

Ch. 4 - Digital
filters

Friday make-up

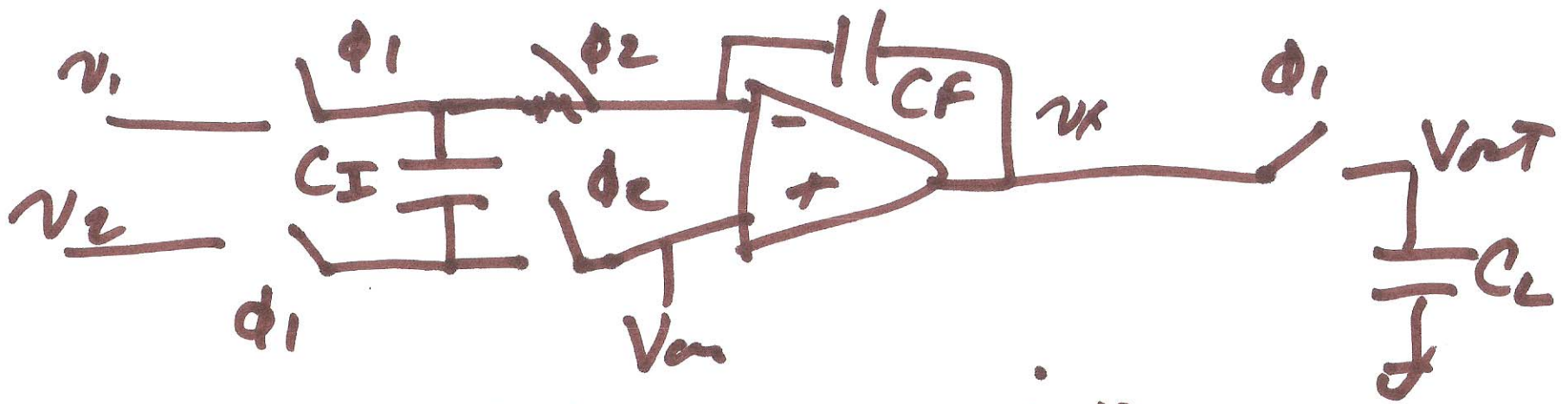
4-5:15 pm

TBE B178

NO H.W due

NO QUIZ

IF YOU CAN'T MAKE IT
WATCH THE VIDEO BEFORE
Monday 9/29



$\phi_1 \downarrow \phi_1$ switches are opening $+v_{in}$

$$C_I (v_1 [(N-1)T] - v_2 [(N-1)T]) = Q_I$$

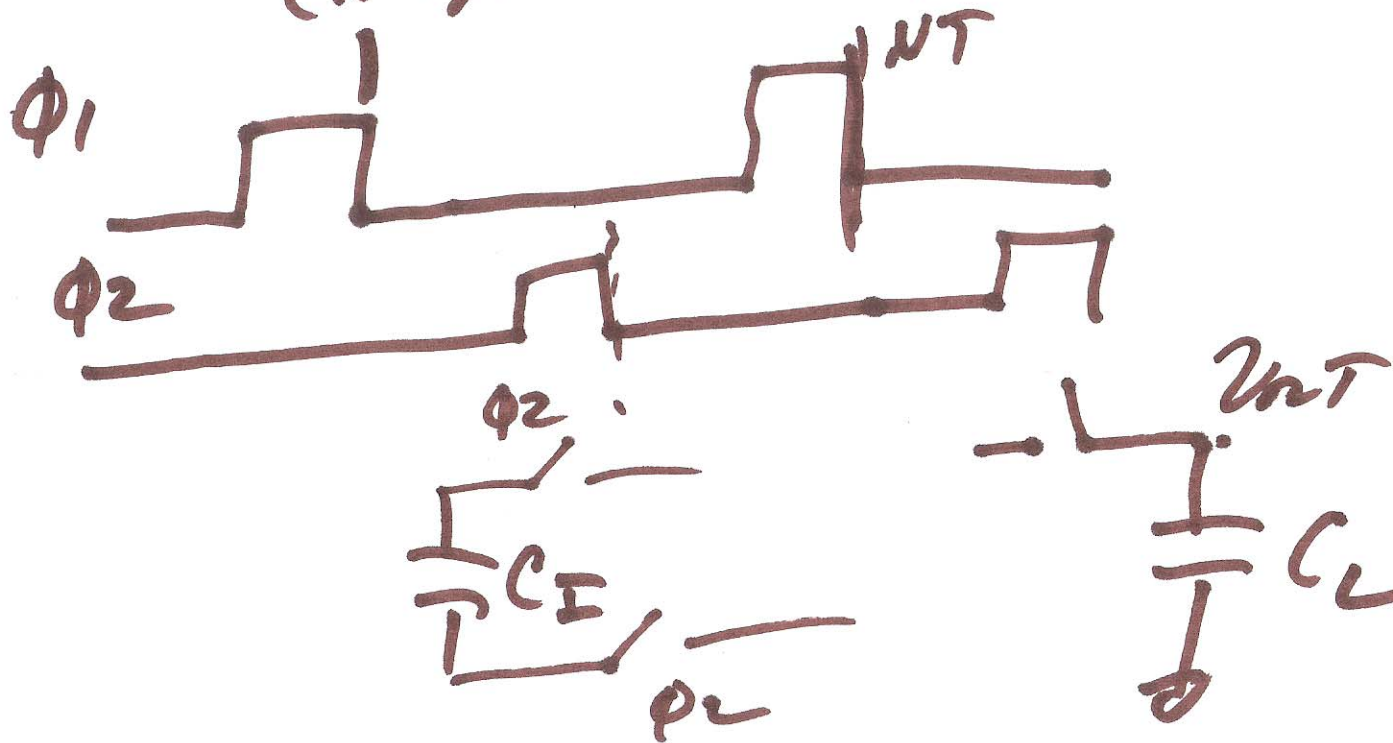
$$(v_{out}(NT) - v_{out}((N-1)T)) \cdot C_F =$$

$$v_{out}(z) (1 - z^{-1}) \cdot C_F = C_I (v_1 [(N-1)T] - v_2 [(N-1)T])$$

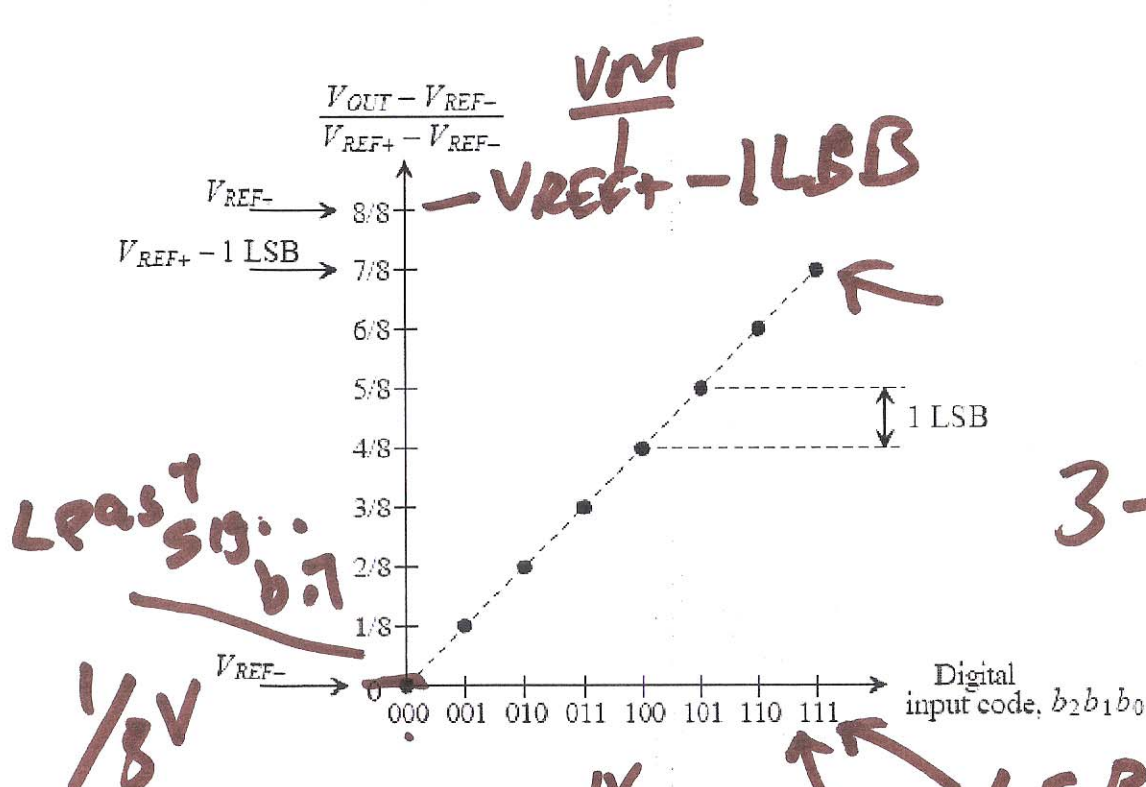
$$= C_I \cdot V_1(z) z^{-1} - v_2 [(N-1)T]$$

$$- v_2(z) \cdot C_I \cdot z^{-1}$$

$$V_{\Omega T}(z) = \frac{CI}{CF} \frac{z^{-1}}{1-z^{-1}} \left(V_1(z) - V_2(z) \right)$$



3)



$$1 \text{ LSB} = \frac{V_{REF+} - V_{REF-}}{2^N}$$

3-bits $2^3 = 8$
 $2^8 = 256$

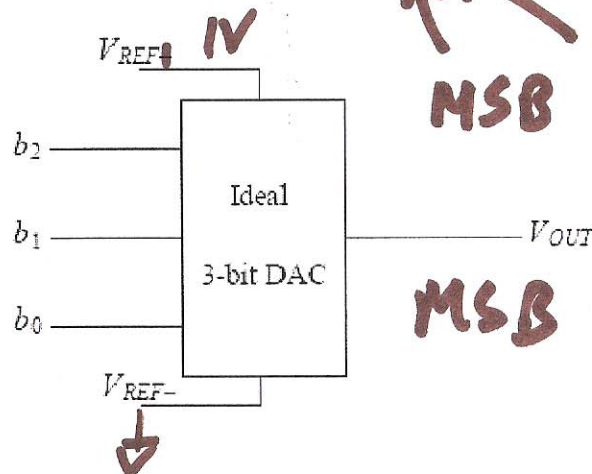


Figure 4.1 An ideal 3-bit DAC.

$$V_{OUT} = (V_{REF+} - V_{REF-}) \cdot \left(\frac{b_{N-1}}{2^1} + \frac{b_{N-2}}{2^2} + \dots + \frac{b_0}{2^N} \right) + V_{REF-}$$

4)

$$\frac{1}{2^8} \approx 4mV$$

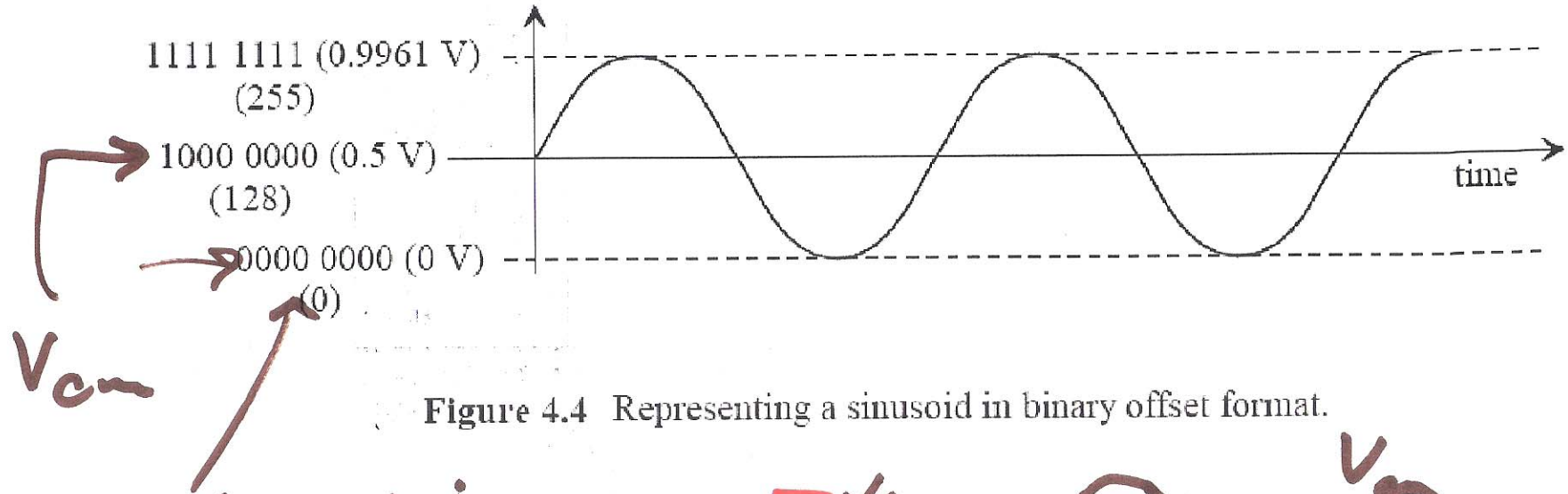
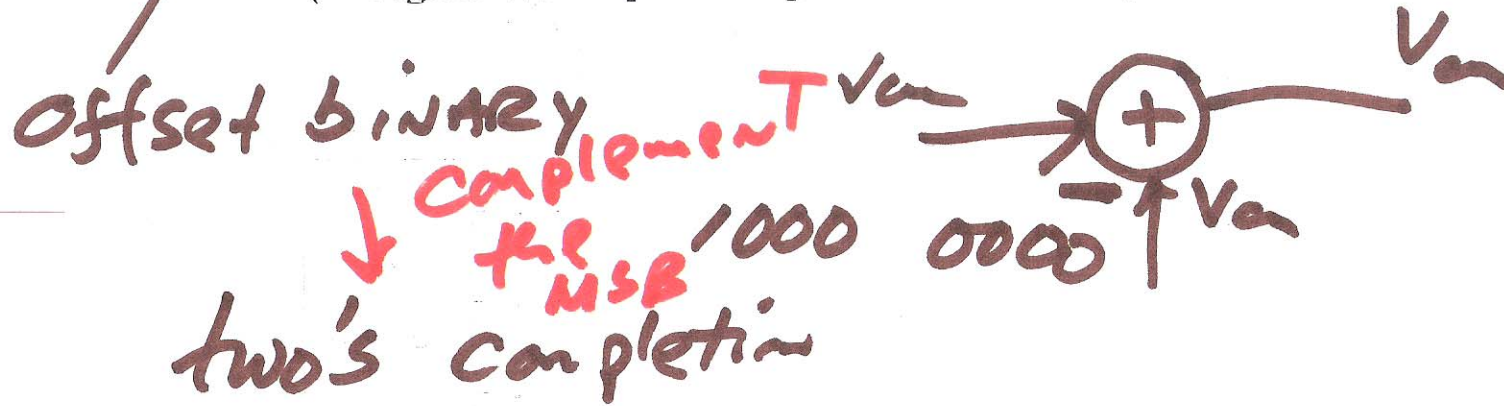


Figure 4.4 Representing a sinusoid in binary offset format.



5)

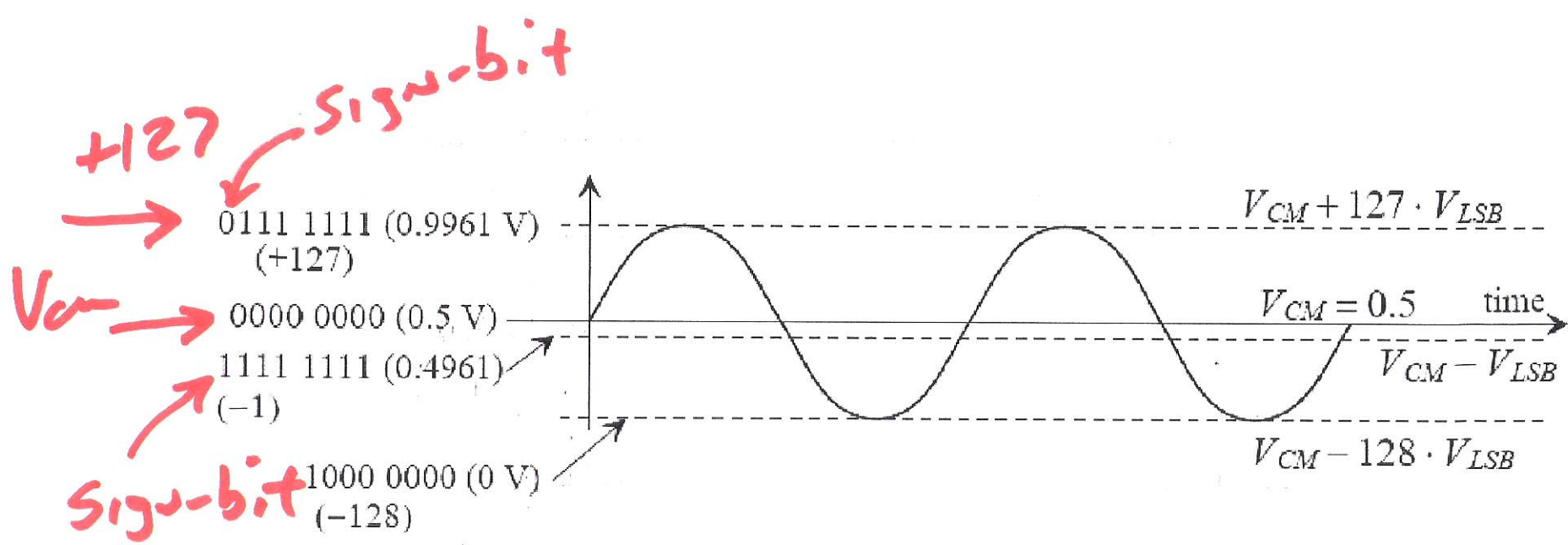
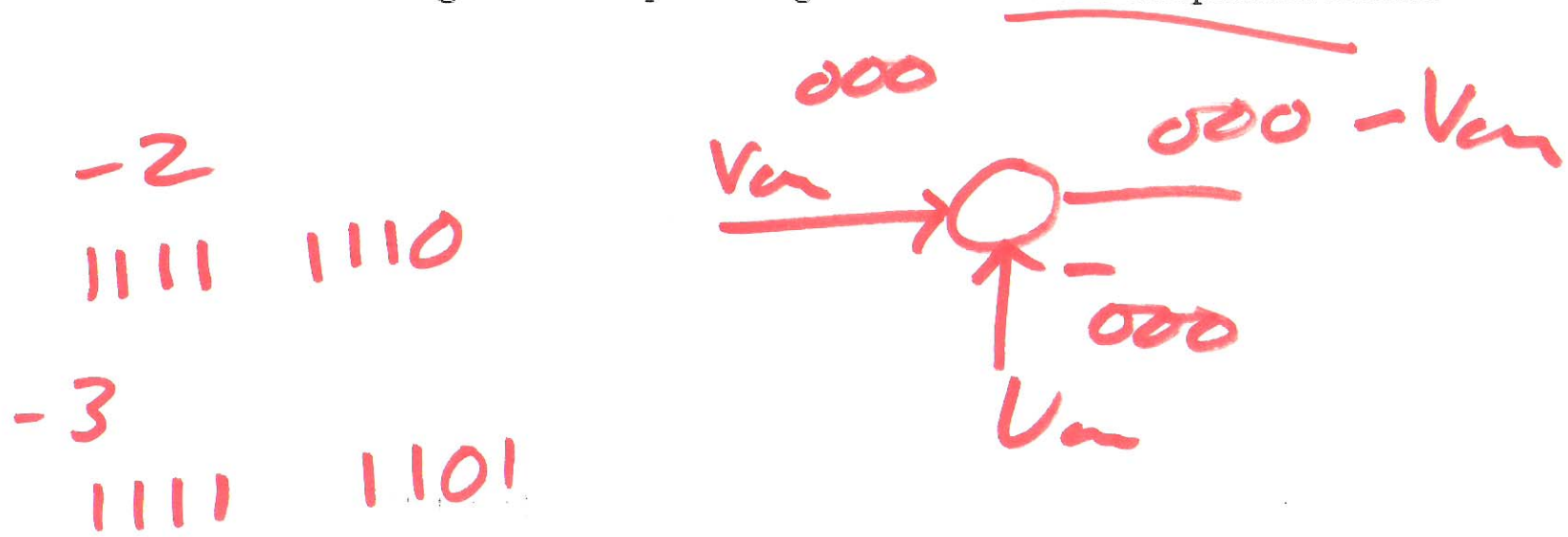


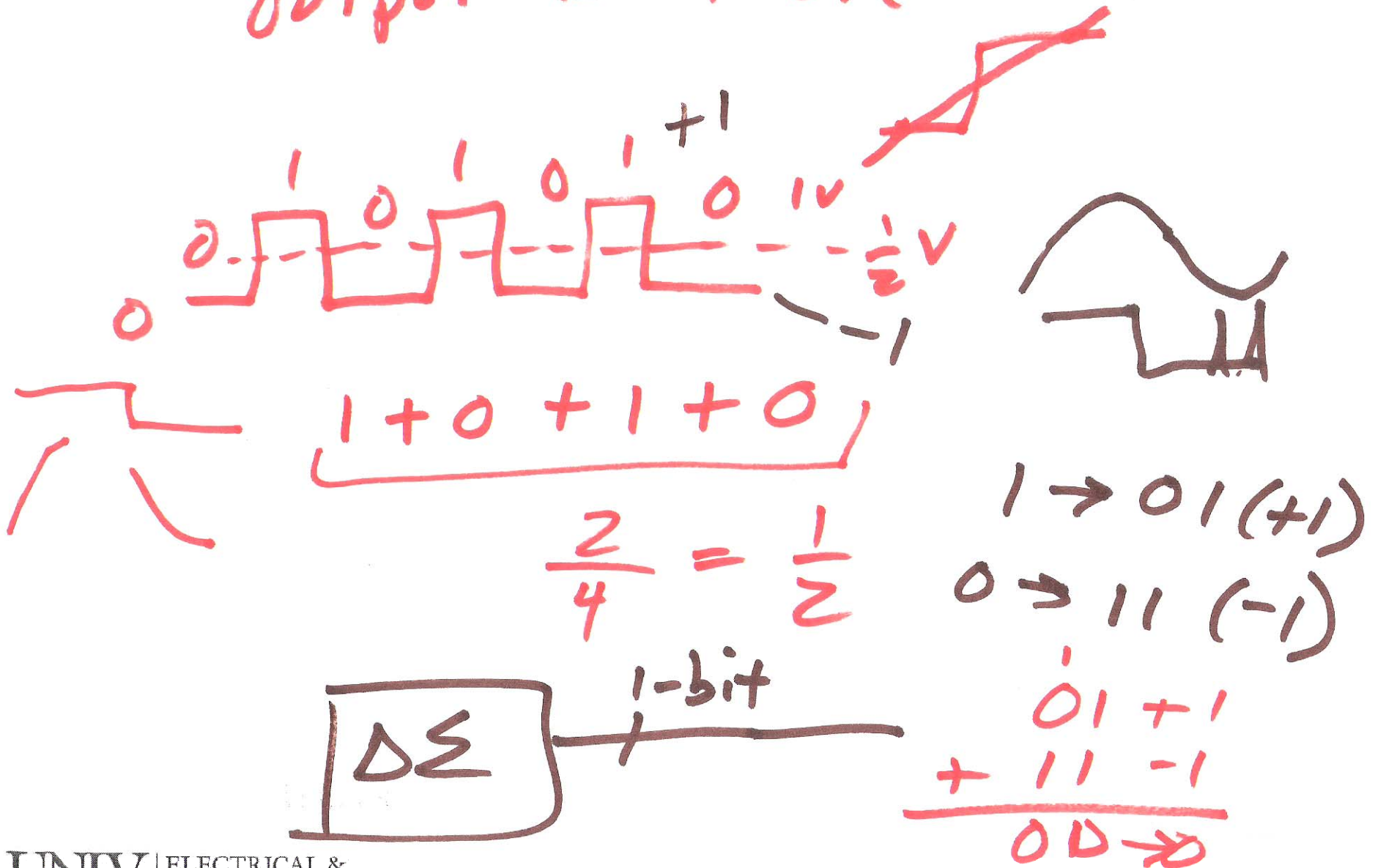
Figure 4.5 Representing a sinusoid in two's complement format.



6)

Delta-Sigma ADC

output is 1-bit



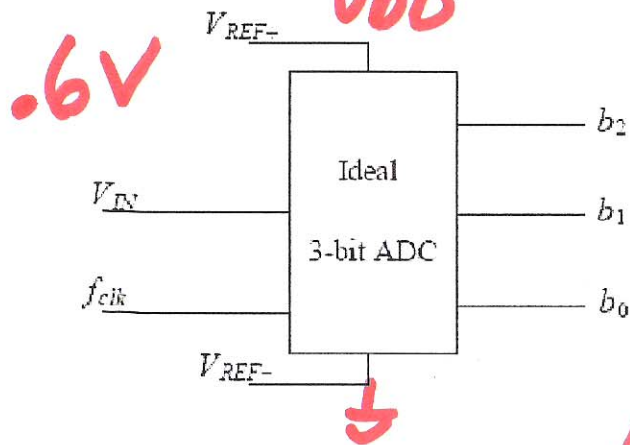
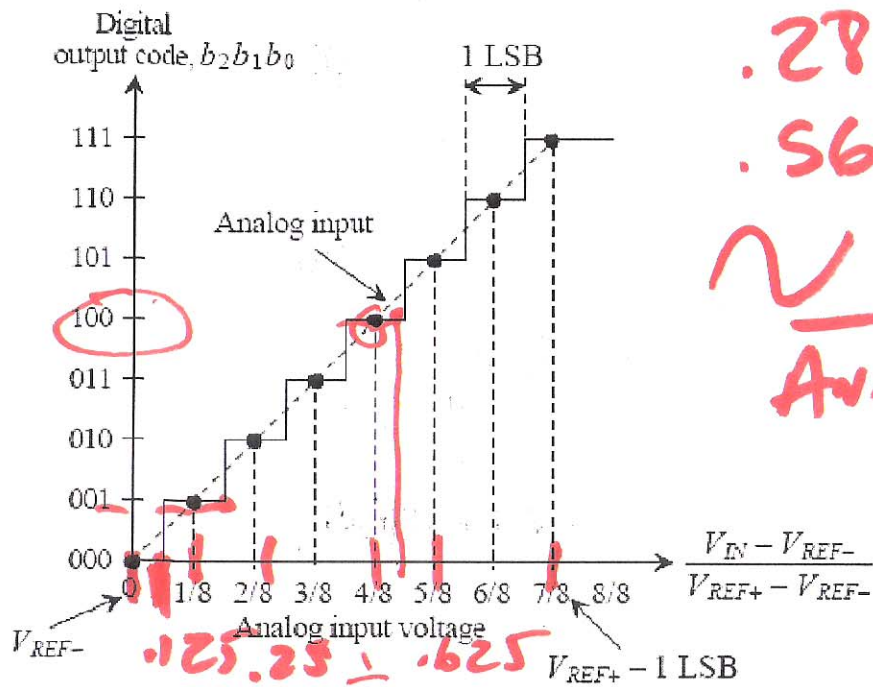


Figure 4.3 An ideal 3-bit ADC.

.14
.28
.56

$b_2 = 0$
 $b_1 = 0$
 $b_0 = 1$
MSB



two's complement

$$V_{an} = \frac{V_{REF+} + V_{REF-}}{2}$$

$$\frac{1}{2}$$

$$b_2 \rightarrow 1$$

$$b_1 \rightarrow 0$$

$$b_0 \rightarrow 0$$

$$\left(0.6 - \frac{1}{2}\right) \times 2 = 200\text{mV}$$

400mV

8)

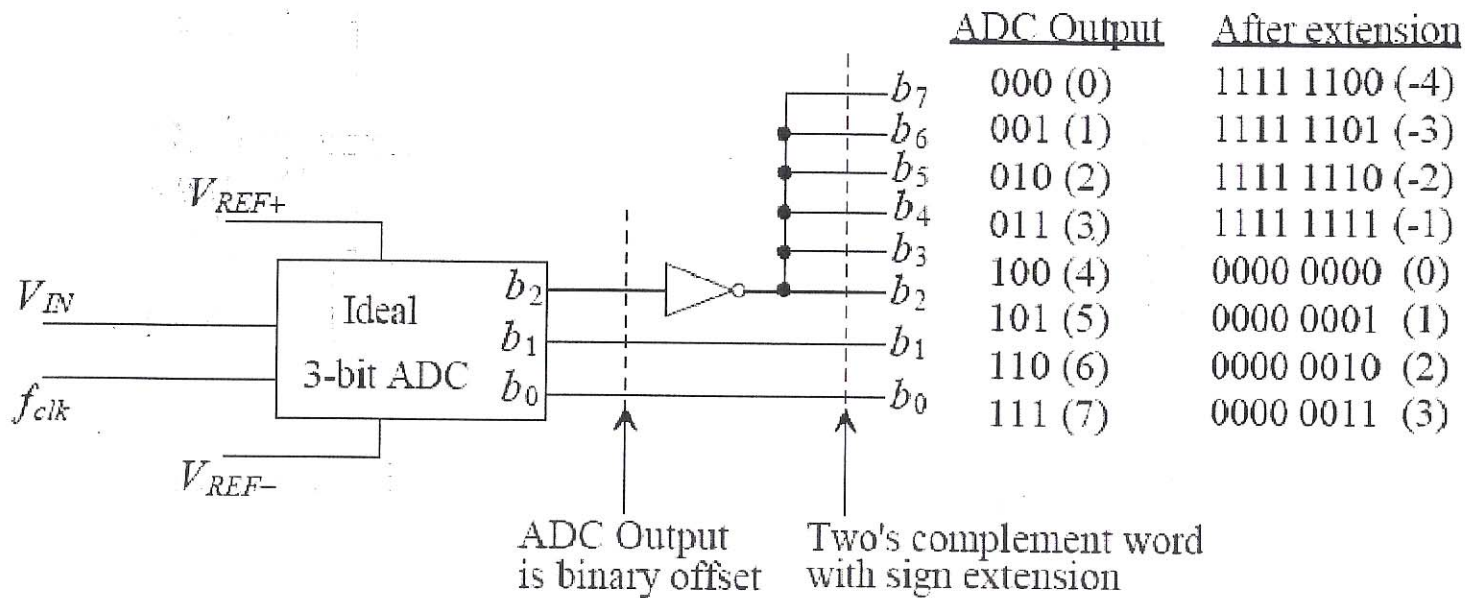
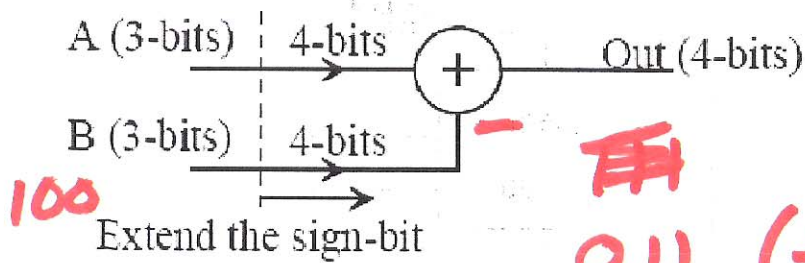


Figure 4.6 Showing how to change the output of an ADC to two's complement and how to extend the sign bit.

1 → 01 → 0000 0001 (+1)
 0 → 11 → 1111 1111 (-1)
 offset binary 2-bit two's complement

9)

011 Adding two's complement numbers



A	B	Out (A+B)
0011 (+3)	0011 (+3)	0110 (+6)
1100 (-4)	1100 (-4)	1000 (-8) OF!
1111 (-1)	0001 (+1)	0000 (0) OF!
0011 (+3)	1111 (-1)	0110 (+2) OF!
0001 (+1)	1100 (-4)	1101 (-3)

111 -1
110 -2
100 -3

$$\begin{array}{r} 1100 (-4) \\ + 1100 (-4) \\ \hline 1\ 1000 (-8) \end{array}$$

$$\begin{array}{r} 1111 (-1) \\ + 0001 (+1) \\ \hline 1\ 0000 (0) \end{array}$$

$100 (-3)$
 $3 - (-3) = 6$

Overflow (OF!), toss into the bit bucket (ignore).

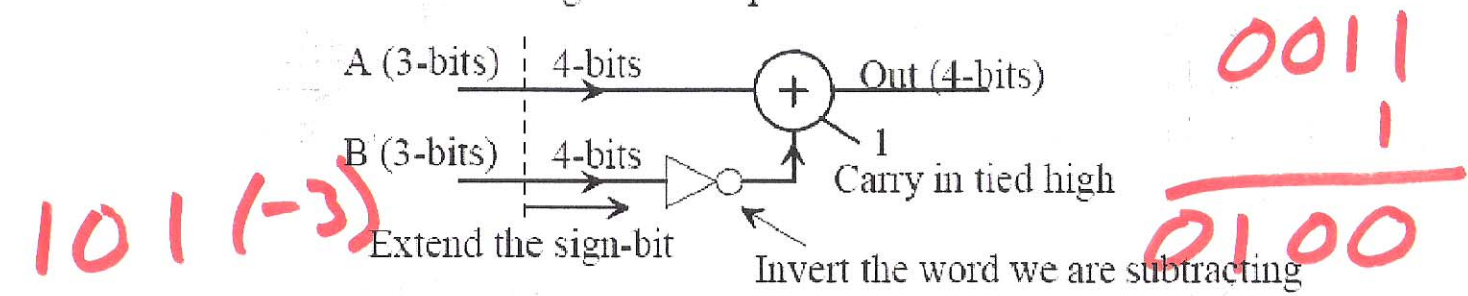
Complement
Subtraction
tie CARRY
High

Figure 4.7 Showing how two's complement numbers are added.

$$\begin{array}{r} 0011 \\ 1100 \rightarrow 0011 \\ + \quad 0 \\ \hline 0110 \rightarrow \end{array}$$

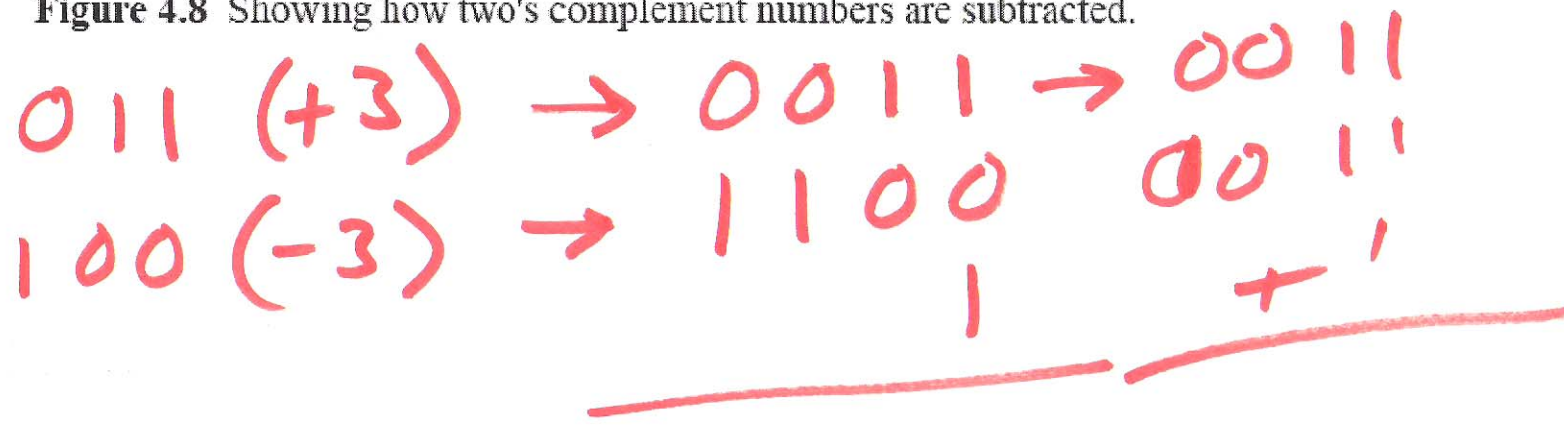
10)

Subtracting two's complement numbers



	A	B	$\bar{B} + 1$	Out (A-B)
111 - 1	0011 (+3)	1100 (-4)	0100 (+4)	0111 (+7) ←
110 - 2	1100 (-4)	1100 (-4)	0100 (+4)	0000 (0) OF!
101 (-3)	1111 (-1)	0001 (+1)	1111 (-1)	1110 (-2) OF!
	0011 (+3)	1111 (-1)	0001 (+1)	0100 (+4)
	0010 (+2)	0001 (+1)	1111 (-1)	0001 (+1) OF!

Figure 4.8 Showing how two's complement numbers are subtracted.



11)

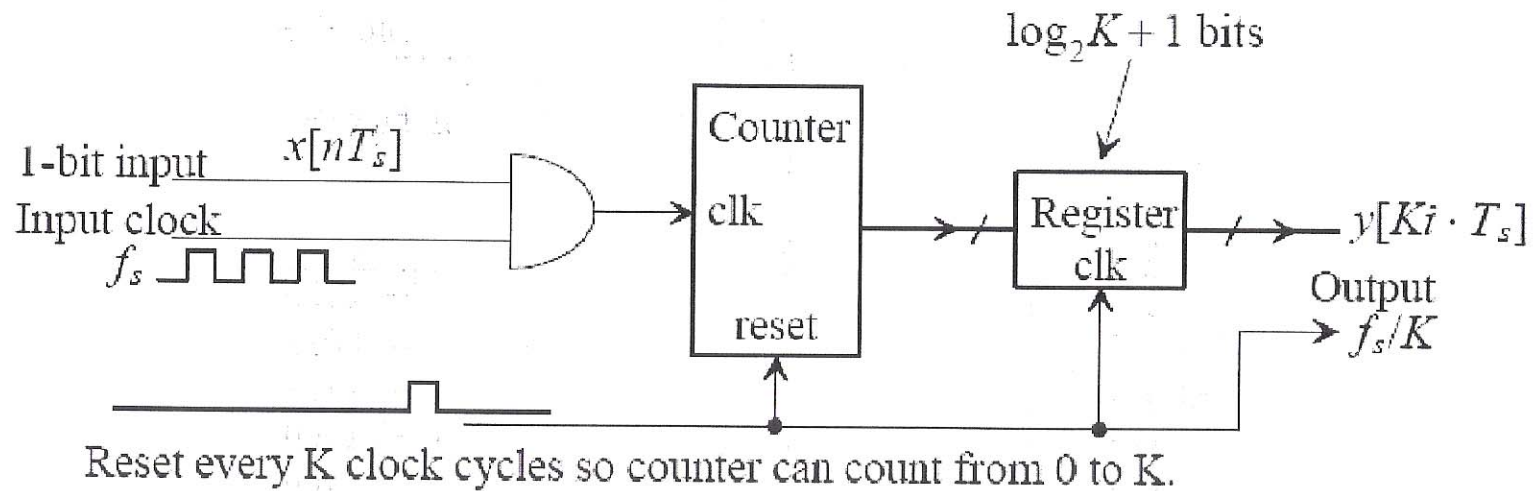


Figure 4.9 Using the counter as a digital filter.

$$Y = \frac{x_1 + x_2 + x_3 + x_4 + \dots + x_N}{N}$$

$$Y(z) = \frac{X(z)}{N} \left(1 + z^{-1} + z^{-2} + z^{-3} + \dots + z^{-(N-1)} \right)$$

$$\frac{Y(z)}{X(z)} = \frac{1}{N} \frac{1 - z^{-N}}{1 - z^{-1}}$$

12)

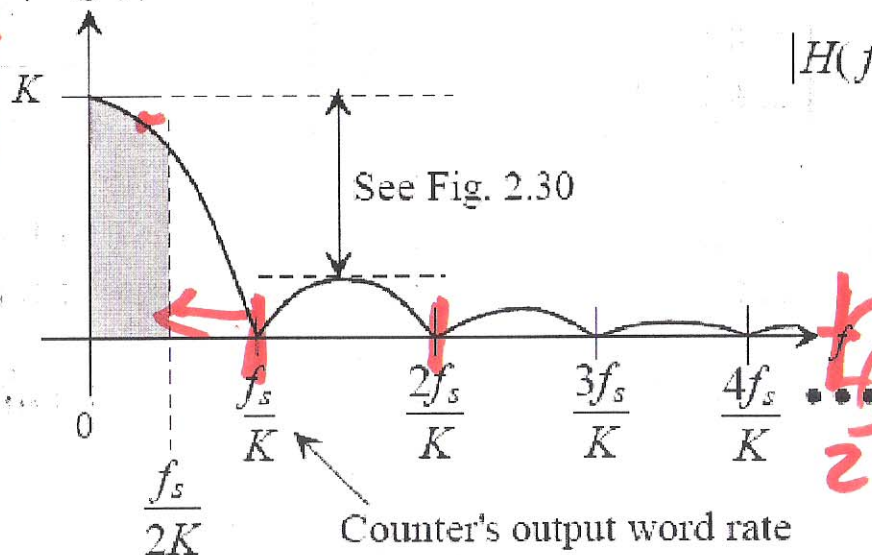
$$H(z) = \frac{1 - z^{-K}}{1 - z^{-1}}$$

$$|H(f)|$$

$$|H(f)| = K \frac{\text{Sinc}\left(K\pi \cdot \frac{f}{f_s}\right)}{\text{Sinc}\left(\pi \cdot \frac{f}{f_s}\right)}$$

1000 1000
1000

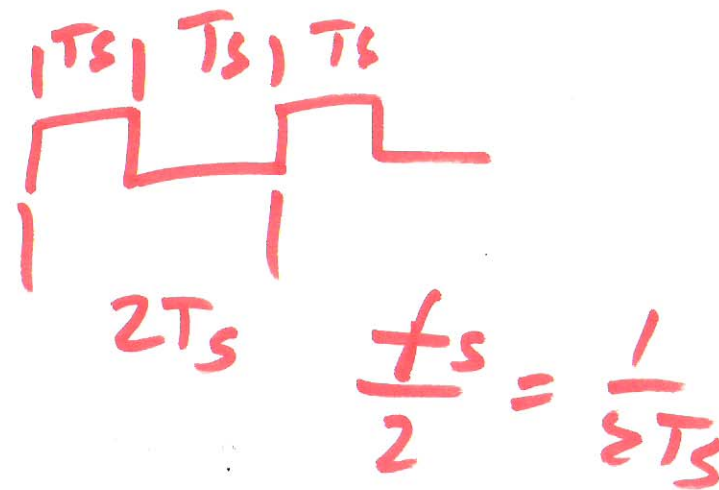
256
K



256

Figure 4.10 Frequency response of a Sinc-shaped digital filter.

1/4
10 10
256
-1 +1 -1 +1



13)

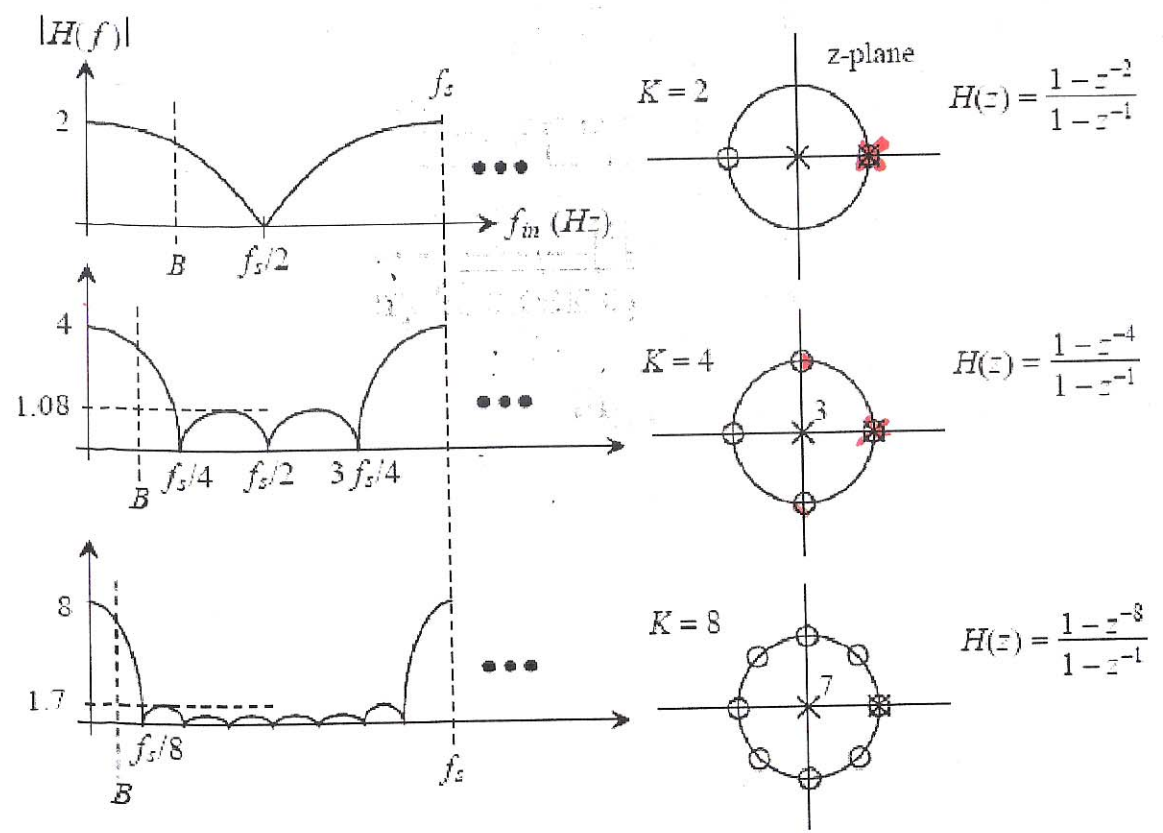
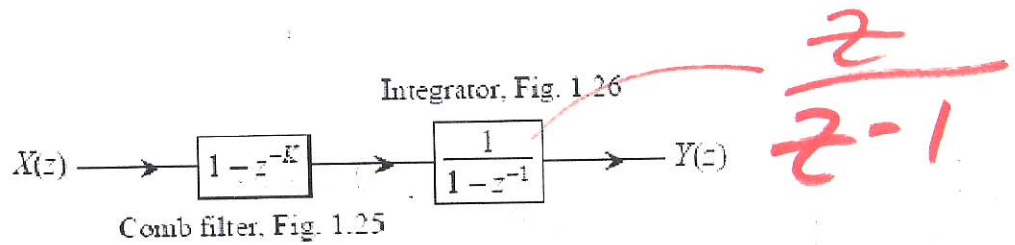


Figure 4.13 Lowpass Sinc-response filters with varying values of K.

14)