{\underline{0}}\end{aligned}$$

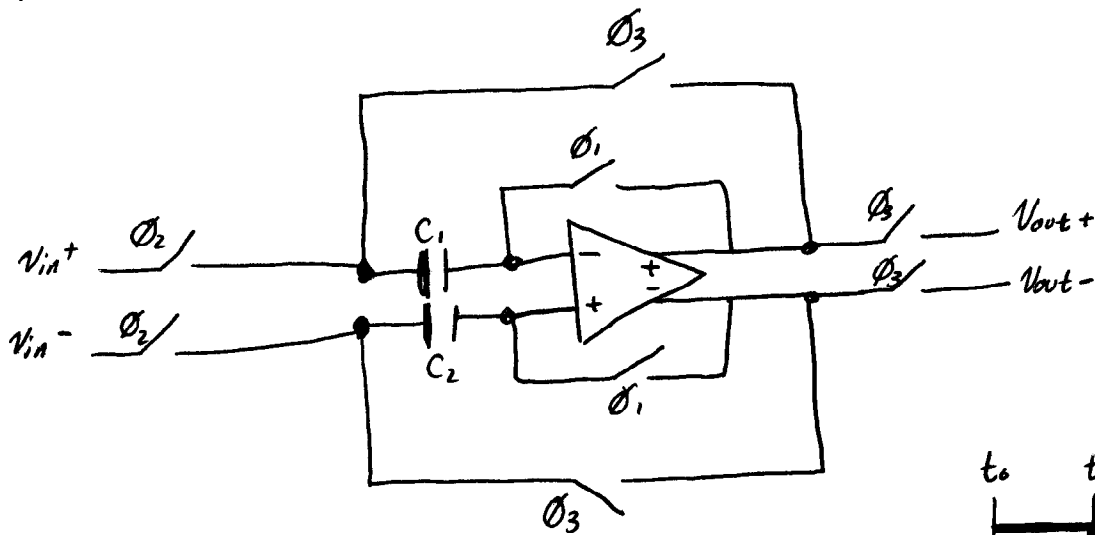
(34.9)

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①

DOES MATCHING OF THE CAPACITORS MATTER IN THE S/H OF FIG 34.31?



To make it easier to analyze. Let's have the input $V_{in} = V_{in}^+ - V_{in}^-$ be zero, or $V_{in}^+ = V_{in}^- = V_{cm}$. So for this example, the circuit will be sampling $v_{in} = 0$.

For $t < t_1$

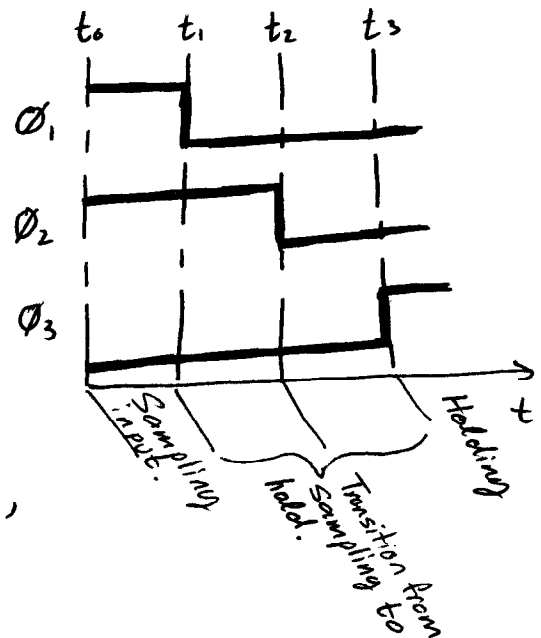
The charge stored on the storage capacitors, C_1 & C_2 is:

$$Q_1 = (V_{in}^+ - V_{cm})C_1 = (V_{cm} - V_{cm})C_1 = 0 = Q_{SAMPLED1}$$

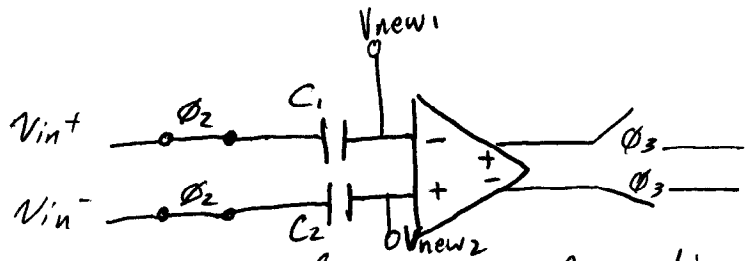
$$Q_2 = (V_{in}^- - V_{cm})C_2 = (V_{cm} - V_{cm})C_2 = 0 = Q_{SAMPLED2}$$

For $t_1 < t < t_2$

At the beginning of this period, the ϕ_1 switches turn off. Since the ϕ_1 switches are the same size and the Bias voltages are all the same, they should both inject the same amount of charge, δQ , to the inputs (the top plates of the capacitor) of the op-amp.



The S/H circuit for this time ($t_1 < t < t_2$) is as shown below.



The op-amp is in open-loop gain configuration, making it very sensitive to differential input voltages.

The charge δQ (from charge injection of Φ_1 switches turning off) is input to both inputs of the differential op-amp.

$$Q_{TOTAL1} = Q_{SAMPLED1} + \delta Q = \delta Q = (V_{in+} - V_{new1})C_1 = (V_{cm} - V_{new1})C_1$$

$$Q_{TOTAL2} = Q_{SAMPLED2} + \delta Q = \delta Q = (V_{in-} - V_{new2})C_2 = (V_{cm} - V_{new2})C_2$$

$$V_{new1} = V_{cm} - \frac{\delta Q}{C_1} \quad + \quad V_{new2} = V_{cm} - \frac{\delta Q}{C_2}$$

If $C_1 = C_2$ then $V_{new1} = V_{new2}$ and the signal from the Φ_1 switches turning off is common to both inputs and would ideally be rejected.

If $C_1 \neq C_2$ then the signals V_{new1} & V_{new2} aren't common & they, being different, could trigger the op-amp before the Φ_3 switches turn on

Answer: Yes, it does matter that the input capacitors are well matched.

A similar case can be made for the Φ_2 switches turning off.

Solution 34.10:

From fig. 34.34:

When ϕ_1 node is closed:

1. if $V_{os}=0V$, in this case

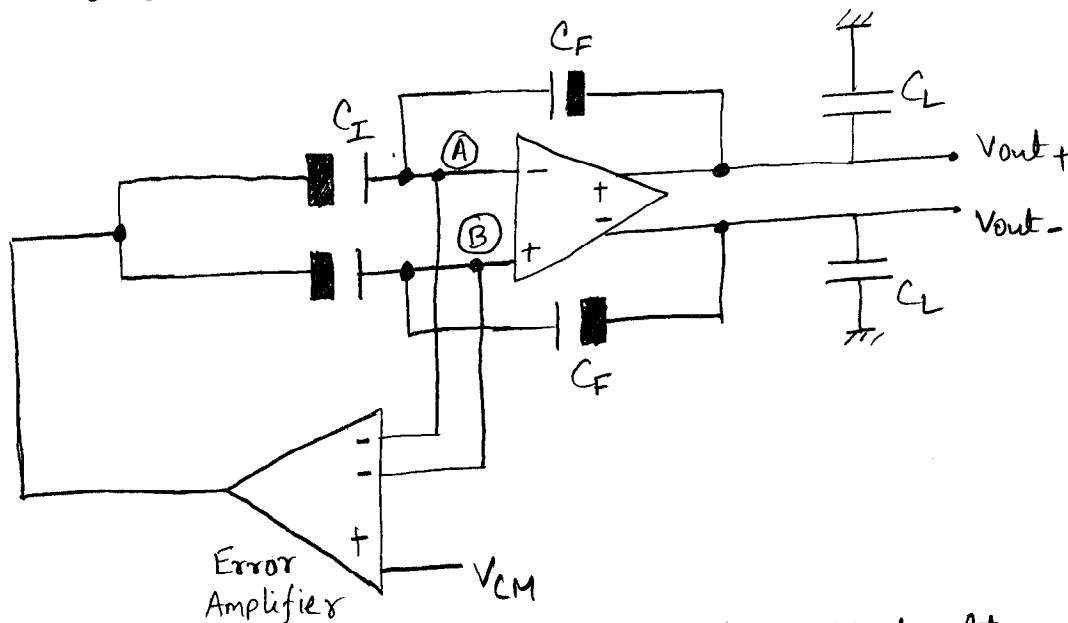
$$A_{ol} \Rightarrow \infty$$

If everything is ideal, input terminal of error amplifier would be at same potential.

2. If things are not ideal, if there is capacitors mismatching and switching imperfection, and op-amp gain is A_{ol} is not ∞ , offset voltage would store on C_I , when ϕ_1 closes. And in this case input terminals of the error amplifier could be at different potential.

Q: When the ϕ_3 switches are closed in Fig. 34.34, is it possible for the inverting inputs of the error amplifier to be at different potentials? Again, neglect offsets.

Ans: With the ϕ_3 switches closed (ϕ_1, ϕ_2 switches open), Fig 34.34 looks like



We see from the above figure that the op-amp is operating in a negative feedback configuration and hence the two inverting inputs of the error amplifier will always be at equal potential if we neglect op-amp offset.

If we have fully differential input signals and no error amplifier, the nodes (A) and (B) are held at V_{CM} .

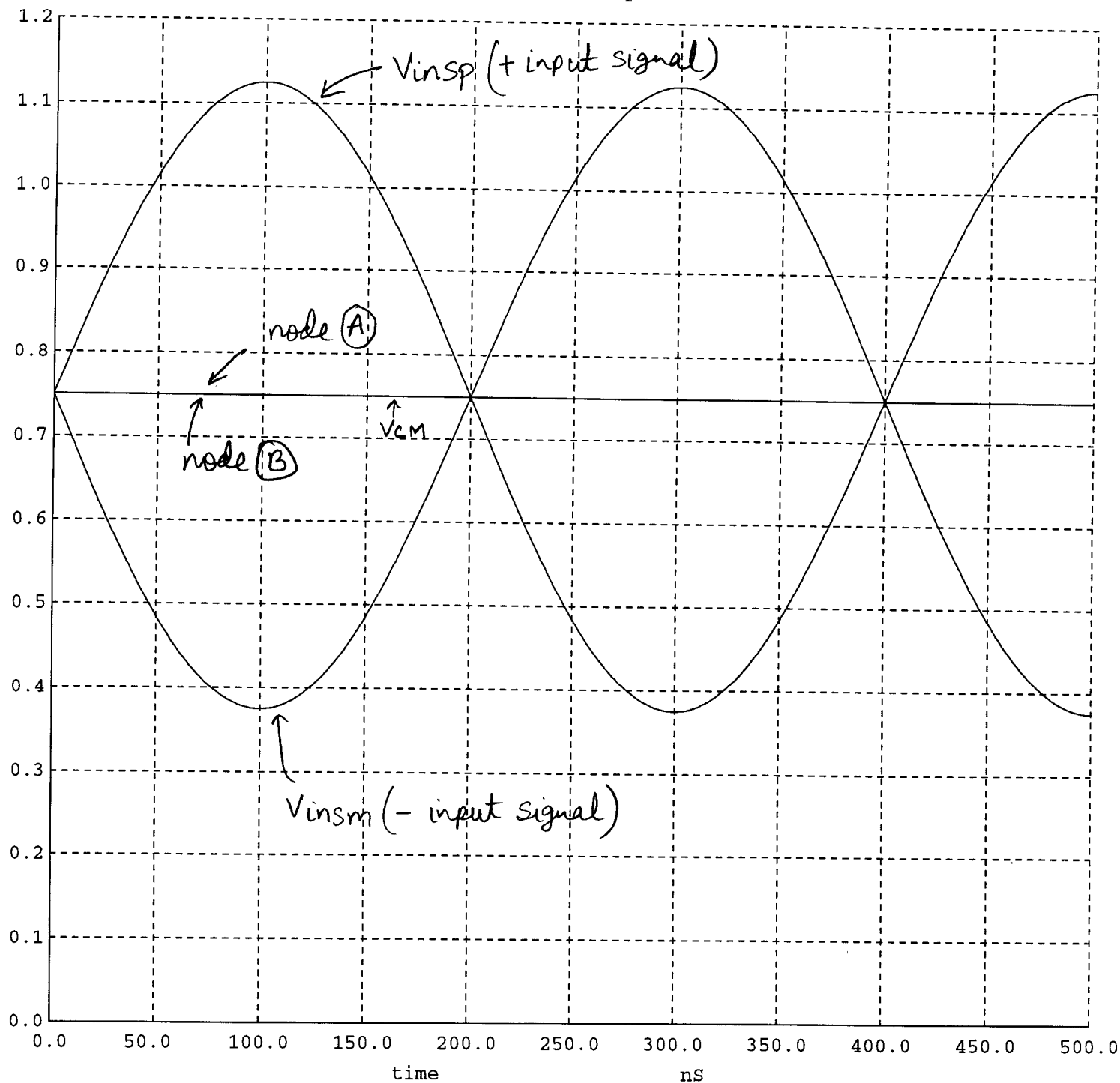
If we have single-ended input signal and an error amplifier with high gain, then also nodes (A) and (B) are at V_{CM} .

If we have single-ended input signal and do not connect the error amplifier, then nodes (A) and (B) vary with the input signal but are still always equal to each other.

The following SPICE plots illustrate the above statements

V ---- Vinm
 - - - Vinsm

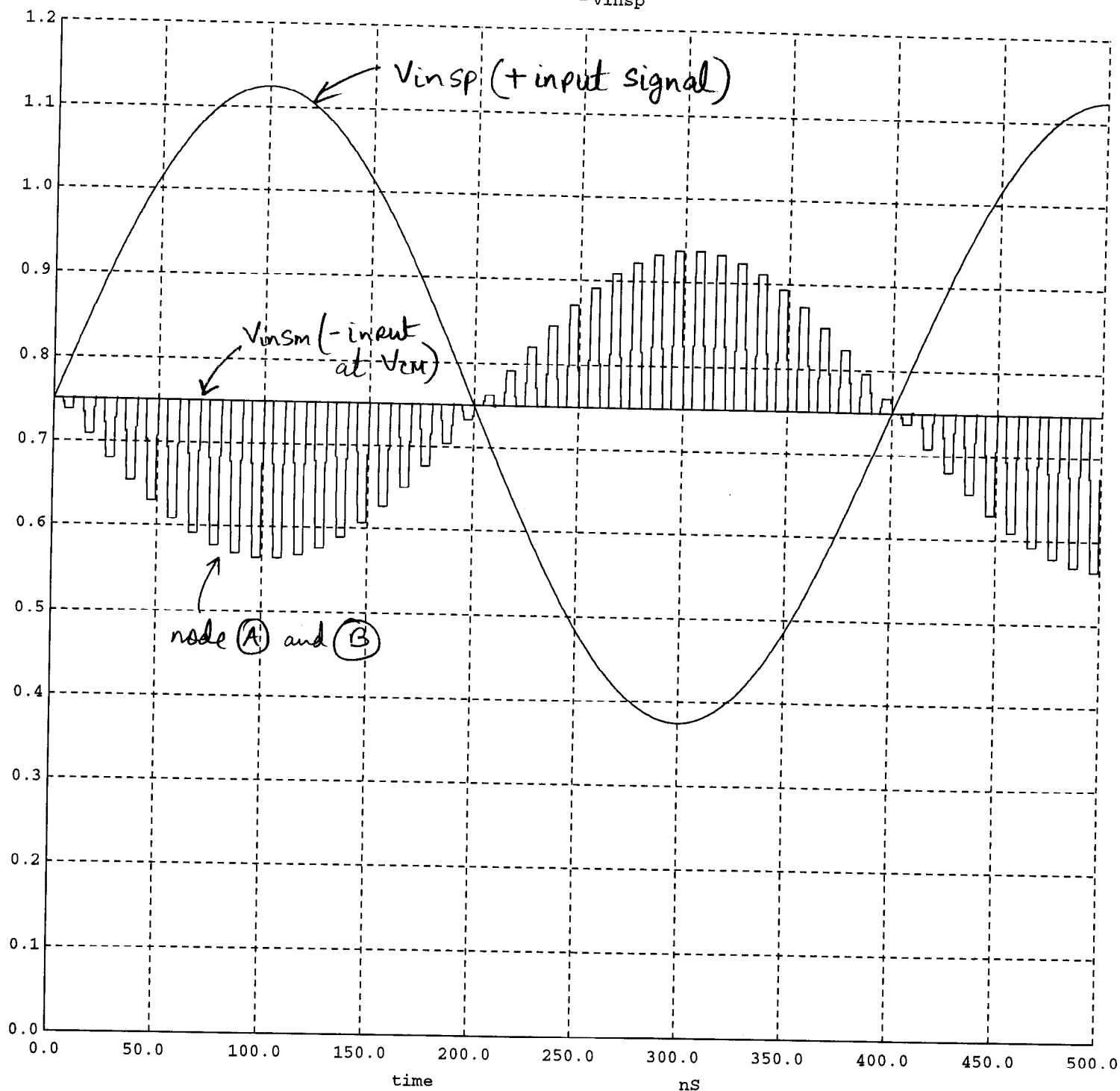
tran1: * Figure 34.34 CMOS: Mixed-Signal Circuit Design *
 ---- Vinp
 - - - Vinsp



PLOT1: No error amplifier, bulky differential inputs
 Nodes (A) and (B) held at $V_{cm} = 0.75V$

V --- Vinm
 --- Vinsm

tran1: * Figure 34.34 CMOS: Mixed-Signal Circuit Design *
 --- Vinp
 --- Vinsp



PLOT2: No error amplifier, Single ended input, other input connected to Vcm.
 Nodes (A) and (B) vary with input signal but are always equal to each other.

Problem 34.12

Question: Determine the output of the cyclic ADC of Fig. 34.38 if the input voltage is 0.41 Volts.

Answer: We follow the algorithm given in Ex. 34.9, in which we follow the cycle of comparing the input to the common-mode voltage, outputting a digital code, multiplying the voltage by 2, and feeding the resulting voltage back to the input to repeat the cycle. Below is a table showing this process, assuming that the converter is an 8-bit ADC. This process could be expanded or shortened depending on the resolution of the converter.

Cycle	Voltage	Vin>VCM?	Digital Output	Subtract VCM?	New Voltage	Multiply by 2
0	0.41	no	0	no	0.41	0.82
1	0.82	yes	1	yes	0.07	0.14
2	0.14	no	0	no	0.14	0.28
3	0.28	no	0	no	0.28	0.56
4	0.56	no	0	no	0.56	1.12
5	1.12	yes	1	yes	0.37	0.74
6	0.74	no	0	no	0.74	1.48
7	1.48	yes	1	yes	0.73	1.46

Thus the 8-bit output word would be 0100 0101 (binary offset) or 69_{10} . This can be verified by finding the LSB ($\text{LSB} = \text{VDD}/2^N$) which is equal to $1.5\text{V}/256 = 0.005889\text{V}$. We then multiply our digital output by 1 LSB to verify our result. Doing this we find that $V_{in} = .404297$. This is less than 1 LSB below our actual input, so we find that our result is correct.

Problem 34.13

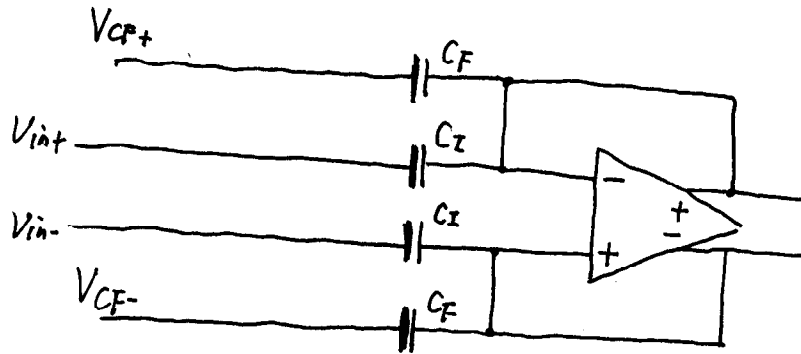
Is kickback noise from the comparator a concern for the circuit of Fig. 34.39?

Yes, the comparator used in Fig. 34.39 has significant kickback noise resulting from the latch positive feedback. The decision circuit of a voltage comparator uses positive feedback to increase the gain of the decision element. The switch noise coming from the positive feedback stage will corrupt the input of the comparator and often limit its performance in the actual circuit.

In the sample-and-hold implementation of Fig. 34.39, if the kickback noise is not isolated, it will feed directly into the input voltage of the op-amp in S/H circuit through the input capacitor C_I , and drastically affect the output of the sample-and-hold. In the Fig. 34.39, the switch ϕ_2 is added in series with the comparator input to disconnect the comparator when comparator is clocked. Since the ϕ_2 and ϕ_3 are nonoverlapping, ideally, kickback noise will have very little effect to the input of the op-amp in the S/H.

Derive the transfer function for the circuit in Fig 34.39.

⇒ when ϕ_s close (ϕ_h open)
[solution]:

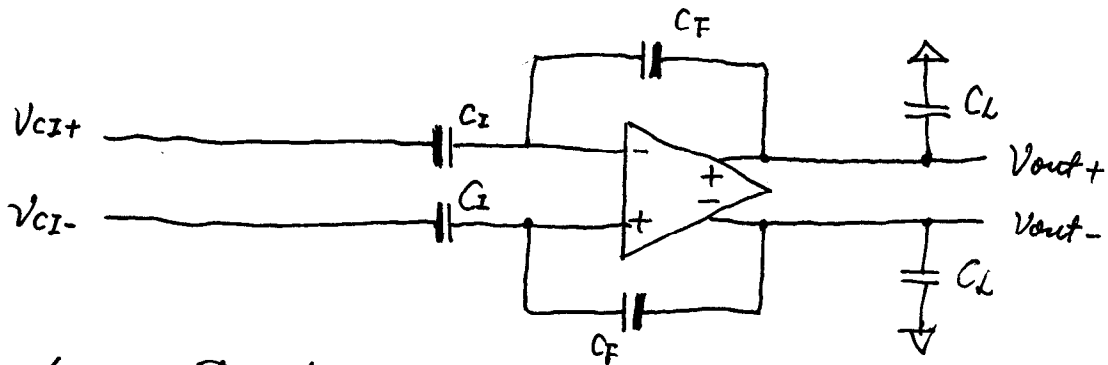


Charge stored on C_I, C_F when ϕ_s close, ϕ_h open:

$$Q_I^{\phi_s} = C_I \cdot (V_{in+} - V_{cm} \pm V_{os})$$

$$Q_F^{\phi_s} = C_F \cdot (V_{CF+} - V_{cm} \pm V_{os}) \quad (\text{only consider + for now})$$

⇒ when ϕ_s open, ϕ_h close



Charge stored on C_I, C_F now

$$Q_I^{\phi_h} = C_I \cdot (V_{CI+} - V_{cm} \pm V_{os})$$

$$Q_F^{\phi_h} = C_F \cdot (V_{out} - V_{cm} \pm V_{os})$$

Since total amount of charge unchanged.

$$Q_I^{\phi_s} + Q_F^{\phi_s} \equiv Q_I^{\phi_h} + Q_F^{\phi_h}$$

(chapter 34.14 cont)

$$\Rightarrow V_{out+} = V_{cm} \pm V_{os} =$$

$$\frac{C_I}{C_F} (V_{in+} - V_{cm} \pm V_{os}) + (V_{cf+} - V_{cm} \pm V_{os}) \\ - \frac{C_I}{C_F} (V_{in-} - V_{cm} \pm V_{os})$$

Considers fully-differential signal now:

$$\Rightarrow V_{out} = V_{out+} - V_{out-}$$

$$= \left[\frac{C_I}{C_F} (V_{in} - V_{cI}) + V_{cF} \right]$$

where

$$\begin{cases} V_{in} = V_{in+} - V_{in-} \\ V_{cI} = V_{cI+} - V_{cI-} \\ V_{cF} = V_{cF+} - V_{cF-} \end{cases}$$

34.15) Repeat Ex. 34.16 for $V_{in}=.41V$. We begin by following the algorithm given in the example.

Refer to Fig 34.50

Input value below $V_{cm}/2$ ($=0.375V$) our comparator outputs, ab , are 00 respectively.

Input value between $V_{cm}/2$ ($=.375V$) and $3V_{cm}/2$ ($=1.125V$) comparator outputs, ab , are 01 respectively.

Input value above $3V_{cm}/2$ ($=1.125V$) comparator outputs, ab , are 11 respectively.

From the book, Eq(34.59) or Eq(34.60), we can find the value for V_{out} . Note from the equations that we can never get a 00 followed by a 00, because we add V_{cm} , or a 11 followed by a 11, because we subtract V_{cm} , we can however, get repeated values for ab of 01.

For the digital outputs we follow Eq (34.67) to Eq (34.72).

V_{in}	Comparator Outputs, ab	V_{out}	Digital Output
.41 (n-1)	a=0 b=1	.07	$b_7=1$ $c_7=0$ $b_8=0$ MSB
.07 (n-2)	a=0 b=0	.89	$b_6=1$ $c_6=0$
.89 (n-3)	a=0 b=1	1.03	$b_5=0$ $c_5=1$
1.03 (n-4)	a=0 b=1	1.31	$b_4=0$ $c_4=1$
1.31 (n-5)	a=1 b=1	.37	$b_3=0$ $c_3=0$
.37 (b_2)	a=0 b=0	1.49	$b_2=1$ $c_2=0$
1.49 (b_1)	a=1 b=1	.73	$b_1=0$ $c_1=0$
.73 (b_0)	a=0 b=1	.71	$b_0=1$ $c_0=0$ LSB

Our digital output word is 0 1100 0101 (197dec)

We subtract V_{cm} -.5LSB from the output word to maintain accuracy 0 0111 1111 (127dec)

Digital Output
0 1100 0101 (binary offset)

V_{cm} -.5LSB
0 0111 1111 (binary offset)

```

0 1100 0101
1 1000 0000
+          1
1 0 0100 0110

```

The 10th bit is a don't care condition and we can throw it out so we are left with the digital output equal to 70(dec).

To verify, we multiply the digital output (70dec) by 1LSB, $70 * 1.5/256 = .41V$. This is the original input so the digital output is correct.

34.16 solution.

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Determine the output of the Cyclic ADC if the input voltage is 0.41v. The comparators switch at 0.4 (a 25mV offset) and 1.05 V (a 75mV offset).

The Digital output is calculated as shown below...

V_{in}	Comparator outputs, ab		V_{out}	Digital Out		
0.41	$a_7=$ 0	$b_7=$ 1	$(0.41-0.375)*2 = 0.07$	$Bit_7=$ 1	$c_7=$ 0	$Bit_8=$ 0
0.07	$a_6=$ 0	$b_6=$ 0	$(0.07*2)+0.75 = 0.89$	$Bit_6=$ 1	$c_6=$ 0	
0.89	$a_5=$ 0	$b_5=$ 1	$(0.89-0.375)*2 = 1.03$	$Bit_5=$ 0	$c_5=$ 1	
1.03	$a_4=$ 0	$b_4=$ 1	$(1.03-0.375)*2 = 1.31$	$Bit_4=$ 0	$c_4=$ 1	
1.31	$a_3=$ 1	$b_3=$ 1	$(1.31-1.125)*2 = 0.37$	$Bit_3=$ 0	$c_3=$ 0	
0.37	$a_2=$ 0	$b_2=$ 0	$(0.37*2) + 0.75 = 1.49$	$Bit_2=$ 1	$c_2=$ 0	
1.49	$a_1=$ 1	$b_1=$ 1	$(1.49-1.125)*2 = 0.73$	$Bit_1=$ 0	$c_1=$ 0	
0.73	$a_0=$ 0	$b_0=$ 1	$(0.73-0.375)*2 = 0.71$	$Bit_0=$ 1	$c_0=$ 0	

The resulting Digital output from MSB to LSB is: 0 1100 0101

The Details for calculating the digital output are shown below, which are from equations 34.67-34.72. Equations 34.67-34.72 for an 8-bit ADC are:

$$Bit_0 = \bar{a}_0 b_0 = 1$$

$$c_0 = a_0 = 0$$

$$Bit_1 = \bar{a}_1 b_1 \oplus c_0 = 0$$

$$c_1 = \bar{a}_1 b_1 c_0 = 0$$

$$Bit_2 = \bar{a}_2 b_2 \oplus a_1 \oplus c_1 = 1$$

$$c_2 = \bar{a}_2 b_2 a_1 + c_1 (\bar{a}_2 b_2 + a_1) = 0$$

$$Bit_3 = \bar{a}_3 b_3 \oplus a_2 \oplus c_2 = 0$$

$$c_3 = \bar{a}_3 b_3 a_2 + c_2 (\bar{a}_3 b_3 + a_2) = 0$$

$$Bit_4 = \bar{a}_4 b_4 \oplus a_3 \oplus c_3 = 0$$

$$c_4 = \bar{a}_4 b_4 a_3 + c_3 (\bar{a}_4 b_4 + a_3) = 1$$

$$Bit_5 = \bar{a}_5 b_5 \oplus a_4 \oplus c_4 = 0$$

$$c_5 = \bar{a}_5 b_5 a_4 + c_4 (\bar{a}_5 b_5 + a_4) = 1$$

$$Bit_6 = \bar{a}_6 b_6 \oplus a_5 \oplus c_5 = 1$$

$$c_6 = \bar{a}_6 b_6 a_5 + c_5 (\bar{a}_6 b_6 + a_5) = 0$$

$$Bit_7 = \bar{a}_7 b_7 \oplus a_6 \oplus c_6 = 1$$

$$c_7 = \bar{a}_7 b_7 a_6 + c_6 (\bar{a}_7 b_7 + a_6) = 0$$

$$Bit_8 = a_7 \oplus c_7 = 0$$

Bit 8 is not used. So the output is 1100 0101₂ (which is 197₁₀).

If you subtract $V_{cm} - \frac{1}{2}LSB$ (127₁₀) from the Digital output the result is:

$$0100\ 0110_2\ (70_{10})$$

To check that the Digital output is correct, the Digital Output- V_{cm} multiplied by 1LSB[Volts] should give the same, or at least $\frac{1}{2}$ LSB difference from the original input, which was 0.41 volts.

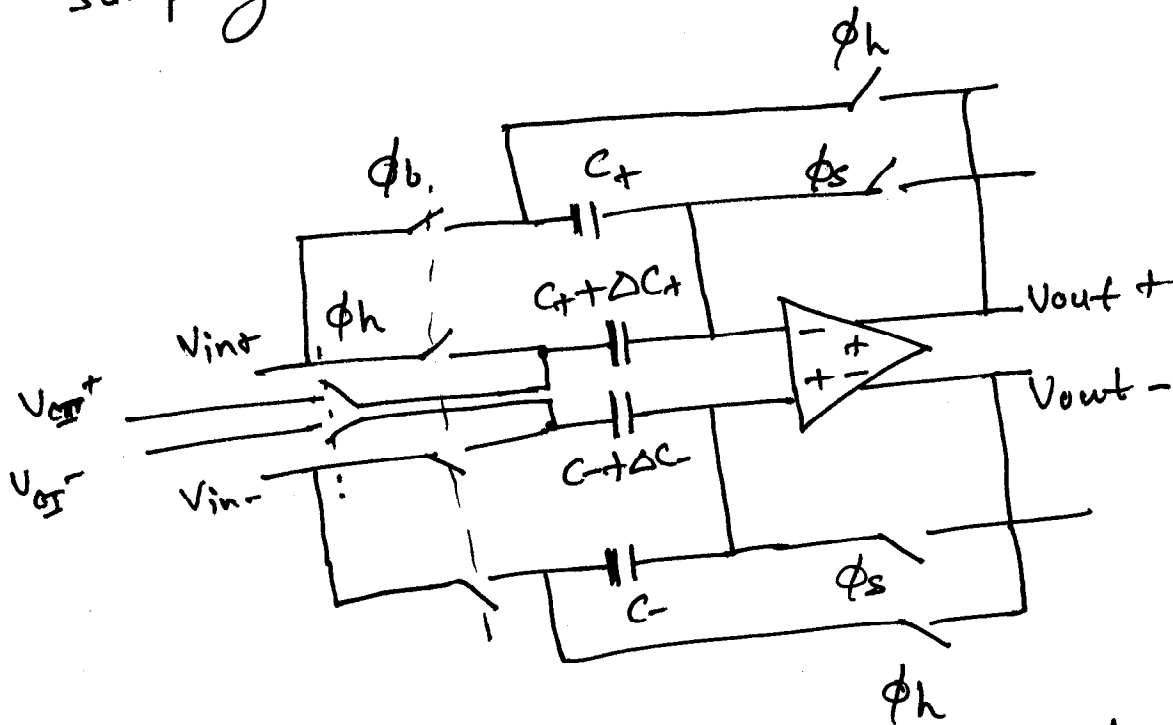
$$V_{1LSB} = 1.5V/2^8 = 5.86mV$$

$$70 * 5.86mV = 0.4102\ Volts\ which\ is\ only\ 0.0002\ greater\ than\ the\ input.$$

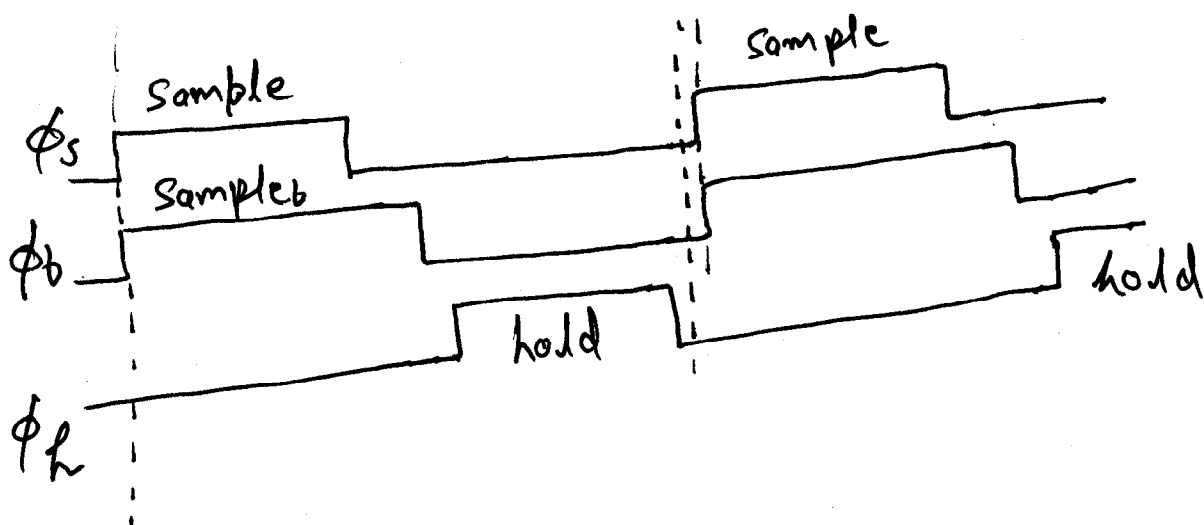
This results even with the comparator offsets not ideal. The ideal comparator for the same input voltage (0.41V) is analyzed in the previous problem.

Solution - 34.17

Fig 34.54 would be as below if bottom plate sampling used.

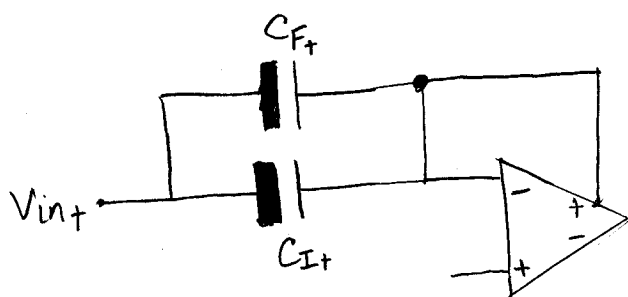


and clock waveform for this ckt would be:



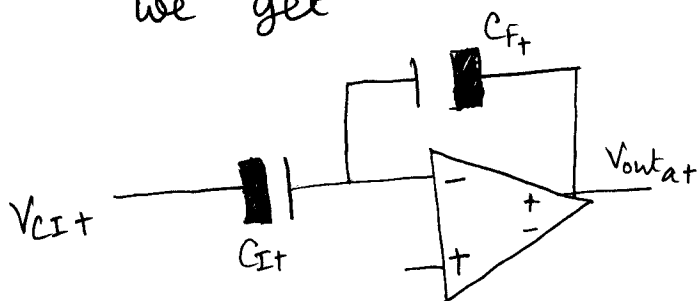
Q: Show the derivation leading up to ^{Eqn} 34.83. Show using practical values for mismatch, how the squared mismatch terms are negligible.

Ans: Redrawing fig. 34.55 for the sampling phase (ϕ_s switches closed) we get the following figure. We draw all circuits only for the positive input knowing that the ^{equations} same holds true for the negative input when the input signals are fully differential.



Sample phase

For the amplify phase, when ϕ_a switches are closed, we get



Amplify phase

The total charge in the sample phase can be written as

$$Q_{\text{sample}} = (C_{I+} + C_{F+})(V_{in+} - V_{CM} + V_{os})$$

where V_{os} = offset voltage of op-amp

The total charge

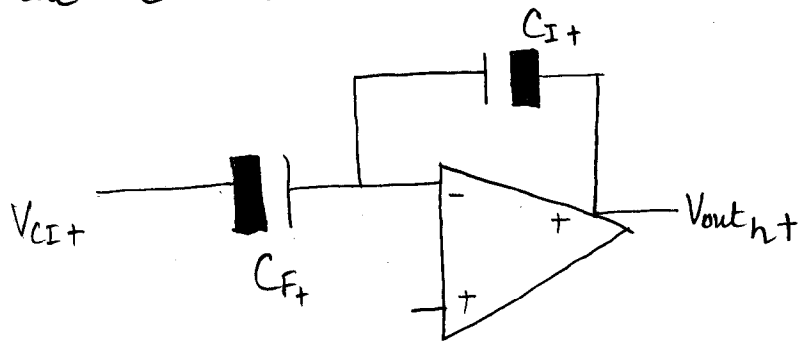
$$Q_{\text{amp}} = C_{I+}(V_{CI+} - V_{CM} + V_{os}) + C_{F+}(V_{out+} - V_{CM} + V_{os})$$

Since charge must be conserved, equating Q_{sample} and Q_{amp} , we get

$$C_{I+}(V_{in+} - V_{CI+}) + C_{F+}(V_{in+} - V_{out+}) = 0$$

$$\text{or } \boxed{V_{in+}(C_{F+} + C_{I+}) - C_{F+} \cdot V_{out+} = C_{I+} V_{CI+}} \quad \text{--- eqn (a)}$$

In the hold phase when the ϕ_n switches are closed, the circuit becomes



Note that the C_{F+} and C_{I+} capacitors have interchanged their positions wot the amplify phase.

Hold phase

Total charge in hold phase

$$Q_{\text{hold}} = C_{I+} (V_{\text{outh+}} - V_{\text{cm}} \mp V_{\text{os}}) + C_{F+} (V_{\text{CI+}} - V_{\text{cm}} \mp V_{\text{os}})$$

Equating Q_{hold} and Q_{amp} we get

$$C_{I+} (V_{\text{CI+}} - V_{\text{outh+}}) + C_{F+} (V_{\text{out+}} - V_{\text{CI+}}) = 0$$

$$\text{or } \boxed{C_{F+} V_{\text{out+}} - C_{I+} V_{\text{outh+}} = V_{\text{CI+}} (C_{F+} - C_{I+})} \quad - \text{eqn (b)}$$

Adding eqns (a) and (b), we eliminate $V_{\text{out+}}$ and get

$$V_{\text{in+}} (C_{F+} + C_{I+}) - \cancel{C_{F+} V_{\text{out+}}} + \cancel{C_{F+} V_{\text{out+}}} - C_{I+} V_{\text{outh+}} = V_{\text{CI+}} (C_{F+} - \cancel{C_{I+}} + C_{I+})$$

$$\text{or } V_{\text{outh+}} = V_{\text{in+}} \cdot \left(1 + \frac{C_{F+}}{C_{I+}}\right) - V_{\text{CI+}} \cdot \frac{C_{F+}}{C_{I+}}$$

Putting $C_{I+} = C_+ + \Delta C_+$ and $C_{F+} = C_+$ where ΔC_+ indicates the mismatch between C_{F+} and C_{I+} we get

$$V_{\text{outh+}} = V_{\text{in+}} \cdot \left(1 + \frac{C_+}{C_+ + \Delta C_+}\right) - V_{\text{CI+}} \cdot \frac{C_+}{C_+ + \Delta C_+}$$

$$\boxed{V_{\text{outh+}} = V_{\text{in+}} \cdot \left(1 + \frac{1}{1 + \frac{\Delta C_+}{C_+}}\right) - V_{\text{CI+}} \cdot \left(\frac{1}{1 + \frac{\Delta C_+}{C_+}}\right)} \quad - \text{eqn (c)}$$

Now we know that $\frac{1}{1+x} = (1+x)^{-1} = 1 - x + x^2 - x^3 + \dots$

(using binomial expansion)

$$\therefore \text{writing } \frac{1}{1 + \frac{\Delta C_+}{C_+}} = \left(1 + \frac{\Delta C_+}{C_+}\right)^{-1} = 1 - \left(\frac{\Delta C_+}{C_+}\right) + \left(\frac{\Delta C_+}{C_+}\right)^2 - \left(\frac{\Delta C_+}{C_+}\right)^3 + \dots$$

But ΔC_+ = mismatch between capacitors and is typically much less than C_+ , the nominal capacitor value.

$$\therefore \frac{\Delta C_+}{C_+} \ll 1$$

Hence, the higher order terms $\left(\frac{\Delta C_+}{C_+}\right)^2$, $\left(\frac{\Delta C_+}{C_+}\right)^3$ etc are small and can be neglected.

Eg: if the typical capacitor mismatch in a process technology is 5%, then $\frac{\Delta C_+}{C_+} = 0.05$, $\left(\frac{\Delta C_+}{C_+}\right)^2 = 0.0025$, $\left(\frac{\Delta C_+}{C_+}\right)^3 = 0.000125$ etc.

Hence we can neglect the higher order terms and write

$$\frac{1}{1 + \frac{\Delta C_+}{C_+}} \approx 1 - \left(\frac{\Delta C_+}{C_+}\right)$$

Making the above substitution in eqn (c), we get

$$V_{out+} = V_{int.} \left(1 + 1 - \frac{\Delta C_+}{C_+}\right) - V_{C_{I+}} \left(1 - \frac{\Delta C_+}{C_+}\right)$$

$$V_{out+} = (2V_{int.} - V_{C_{I+}}) - \frac{\Delta C_+}{C_+} (V_{int.} - V_{C_{I+}})$$

Which is Eq. 34.83.

Problem 34.19

What happens to the error adjustment term in Equation 34.92 if the capacitors in the Sample-and-Hold are perfectly matched?

Solution

First of all, here is Equation 34.92 from the book:

$$V_{avg+} - V_{avg-} = V_{out+} - V_{out-} - \underbrace{\frac{(V_{out+} - V_{outh+}) - (V_{out-} - V_{outh-})}{2}}_{\text{Error adjustment term}} \quad (34.92)$$

It says in the paragraph above the equation in the book that if the capacitors are perfectly matched then $V_{out+} = V_{outh+}$, and $V_{out-} = V_{outh-}$, making the error adjustment term disappear, but let's verify with previous equations.

The V_{out+} and V_{outh+} terms refer to V_{out} of the sample-and-hold when two of the three phases of the clock are high ($V_{out+} = V_{out}$ during amplify phase ϕ_a , and $V_{outh+} = V_{out}$ during the hold phase ϕ_h). Let's see how V_{out+} and V_{outh+} relate to each other. Also I will neglect V_{CI} terms, assuming $V_{CI+} = V_{CI-} = V_{cm}$.

(Equation 34.80) $\rightarrow V_{outh+} = \cancel{V_{CI+}} + \frac{C_+}{C_+ + \Delta C_+} V_{out+} + \frac{C_+}{C_+ + \Delta C_+} \cancel{V_{CI+}}$

Thus $V_{outh+} = \frac{C_+}{C_+ + \Delta C_+} V_{out+}$ (the same equation applies to V_{outh-} and V_{out-})

If the capacitors are perfectly matched, $\Delta C_+ = \Delta C_- = 0$, and $V_{outh+} = V_{out+}$, thus making our error adjustment term equal to zero.

Q. 34.20

P. K. Irkar

HW 1

prob: 34.20

Repeat Ex. 34.18 if all capacitors are 1 pF (the ideal sim.) and verify that the error out of the stage is zero.

Ans:

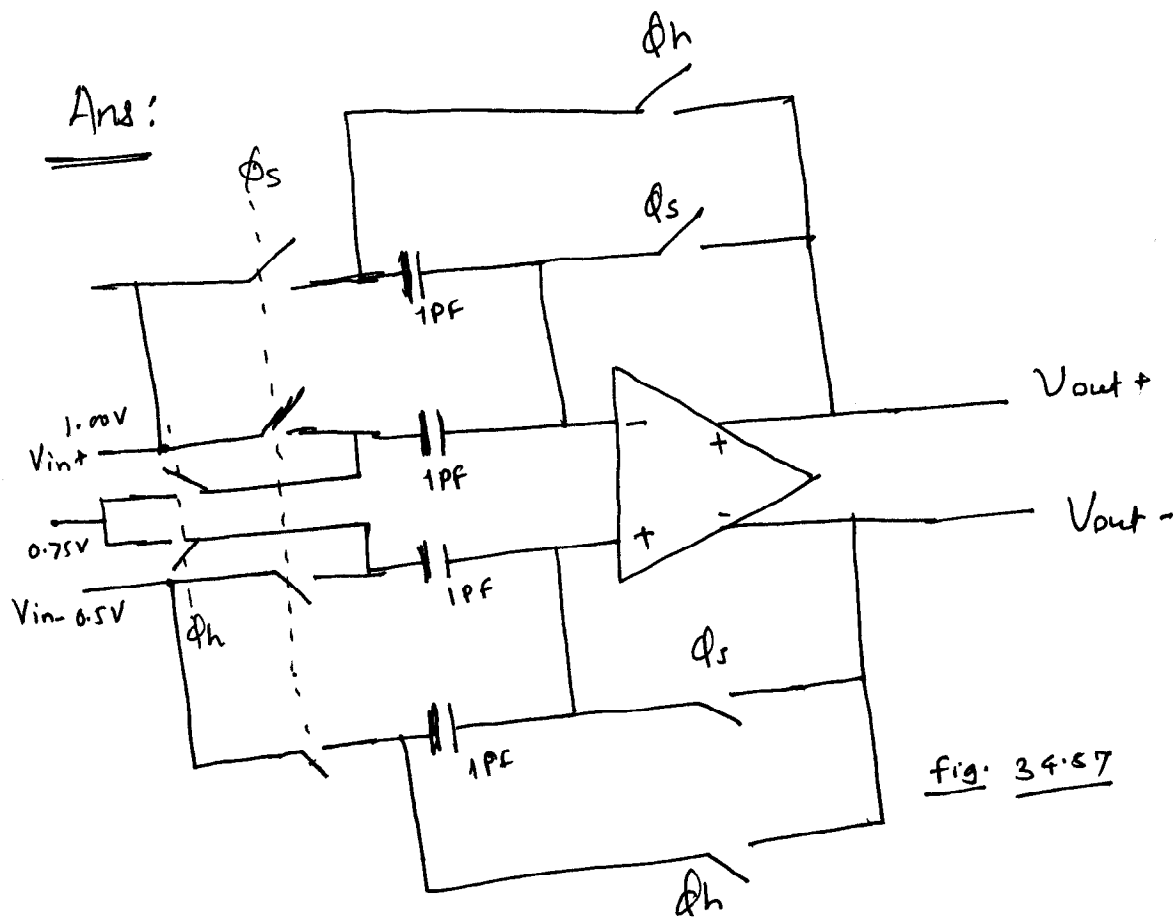


fig. 34.57

The capacitors are equal and input voltage $V_{in+} - V_{in-} \Rightarrow 0.5V$
Ideally the output is then 1V.

$$\begin{aligned} V_{out+} &\approx 1.25V \\ V_{out-} &= 0.25V \end{aligned}$$

$$\underline{V_{out+} - V_{out-} \Rightarrow 1V}$$

$$\text{Then Error} \approx \text{Ideal output} - (V_{out+} - V_{out-})$$

$$= 1V - 1V$$

$$= \underline{\underline{0}}$$

Simulation also shows the same as I've described above.

Sample and hold circuit

```
.control  
destroy all  
run  
.endc
```

*Op-amp using VCVS

```
E1 Voutp Vcom Vplus Vminus 100MEG  
E2 Vcom Voutm Vplus Vminus 100MEG
```

*Sample and hold capacitors

```
Cim Vminus Vcomm 1p  
Cfm Vminus Vcenm 1p  
Cip Vplus Vcomp 1p  
Cfp Vplus Vcenp 1p
```

*switches

```
S1 Vcenm Vsigg phis Vswit switmod  
S2 Vcomm Vsigg phis Vswit switmod  
S3 Vcomm Vcom phih Vswit switmod  
S4 Vcomp Vcom phih Vswit switmod  
S5 Vcomp Vsigg phis Vswit switmod  
S6 Vcenp Vsigg phis Vswit switmod  
S7 Voutp Vcenm phih Vswit switmod  
S8 Voutp Vminus phis Vswit switmod  
S9 Voutm Vplus phis Vswit switmod  
S10 Voutm Vcenp phih Vswit switmod
```

*supply voltage

```
VDD VDD 0 DC 1.5
```

*reference voltages

```
Vswit Vswit 0 DC 0.75  
Vcom Vcom 0 DC 0.75
```

*Input Signals

```
Vsigg Vsigg 0 DC 0 sin 0.75 1.0 2.5MEG  
Vsigm Vsigm 0 DC 0 sin 0.75 0.5 2.5MEG
```

*Ideal signal

```
Videal Videal 0 DC 0 sin 0 1.0 2.5MEG
```

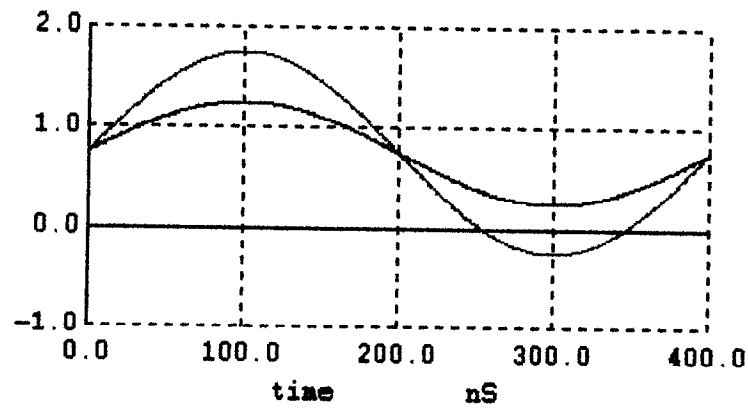
*Clock Signals

```
vphis phis 0 DC 0 Pulse 0 1.5 0 200p 200p 4n 10n  
Vphih phih 0 DC 0 Pulse 0 1.5 5n 200p 200p 4n 10n  
Rs phis 0 1MEG  
Rh phih 0 1MEG
```

```
.tran 2n 400n 0 2n UIC  
.model switmod SW RON=1  
.end
```

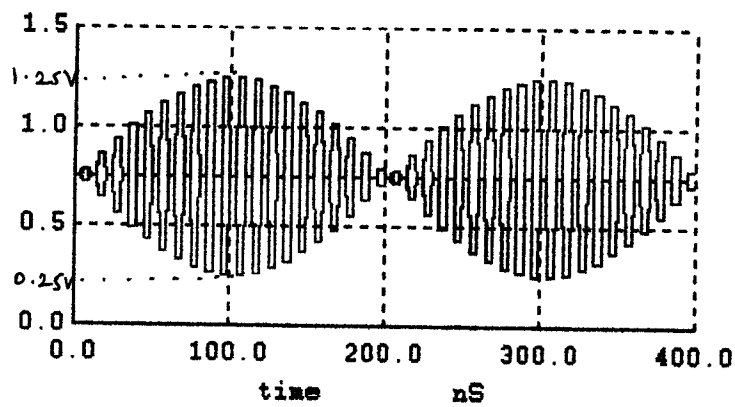
Input Signals

V — vsiga — vsigp



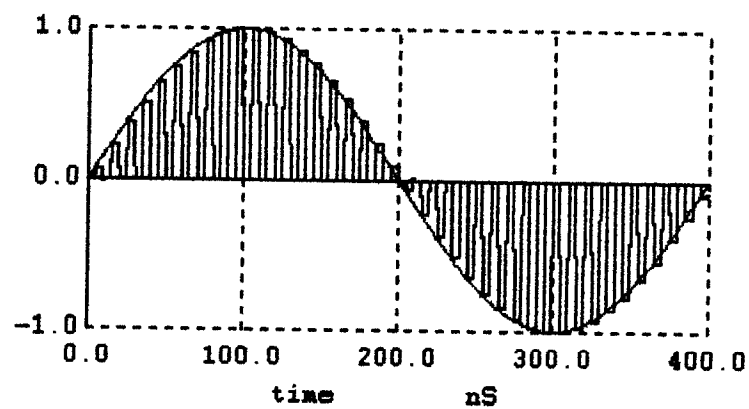
Output signals

V — voutp — voutm



Ideal signal and Actual (Voutp-Voutm) Signal

V — voutp-voutm — videal



Here the Ideal o/p and Actual ($V_{outp} - V_{outm}$) are same means that the error

$$= \text{Ideal} - (V_{outp} - V_{outm})$$

$$= 0$$

Chapter 34.21. Sketch a circuit to provide the inputs for the four-phase, non-overlapping clock generator shown in Figure. 34.81

[Solution]: Based on state diagram / truth table as following, one of the possible solution circuits can be designed as indicated: (attached simulation results). Note: this circuit can be triggered on both clock rising and falling edges in real design to improve speed. For demo purpose, we only use rising-edge triggering scheme.

Clock Pulse	Q_1	Q_0
Initially	0	0
1	0	1
2	1	0
3	1	1
4 (recycles)	0	0

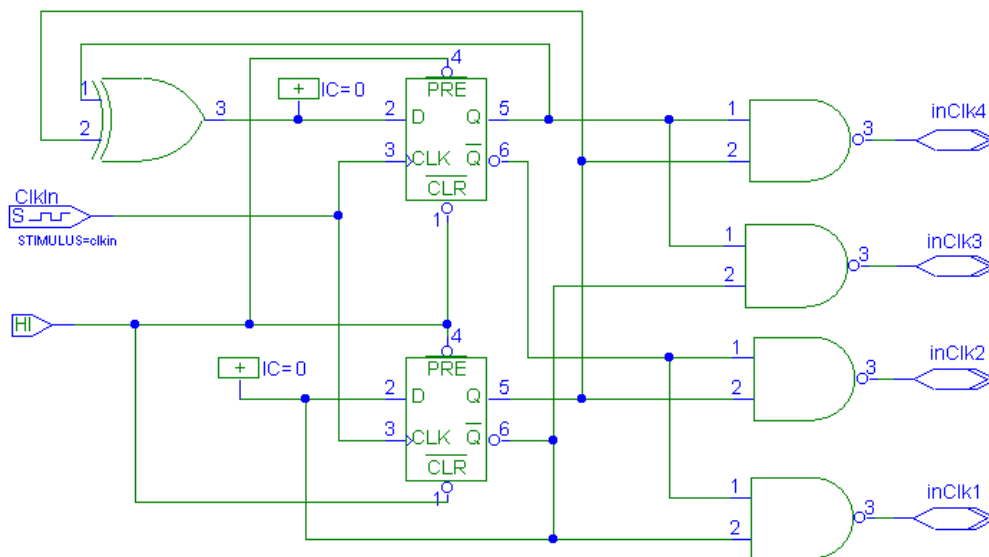
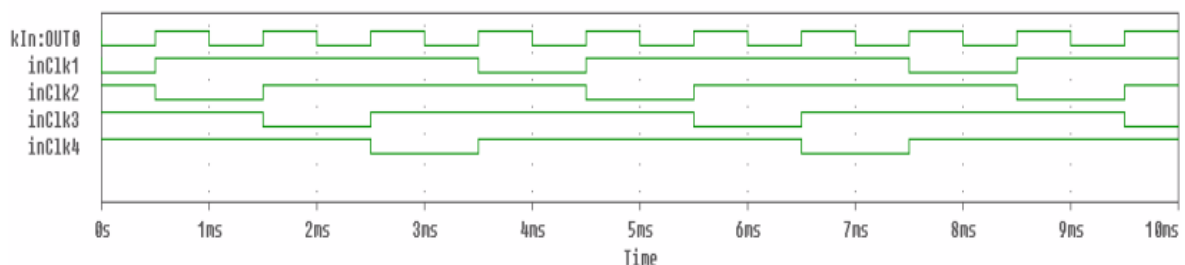


Figure 1: Four-Phase overlapping Clock Generator Circuit Design

Circuit Simulation for Four-Phase, non-overlapping clock Generator Inputs:
(First line: system input clock)



Chapter 34.22. What is the main advantage of using dynamic CMFB over other CMFB circuits? What is the main disadvantage?

[Answer]: Generally speaking, CMFB circuit is required to precisely balance the outputs of differential amplifier. In the meantime, issues associated with CMFB would be problems if a. CMFB doesn't affect each output the same → causing a difference mode signal; b. circuit becomes complex, CMFB circuit CMR would impose some limitation on initial Op-Amp design if not careful.

Main advantage of dynamic CMFB: It allows a lower CMFB loop gain thus making it easier to stabilize, without degrading common-mode voltage accuracy theoretically. It does not employ an amplifier. Unlike other CMFB circuits, dynamic CMFB makes it feasible op-amp's outputs not necessarily to settle down to the final accuracy of ADC by the falling edge of all three clock-phases. More exactly, during sampling phase settling time is meaningless since Op-Amp operates in open loop; inputs of Op-Amp are shorted to common-mode voltage, instead.

Main disadvantage of dynamic CMFB:

Dynamic CMFB would require to be compensated depending on the design parameters. It practically limits the gain of first stage otherwise would saturate outputs, causing offset storage mechanism fail.

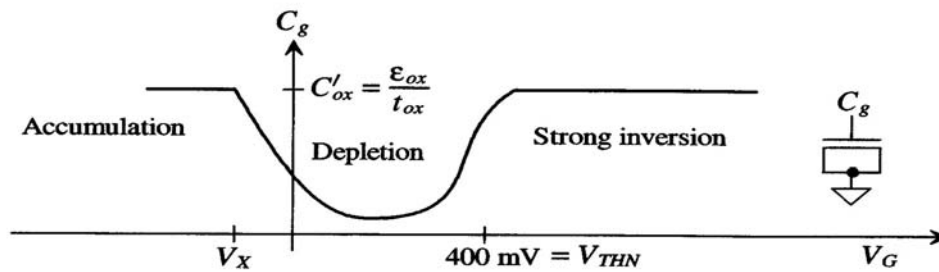
Problem 34.23

Can MOSFETs be used to implement the on-chip decoupling capacitors in Fig 34.77?

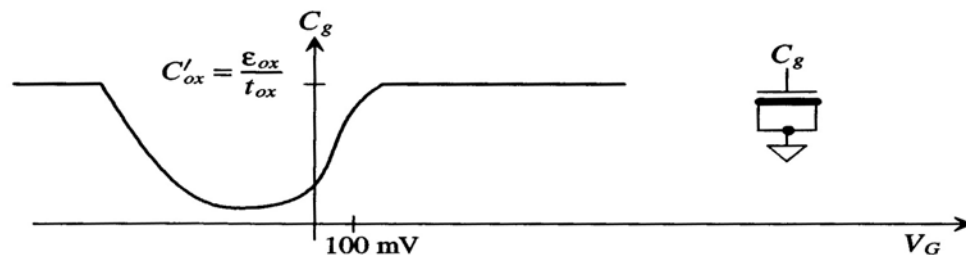
MOSFETs can be used as the on-chip decoupling capacitors for the power and ground supplies. MOSFETs usually make a very good capacitor when $V_{GS} > V_{THN} + \text{a few hundred mV}$. In Fig 34.77, the gate of decoupling MOSFET capacitor is connected to the VDD and source/drain connected to the ground. So, $V_{GS} (=VDD)$ is sufficient large to have MOSFET working in the strong inversion region (good cap area).

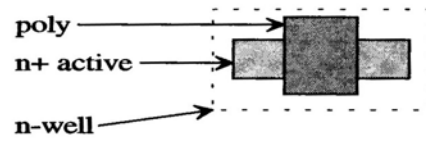
The figure below shows the CV curve for an NMOS transistor with source, drain and body connected to ground. The capacitance between the gate and source/drain is

$$C_{tot} = C_{ox} \cdot W \cdot L$$

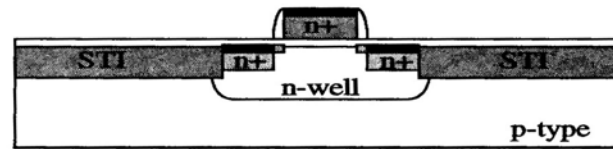


The following figure gives the CV curve for an NMOS with an n-well placed below. This MOSFET capacitor has a very low V_{THN} . To behave as a capacitor, MOSFET required operating in the strong inversion region. If V_{GS} is close to the V_{THN} , then the capacitance can become nonlinear and distortion can happen in the circuit.





(a) Layout view

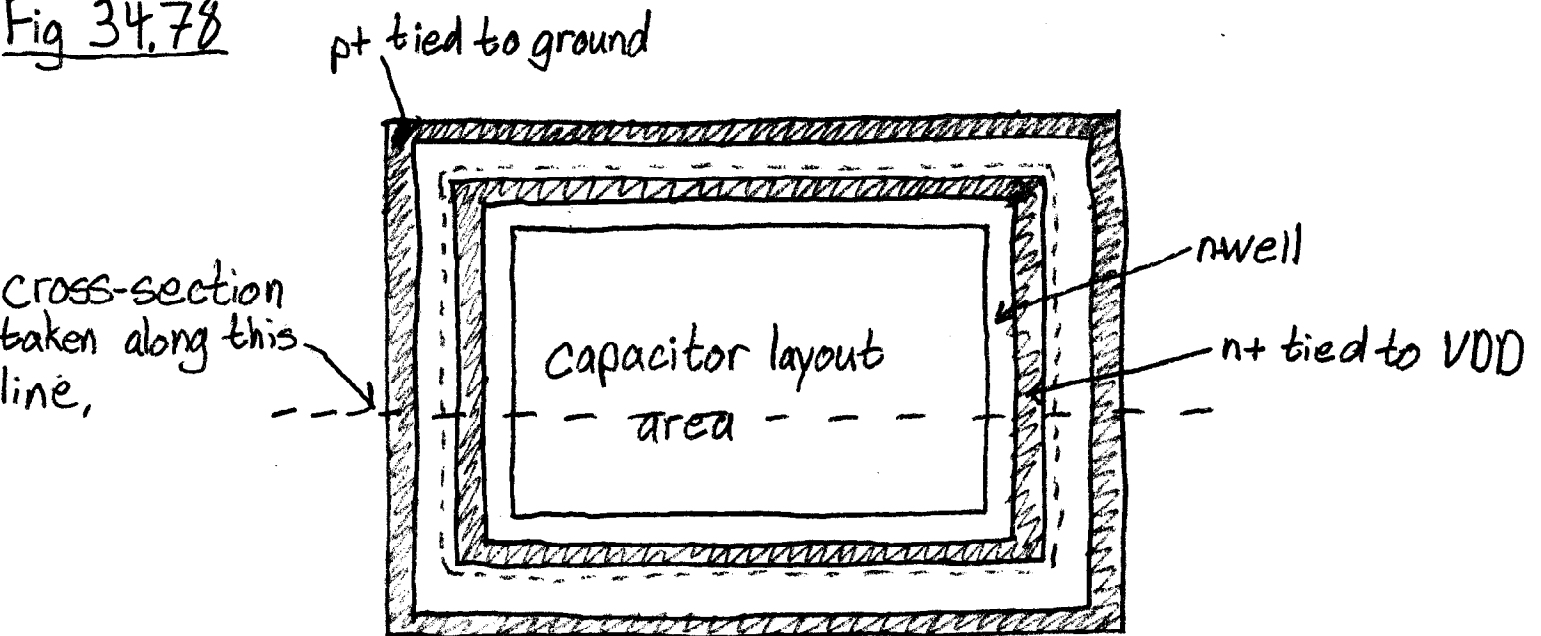


(b) Cross-sectional view

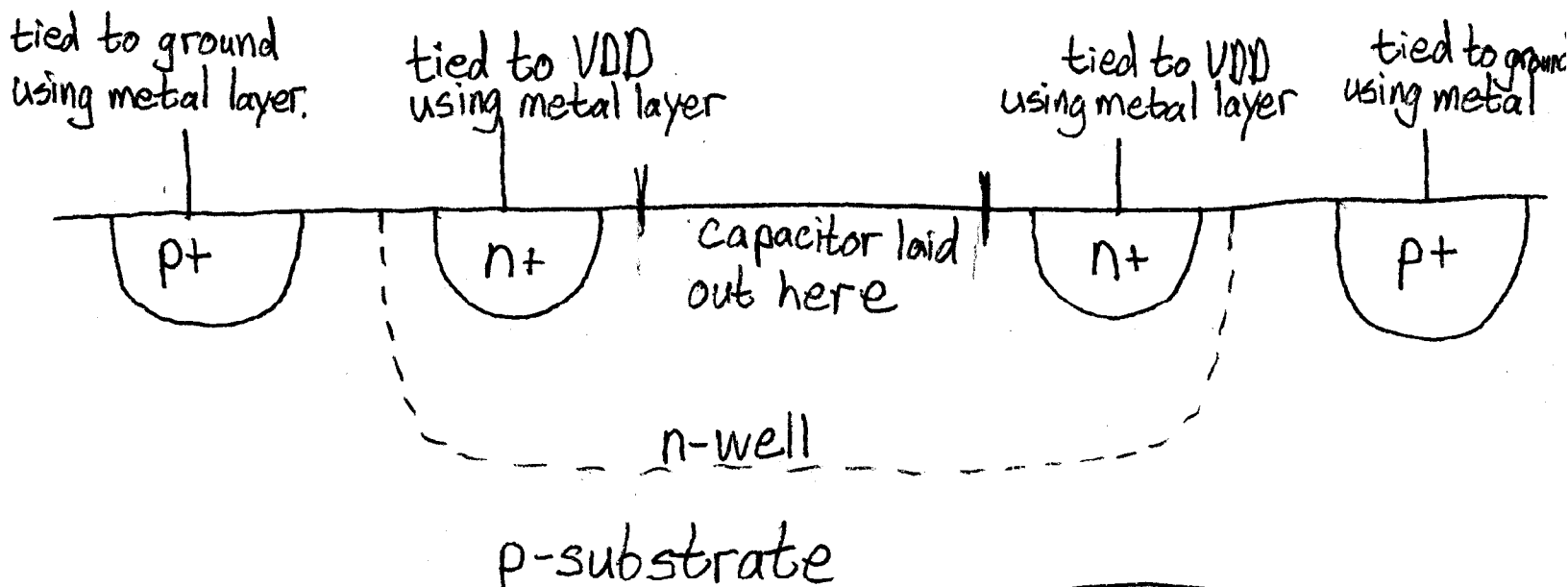
The layout and cross-section of the n-well NMOS capacitor shows above. This type of capacitor is widely used as decoupling capacitor when two layers of poly (like a Flash memory process) are not available in the process.

Problem 34.24

Sketch the cross-sectional view of layout in Fig. 34.78,
Fig 34.78



This cross-sectional area will be drawn assuming no metal has been connected on the chip, though I will indicate where metal will be put.

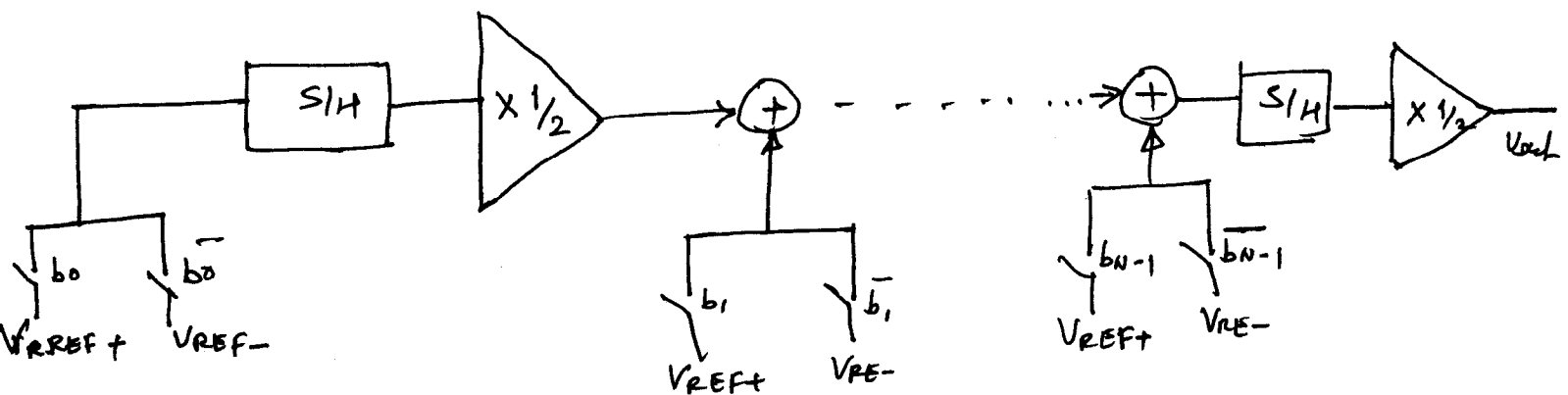


By setting the $c_f/c_f = 1/2$; we can achieve the gain is equal to $1/2$ which is required DAC pipeline implementation.

Since $c_f = 2c_f$; in ϕ_1/ϕ_2 the v_{tg} across $c_f = v_{in} - v_{cm}$ and

the ϕ_3 the output becomes $1/2(v_{in} - v_{cm})$.

The DAC implementation fig. 34.82



The net list and simulated results are on the following pages.

Amplifier with gain 1/2

```
.control
destroy all
run
plot vop vom vsigp vsigm
.endc
```

```
.options scale=1u ABSTOL=10u VNTOL=10m RELTOL=.01
```

***Op-amp using VCVS**

```
E1 Voutp Vcom Vplus Vminus 100MEG
```

```
E2 Vcom Voutm Vplus Vminus 100MEG
```

***Sample and hold capacitors**

```
C1m Vminus Vcomm 1p
```

```
C1m Vminus Vcenm 2p
```

```
C1p Vplus Vcomp 1p
```

```
C1p Vplus Vcenp 2p
```

***Switches**

```
S1 Vcenm Vcom phi2 Vswit switmod
```

```
S2 Vcomm Vsigp phi2 Vswit switmod
```

```
S3 Vcomm Vcom phi3 Vswit switmod
```

```
S4 Vcomp Vcom phi3 Vswit switmod
```

```
S5 Vcomp Vsigm phi2 Vswit switmod
```

```
S6 Vcenp Vcom phi2 Vswit switmod
```

```
S7 Voutp Vcenm phi3 Vswit switmod
```

```
S8 Voutp Vminus phi1 Vswit switmod
```

```
S9 Voutm Vplus phi1 Vswit switmod
```

```
S10 Voutm Vcenp phi3 Vswit switmod
```

```
S11 Voutp Vop phi3 Vswit switmod
```

```
S12 Voutm Vom phi3 Vswit switmod
```

***Load capacitor**

```
Cop Vop 0 10p
```

```
Com Vom 0 10p
```

***Supply voltage**

```
VDD VDD 0 DC 1.5
```

***Reference voltages**

```
Vswit Vswit 0 DC 0.75
```

```
Vcom Vcom 0 DC 0.75
```

***Input Signals**

```
Vsigp Vsigp 0 DC 0 Sin 0.75 0.375 2.5MEG
```

```
Vsigm Vsigm 0 DC 0 Sin 0.75 -0.375 2.5MEG
```

***Clock Signals**

```
Vphi1 phi1 0 DC 0 Pulse 0 1.5 0 200p 200p 4n 10n
```

```
Vphi2 phi2 0 DC 0 Pulse 0 1.5 0 200p 200p 4n 10n
```

```
Vphi3 phi3 0 DC 0 Pulse 0 1.5 5n 200p 200p 4n 10n
```

```
R1 phi1 0 1MEG
```

```
R2 phi2 0 1MEG
```

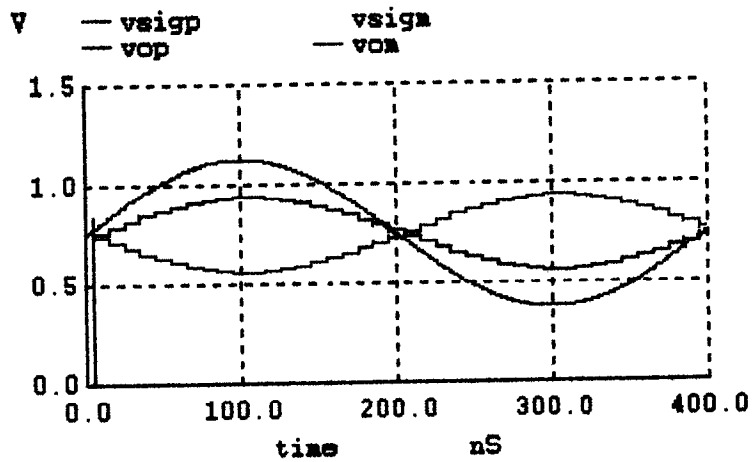
```
R3 phi3 0 1MEG
```

```
.tran 2n 400n 0 2n UIC
```

```
.model switmod SW RON=1
```

```
.end
```

-



Vsigp = 1.25
Vsign = 0.25

Vcom = 0.75

The outputs are as below:

Vop = 0.95
Vom = 0.5

Hence gain is 1