

**PROBLEM # 11.1**

**Meshack Pavan.A**

**Estimate the noise margins for the inverters used to generate Fig. 11.4.**

**Solution: -**

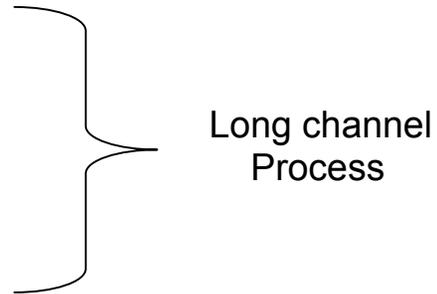
From the graphs in Fig. 11.4 and the text in p11.3, we have

$$\begin{aligned}V_{OH} &= 5V \\V_{OL} &= 0V \\V_{IL} &= 1.8V \\V_{IH} &= 2.1V\end{aligned}$$

Therefore, noise margins,

$$NM_H = V_{OH} - V_{IH} = 5 - 2.1 = 2.9V$$

$$NM_L = V_{IL} - V_{OL} = 1.8 - 0 = 1.8V$$



Long channel  
Process

For the short channel process, similarly from the graph and text in the book,

$$\begin{aligned}V_{OH} &= 1V \\V_{OL} &= 0V \\V_{IL} &= 400mV \\V_{IH} &= 500mV\end{aligned}$$

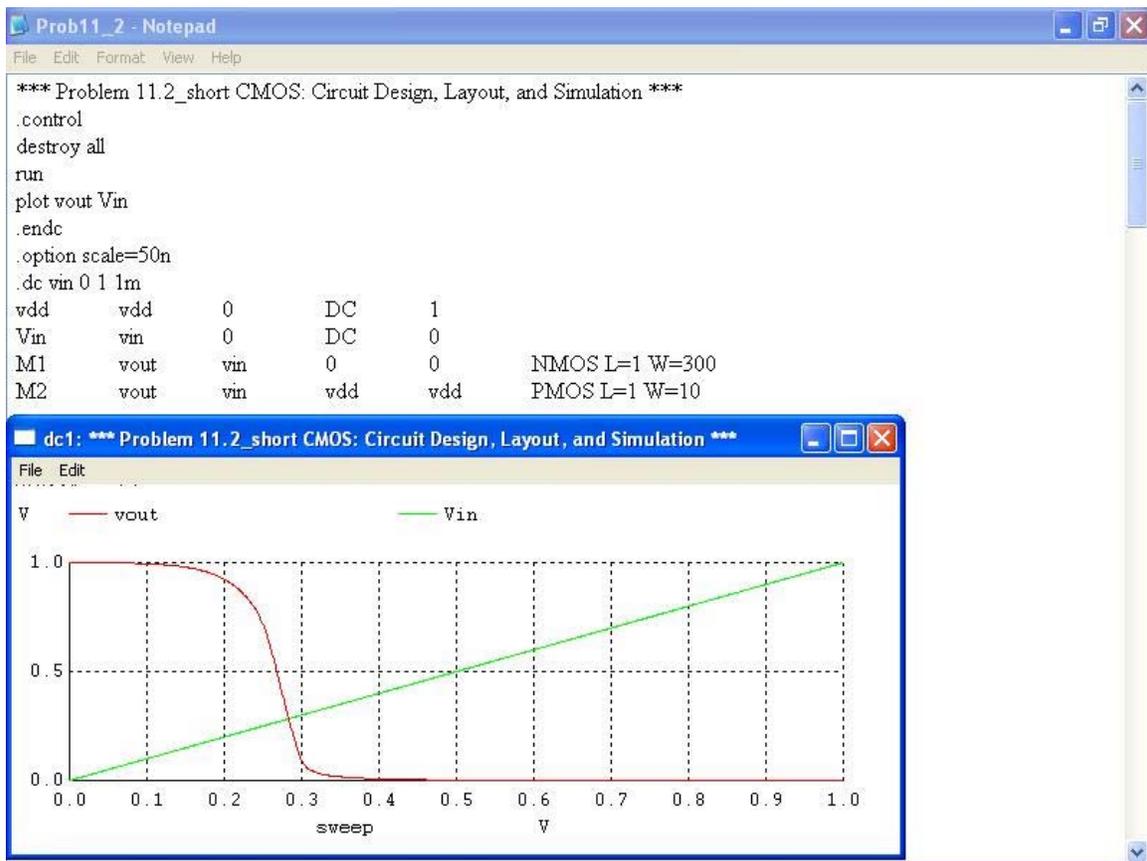
So, noise margins,

$$NM_H = V_{OH} - V_{IH} = 1 - 0.5 = 0.5V = 500mV$$

$$NM_L = V_{IL} - V_{OL} = 0.4 - 0 = 0.4V = 400mV$$

**11.2) Design and simulate the DC characteristics of an inverter with  $V_{sp}$  approximately equal to  $V_{THn}$ . Estimate the resulting noise margins for the design.**

For  $V_{sp} \sim V_{thn}$ ,  $(\beta_n / \beta_p)$  should be as large as possible.  
Let  $(\beta_n / \beta_p) = 90$   
So  $W_n = 30 * W_p$  assuming  $K_{Pn} = K_{Pp}$  and  $L_n = L_p$   
If  $W_p = 10$ , then  $W_n = 300$ .  
Substituting these values in equation 11.4,  
we get  $V_{sp} = 0.32$ . ( $V_{tn} = V_{tp} = 0.28V$  from Table 6.3)  
From simulations we obtain  $V_{sp} \sim 0.27V$ .



### Problem 11.3

Satish Dulam

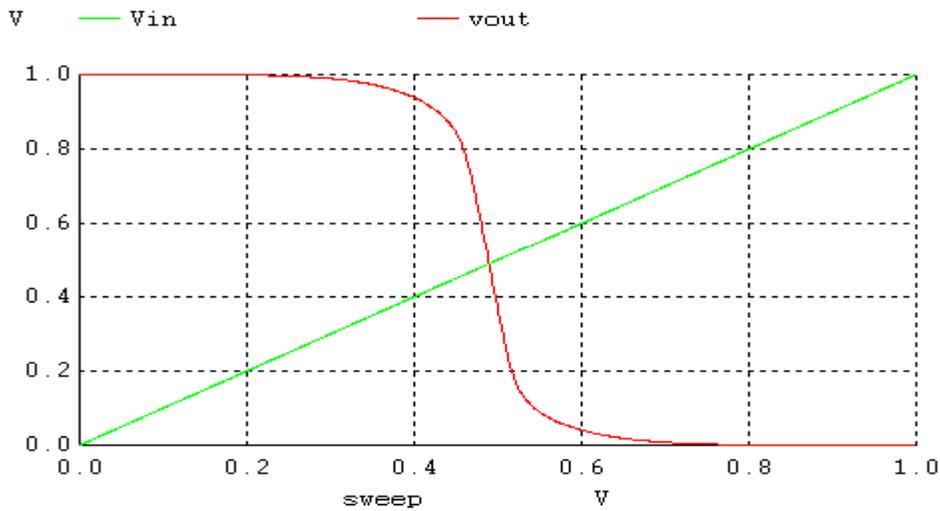
#### Question:

Show that the switching point of three inverters in series is dominated by the  $V_{sp}$  of the first inverter.

#### Solution:

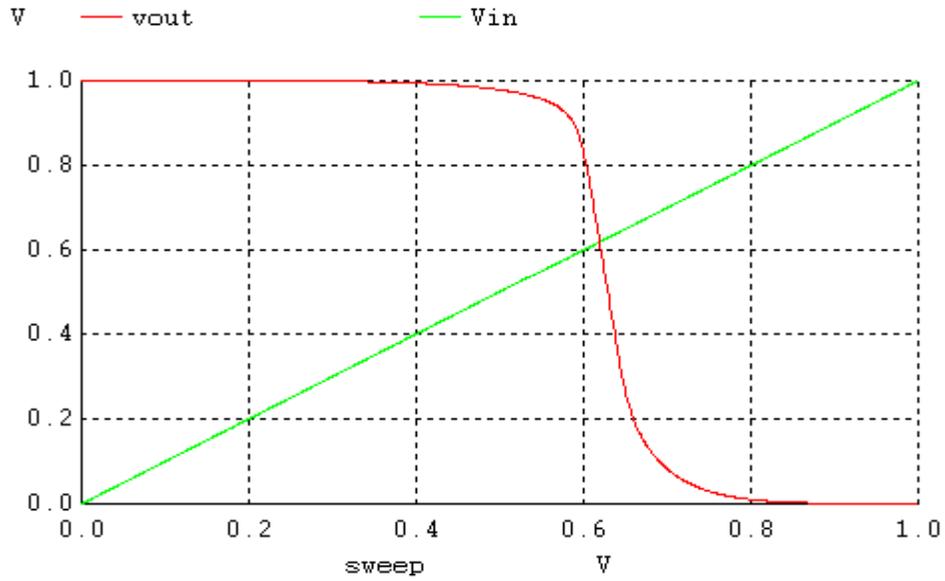
Three inverters with different switching points are simulated as below:

```
*Stage1 Inverter
.control
destroy all
run
let Icross=-i(vdd)
*plot Icross
plot vout Vin
.endc
.option scale=50n
.dc vin 0 1 1m
vdd vdd 0 DC 1
Vin vin 0 DC 0
M1 vout vin 0 0 NMOS L=1 W=10
M2 vout vin vdd vdd PMOS L=1 W=20
* 50nm BSIM4 models
```



Stage2 and 3 Inverters with different Switching points:

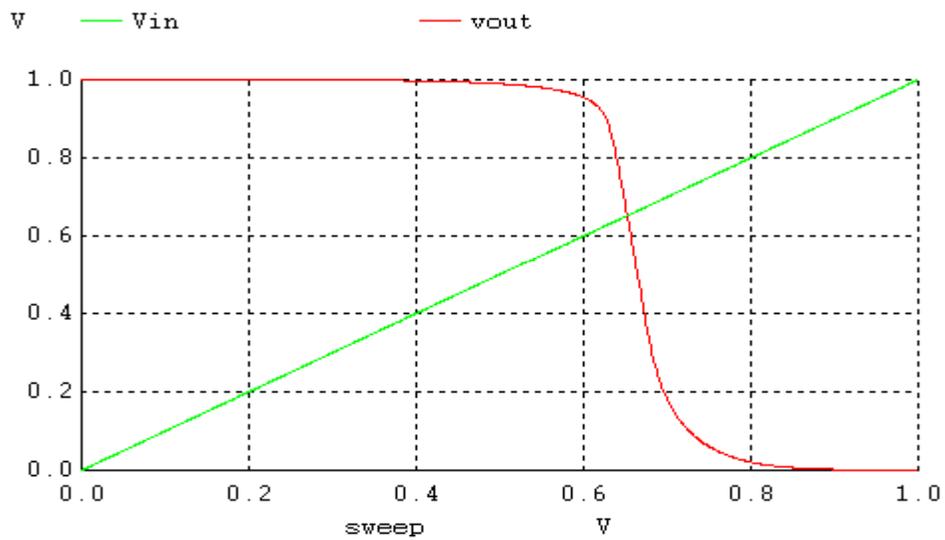
```
*Stage2 Inverter
.control
destroy all
run
let Icross=-i(vdd)
*plot Icross
plot vout Vin
.endc
.option scale=50n
.dc vin 0 1 1m
vdd vdd 0 DC 1
Vin vin 0 DC 0
M1 vout vin 0 0 NMOS L=1 W=10
M2 vout vin vdd vdd PMOS L=1 W=200
* 50nm BSIM4 models
```



```

*Stage3 Inverter
.control
destroy all
run
let Icross=-i(vdd)
*plot Icross
plot vout Vin
.endc
.option scale=50n
.dc vin 0 1 1m
vdd vdd 0 DC 1
Vin vin 0 DC 0
M1 vout vin 0 0 NMOS L=1 W=10
M2 vout vin vdd vdd PMOS L=1 W=400
* 50nm BSIM4 models

```



```
*Three inverters in series
```

```
.control  
destroy all  
run  
plot vout vin  
.endc  
.option scale=50n  
.dc vin 0 1 1m
```

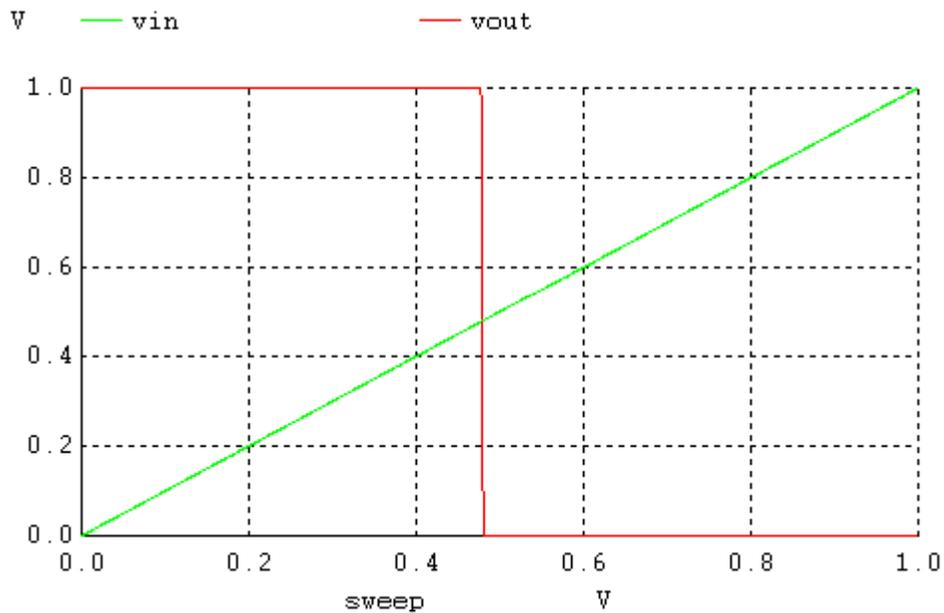
```
vdd    vdd    0      DC    1  
Vin    vin    0      DC    0  
M1     vout1  vin    0      0      NMOS L=1 W=10  
M2     vout1  vin    vdd   vdd   PMOS L=1 W=20  
M3     vout2  vout1  0      0      NMOS L=1 W=10  
M4     vout2  vout1  vdd   vdd   PMOS L=1 W=200  
M5     vout   vout2  0      0      NMOS L=1 W=10  
M6     vout   vout2  vdd   vdd   PMOS L=1 W=400
```

```
* 50nm BSIM4 models
```

```
* Don't forget the .options scale=50nm if using an Lmin of 1
```

```
* 1<Ldrawn<200 10<Wdrawn<10000 Vdd=1V
```

```
* Change to level=54 when using HSPICE
```

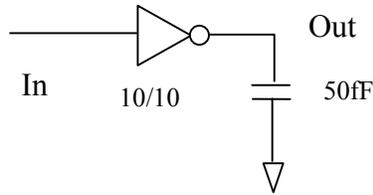


Notice the switching point of the 3stage inverters is around 500mv though the second and third stage switching points are at 640mv.

**Problem 11.4**

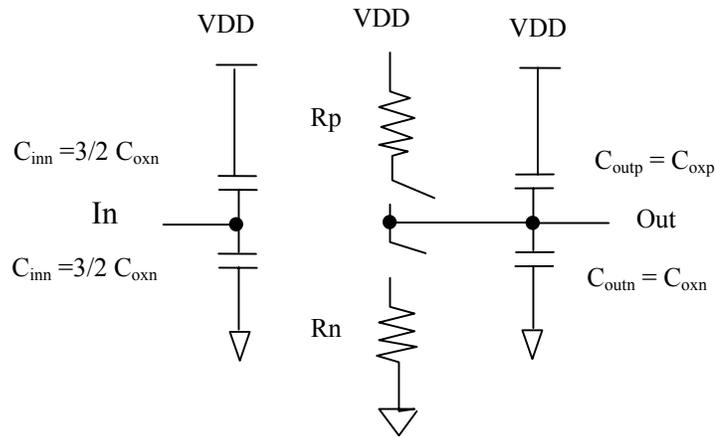
**Rahul Mhatre**

**Repeat Ex. 11.6 using a PMOS device with a width of 10.**



**Figure 11.11** Circuit Schematic for Problem 11.4.

**Solution:** Replacing the inverter with its digital model, we get the following schematic



**Figure 11.4.1** Digital Model for the inverter

The total capacitance at the output node is given as

$$\begin{aligned}
 C_{\text{tot}} &= C_{\text{oxp}} + C_{\text{oxn}} + C_{\text{load}} \\
 C_{\text{tot}} &= (62.5\text{aF}) [W_n L_n + W_p L_p] + 50\text{fF} \\
 &= (62.5\text{aF}) [10 + 10] + 50\text{fF} \\
 C_{\text{tot}} &= 51.25 \text{ fF}
 \end{aligned}$$

From Table 10.1,

$$\begin{aligned}
 R_p &= 68\text{k}/W_p \\
 R_p &= 68\text{k}/10 = 6.8\text{K}
 \end{aligned}$$

$$\begin{aligned}
 R_n &= 34\text{K}/W_n \\
 R_n &= 34\text{K}/10 = 3.4\text{K}
 \end{aligned}$$

$$\begin{aligned}
 T_{\text{pHL}} &= 0.7 R_n C_{\text{tot}} \\
 T_{\text{pHL}} &= 0.7 * 3.4\text{K} * 51.25\text{fF} \\
 \mathbf{T_{\text{pHL}} = 122 \text{ pS}}
 \end{aligned}$$

$$T_{pLH} = 0.7 R_p C_{tot}$$

$$T_{pLH} = 0.7 * 6.8K * 51.25fF$$

$$T_{pLH} = 244pS$$

The SPICE Simulation netlist is as shown in Figure 11.4.2

```

*** Problem 11.4 CMOS: Circuit Design, Layout, and Simulation ***
.control
destroy all
run
plot vin vout
.endc

.option scale=50n
.tran 10p 2n UIC

vdd vdd 0 DC 1
Vin vin 0 DC 0 pulse 0 1 500p 0 0 1n 2n

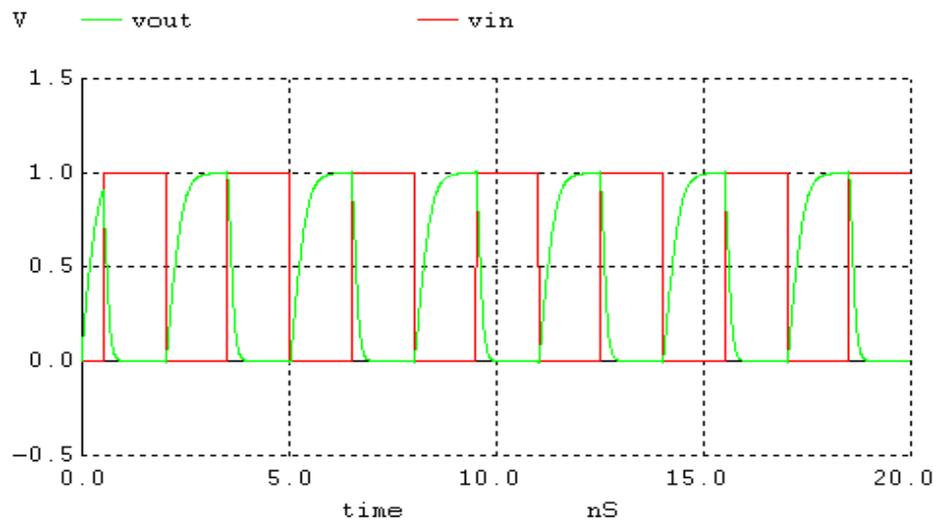
M1 vout vin 0 0 NMOS L=1 W=10
M2 vout vin vdd vdd PMOS L=1 W=10
Cl vout 0 50f

***Models for NMOS and PMOS****
.end

```

**Figure 11.4.2** SPICE netlist for Figure 11.11

The SPICE simulations are as shown in Figure 11.4.3. The  $T_{pHL}$  was found to be **100 ps** and  $T_{pLH}$  was found out to be around **200ps**.



**Figure 11.4.3** SPICE Simulation for the netlist of Figure 11.1

**Problem 11.5: Repeat Ex. 11.6 using the long channel process with a 30/10 inverter.**

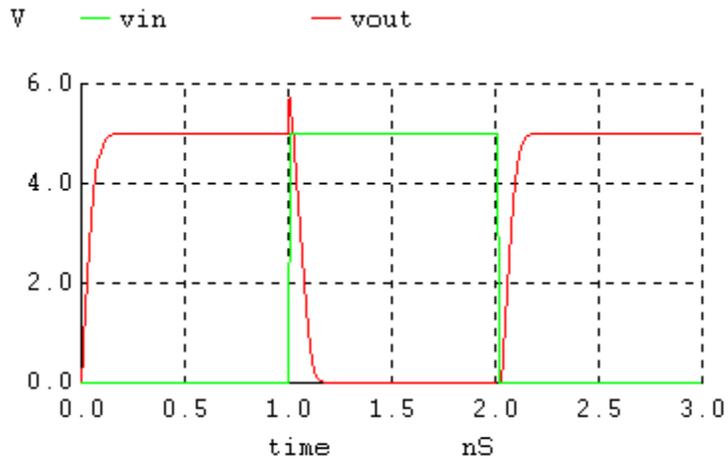
**Solution:**

\*\*\* Top Level Netlist \*\*\*

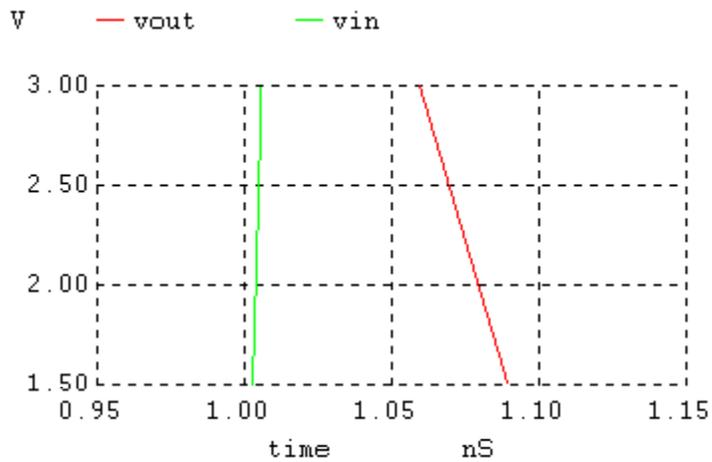
```
vdd      vdd      0      DC      5
Vin      vin      0      DC      0      pulse 0 5 1n 0p 0p 1n 2n
C1       Vout 0 50fF
```

```
M1       Vout Vin 0 0  NMOS L=1u W=10u
M2       Vout Vin Vdd Vdd PMOS L=1u W=30u
```

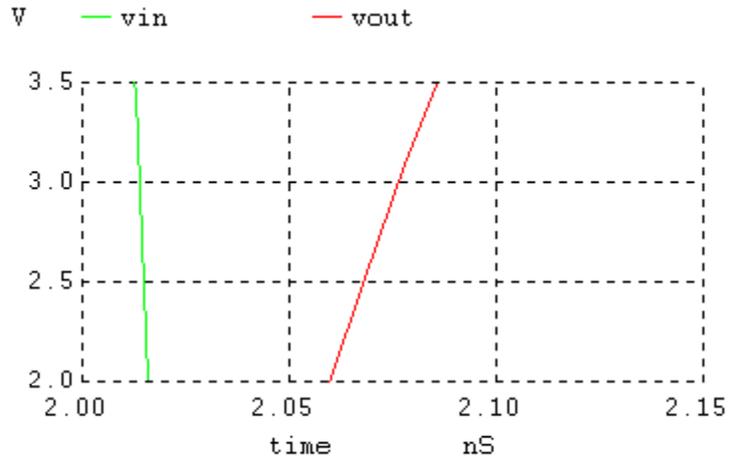
Simulation:



tPHL=69ps



tPLH=67ps



Taking into account the output capacitance of the MOS's:

$$t_{PHL} = 7 * R_p * C_{tot} = 7 * 45k / 30 * (1.75f * (30 + 10) + 50f) = 126ps$$
$$t_{PLH} = 7 * R_n * C_{tot} = 7 * 15k / 10 * (1.75f * (30 + 10) + 50f) = 126ps$$

W/O taking into account the output capacitance of the MOS's:

$$t_{PHL} = 7 * R_p * C_{tot} = 7 * 45k / 30 * (50f) = 52.5ps$$
$$t_{PLH} = 7 * R_n * C_{tot} = 7 * 15k / 10 * (50f) = 52.5ps$$

W/O appears to be closer to the sim.

**Problem 11.6**

Steve Bard

*Estimate the oscillation frequency of an 11-stage ring oscillator using 30/10 inverters in the long-channel CMOS process. Compare your hand calculations to the simulation results.*

Solution: The frequency is about 250 MHz when calculated by hand. When simulated on SPICE, the frequency is higher, reaching 495 MHz. The hand calculations are shown below, and the SPICE simulation is shown on the next page.

$$f_{rosc} = \frac{1}{n(t_{PHL} + t_{PLH})}$$

$$t_{PHL} + t_{PLH} = 0.7(R_n + R_p)C_{TOT}$$

$$C_{TOT} = \frac{5}{2}(C_{oxP} + C_{oxN})$$

$$R_n = 15k \frac{L}{W} = \frac{15k}{10} = 1.5k$$

$$R_p = 45k \frac{L}{W} = \frac{45k}{30} = 1.5k$$

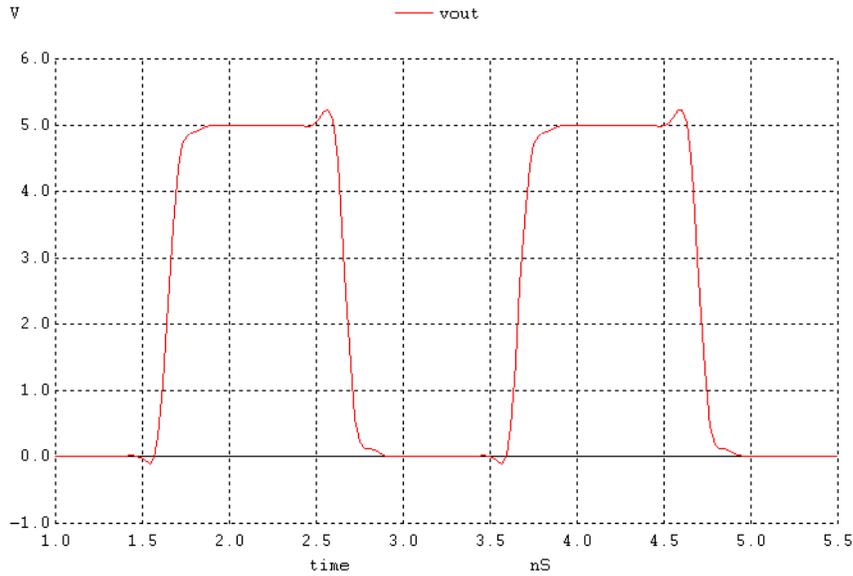
$$C_{oxP} = 1.75 fF \cdot W \cdot L = 52.5 fF$$

$$C_{oxN} = 1.75 fF \cdot W \cdot L = 17.5 fF$$

$$C_{TOT} = \frac{5}{2}(52.5 fF + 17.5 fF) = 175 fF$$

$$t_{PHL} + t_{PLH} = 0.7(1.5k + 1.5k)(175 fF) = 368 ps$$

$$f_{rosc} = \frac{1}{11(368 ps)} \approx 250 MHz$$



From the SPICE simulation, the frequency =  $1/T = 1 / 2.02 \text{ ns} = 495 \text{ MHz}$ .

\*\*\* Problem 11.6 \*\*\*

```
.control
destroy all
run
plot vout
.endc
.option scale=1u
.tran .ln 6.5n uic
```

```
vdd      vdd      0      DC      5
R1       vout     0      1MEG
```

\*\*\*\*\*D\*\*\*\*\*G\*\*\*\*\*S\*\*\*\*\*B\*\*\*\*

```
M1      vout1    vout    0      0      NMOS L=1 W=10
M2      vout1    vout    vdd    vdd    PMOS L=1 W=30
M3      vout2    vout1    0      0      NMOS L=1 W=10
M4      vout2    vout1    vdd    vdd    PMOS L=1 W=30
M5      vout3    vout2    0      0      NMOS L=1 W=10
M6      vout3    vout2    vdd    vdd    PMOS L=1 W=30
M7      vout4    vout3    0      0      NMOS L=1 W=10
M8      vout4    vout3    vdd    vdd    PMOS L=1 W=30
M9      vout5    vout4    0      0      NMOS L=1 W=10
M10     vout5    vout4    vdd    vdd    PMOS L=1 W=30
M11     vout6    vout5    0      0      NMOS L=1 W=10
M12     vout6    vout5    vdd    vdd    PMOS L=1 W=30
M13     vout7    vout6    0      0      NMOS L=1 W=10
M14     vout7    vout6    vdd    vdd    PMOS L=1 W=30
M15     vout8    vout7    0      0      NMOS L=1 W=10
M16     vout8    vout7    vdd    vdd    PMOS L=1 W=30
M17     vout9    vout8    0      0      NMOS L=1 W=10
M18     vout9    vout8    vdd    vdd    PMOS L=1 W=30
M19     vout10   vout9    0      0      NMOS L=1 W=10
M20     vout10   vout9    vdd    vdd    PMOS L=1 W=30
M21     vout     vout10   0      0      NMOS L=1 W=10
M22     vout     vout10   vdd    vdd    PMOS L=1 W=30
```

```
.MODEL NMOS NMOS LEVEL = 3
.MODEL PMOS PMOS LEVEL = 3
.end
```

## Problem 11.7

Lincoln Bollschweiler

Using the long channel process design a buffer with minimum delay ( $A=e$ ) to insert in between a 30/10 inverter and a 50pF load capacitance. Simulate the operation of the design.

### Solution:

For a 30/10 long channel process using table 10.2;

using (11.6),  $C_{IN, 30/10 INV} = 105fF$ ;

using (11.7),  $C_{OUT, 30/10 INV} = 70fF$ .

$$\text{Using (11.20), } e = \left[ \frac{50p}{105f} \right]^{\frac{1}{N}} \Rightarrow N = \ln \left[ \frac{50p}{105f} \right] = 6.17.$$

Although we are creating a buffer to go *in between* the 30/10 inverter and a load capacitance, we are including the 30/10 inverter in the calculations above. Therefore, in order to keep the required inversion we must choose  $N$  equal to the closest *odd* number of gates. As  $N=6.17$ , we choose to round to  $N=7$ .

$$\text{Using (11.20), } A = \left[ \frac{50p}{105f} \right]^{\frac{1}{7}} \Rightarrow A = 2.41.$$

The first gate in the buffer will be sized up by  $A=2.41$  compared to the 30/10 inverter. The second gate will have  $A=2.41^2$ , the third  $A=2.41^3$  and so on until the last, gate 6 of the buffer, will have  $A=2.41^6$ .

Using  $t_{PHL} + t_{PLH} = 0.7 \cdot N (R_N + R_p)(C_{OUT1} + A \cdot C_{IN1})$ , we have

$$t_{PHL} + t_{PLH} = 0.7 \cdot 7 (1500 + 1500)(70E-15 + 2.41 \cdot 105E-15) = 4.75ns \text{ with a 6 gate buffer (7 gates total including the 30/10 inverter).}$$

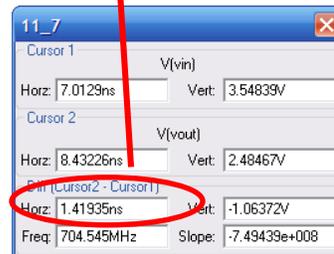
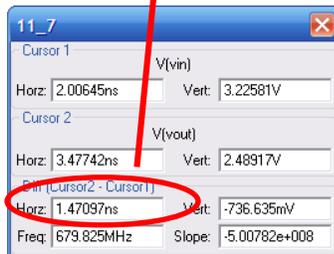
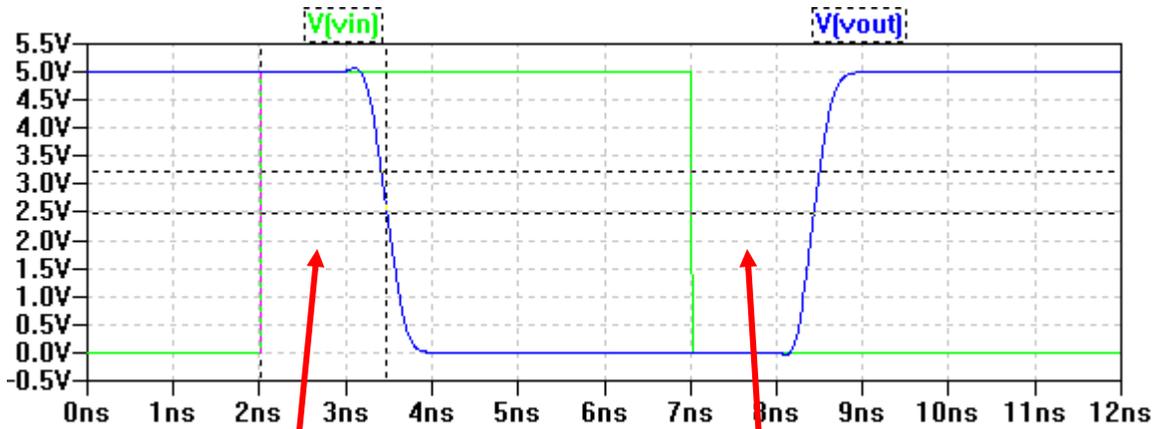
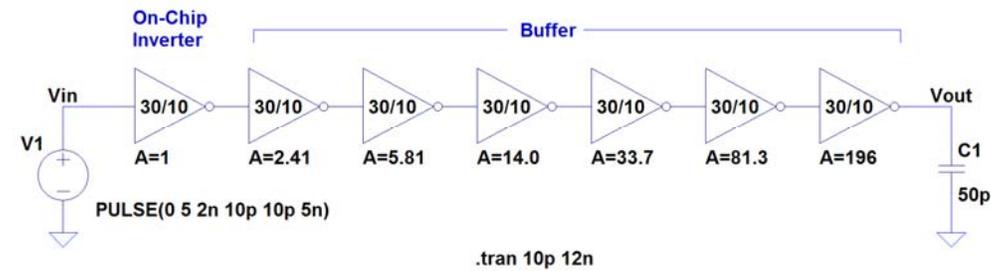
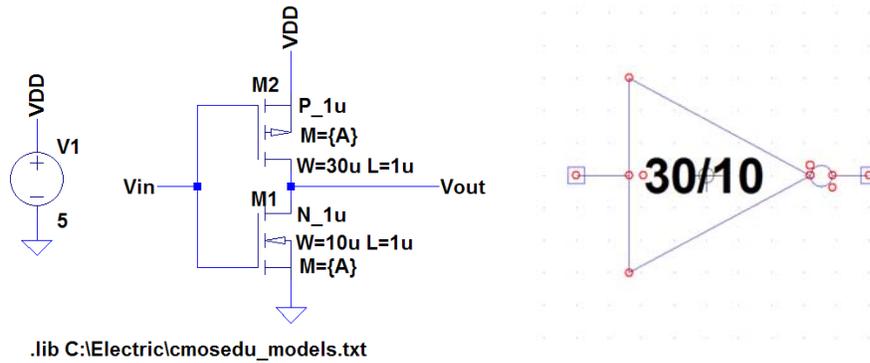
To find the delay for the 30/10 inverter with no buffer we can use

$$t_{PHL} + t_{PLH} \approx 0.7 (R_N + R_p)(C_L), \text{ and so we find}$$

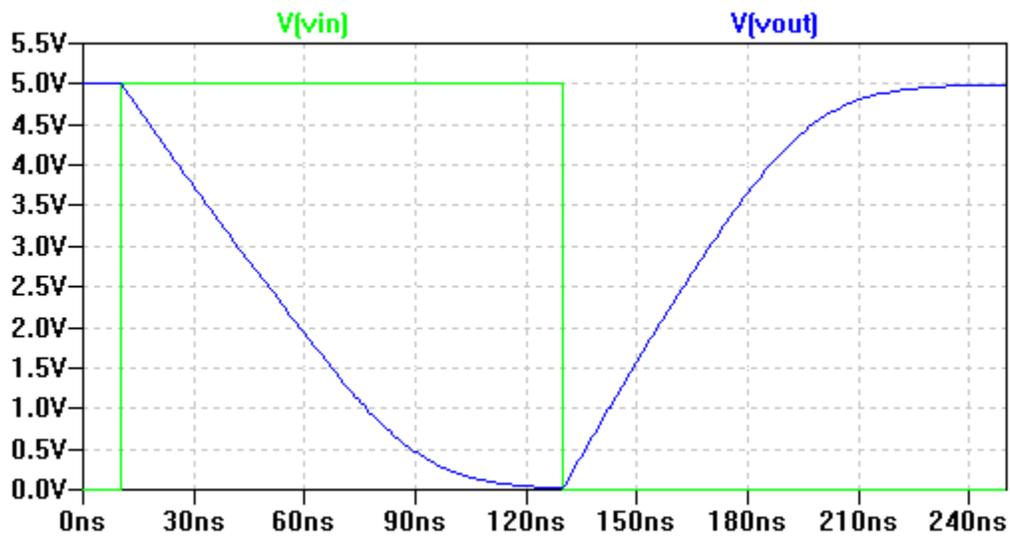
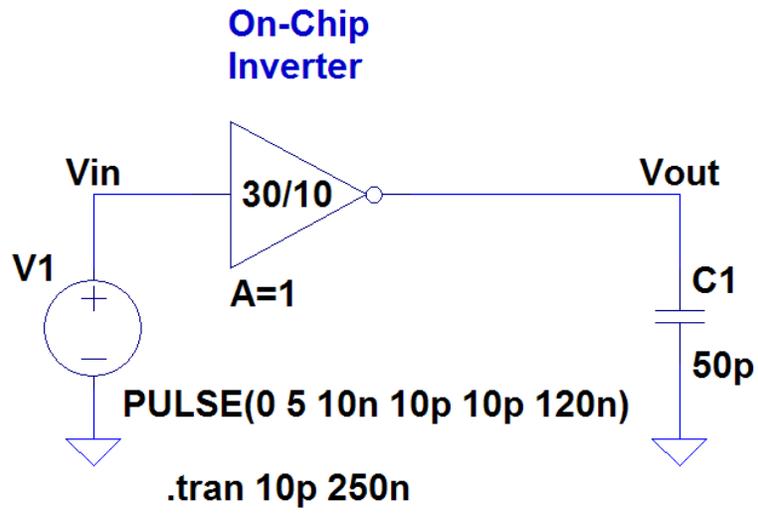
$$t_{PHL} + t_{PLH} \approx 0.7 (1500 + 1500)(50p) = 105ns \text{ for no buffer.}$$

The simulations show our approximations are, just that, approximations. The simulated results are about 60% of the calculated values. The biggest cause for the discrepancy is most likely lower actual channel resistances ( $R_N$  and  $R_p$ ) than the values used in calculations.

The following are the simulation results:



Simulation results:  $t_{PLH} + t_{PHL} = 1.47\text{ns} + 1.42\text{ns} = 2.89\text{ns}$ .



Simulation results:  $t_{PLH} + t_{PHL} = 40n + 32n = 72n$ . ■

## Problem 11.8

Harish Reddy Singidi

Repeat problem 11.7 using an area factor, A of 8.

Solution: -

$$A = 8$$

$$C_{in1} = 1.5 (C_{oxn} + C_{oxp})$$

$$C_{in1} = 1.5 ((1.75)(30) + (1.75)(10))$$

$$C_{in1} = 105\text{fF}$$

$$C_{out1} = C_{oxn1} + C_{oxp1} = (1.75)(10)(1.75)(30) = 70\text{fF}$$

$$R_{n1} = 15\text{K}/10 = 1.5\text{K}$$

$$R_{p1} = 15\text{K}/10 = 1.5\text{K}$$

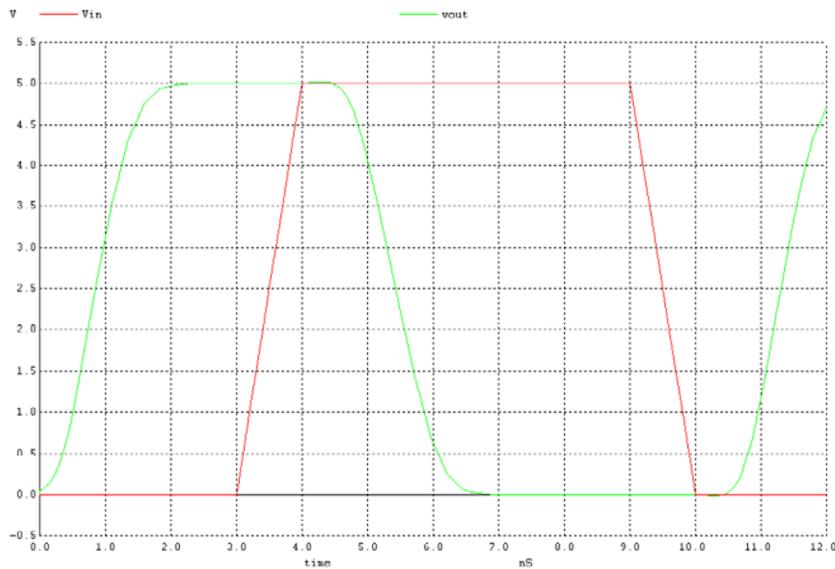
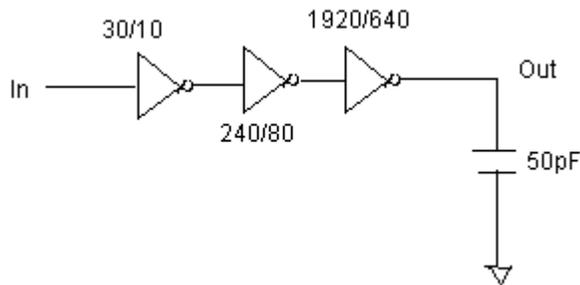
$$N = \ln((C_{load}/C_{in1}))/\ln(A)$$

$$N = 2.9 \text{ Stages} = 3 \text{ Stages}$$

$$(T_{PHL} + T_{PLH}) = 0.7 * N(R_{n1} + R_{p1})(C_{out1} + A * C_{in1})$$

$$(T_{PHL} + T_{PLH}) = 0.7 * 3 * (1.5\text{K} + 1.5\text{K})(70\text{fF} + 8 * 105\text{fF})$$

$$(T_{PHL} + T_{PLH}) = 5.73\text{ns}$$



11.9) Derive an equation for the switching point voltage, similar to the derivation of Eq(11.4), for the NMOS inverter seen in Fig 11.24a.

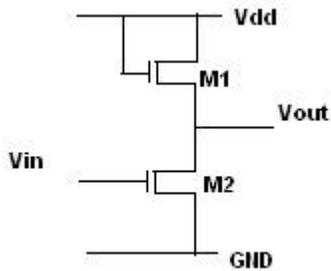


Fig 11.24a

To find the switching point voltage, let  $V_{in} = V_{out} = V_{sp}$

At the conditions mentioned above, both M1 and M2 are in saturation.

So  $I_{ds1}(\text{sat}) = I_{ds2}(\text{sat})$

$$(\beta_{n2}/2)(V_{sp} - V_{thn2})^2 = (\beta_{n1}/2)(V_{dd} - V_{sp} - V_{thn1})^2$$

Solving for  $V_{sp}$  gives

$$V_{sp} = [V_{thn1} \sqrt{(\beta_{n1}/\beta_{n2})} + (V_{dd} - V_{thn2})] / [1 + \sqrt{(\beta_{n1}/\beta_{n2})}]$$

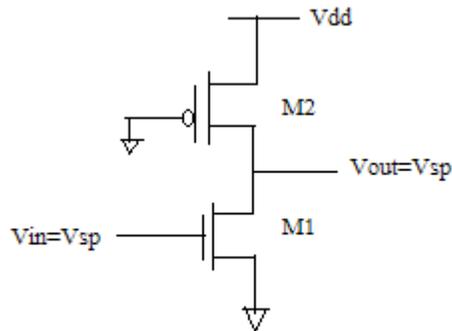
If  $\beta_{n1} = \beta_{n2}$  and  $V_{thn2} = V_{thn1}$ , then

$$V_s = V_{dd}/2.$$

**Problem 11.10**

**Rupa Balan**

Repeat problem 11.9 for the inverter in fig 11.24c. Note that the PMOS transistor is operating in the triode region when the input/output are at  $V_{sp}$ .



Given M2 is in the triode region.

For M1 to be in saturation,  $V_{ds} > V_{gs} - V_{thn}$   
 $V_d > V_g - V_{thn}$   
 $V_{thn} > 0$  ( $V_d = V_g = V_{sp}$ )

Hence M1 is in saturation.

Current through M1,  $I_{DN} = (\beta_n/2) (V_{sp} - V_{thn})^2$

Current through M2,  $I_{DP} = \beta_p [((V_{dd} - V_{thp})(V_{dd} - V_{sp})) - ((V_{dd} - V_{sp})^2/2)]$

Equating the currents,  $(\beta_n/2) (V_{sp} - V_{thn})^2 = \beta_p [((V_{dd} - V_{thp})(V_{dd} - V_{sp})) - ((V_{dd} - V_{sp})^2/2)]$

Simplifying we get,

$$\beta_p/\beta_n = (V_{sp} - V_{thn})^2 / [(V_{dd} - V_{thp})^2 - (V_{sp} - V_{thp})^2]$$

Cross-multiplying we get an equation of the form

$$aV_{sp}^2 + bV_{sp} + c = 0$$

where

$$a = \beta_n + \beta_p$$

$$b = -2(\beta_n V_{thn} + \beta_p V_{thp})$$

$$c = \beta_n V_{thn}^2 - \beta_p V_{dd}^2 + 2\beta_p V_{dd} V_{thp}$$

This is a quadratic equation which can be solved to get  $V_{sp}$ .