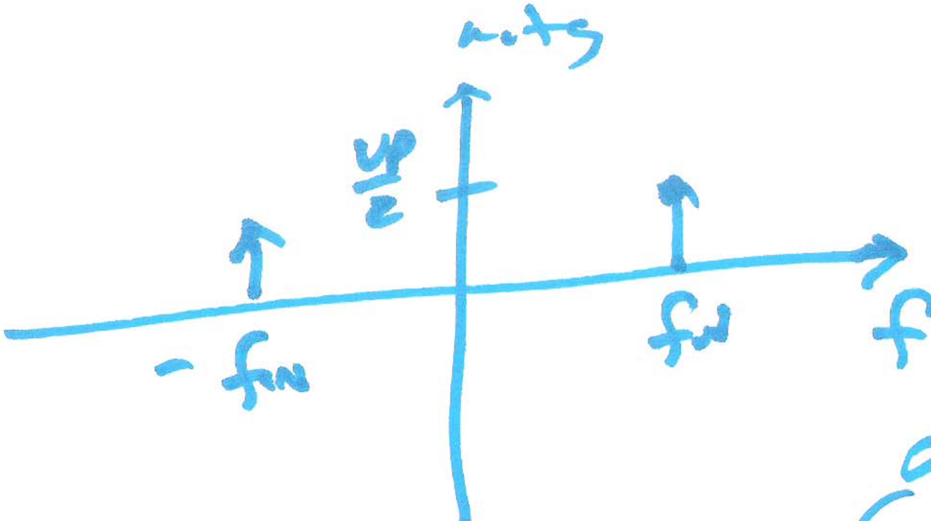
# ECG 722

## Interpolation & K-path Sampling

$v_{in}(t) = V_p \sin 2\pi f_{in} t$

Sept. 10, 2014



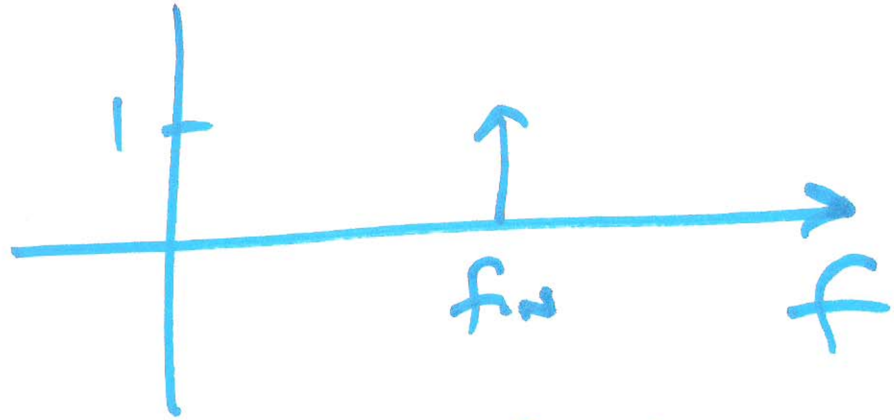

$$|e^{-j2\pi f_{in} t}| = 1$$

$$\angle e^{j2\pi f_{in} t} = 2\pi f_{in} t$$

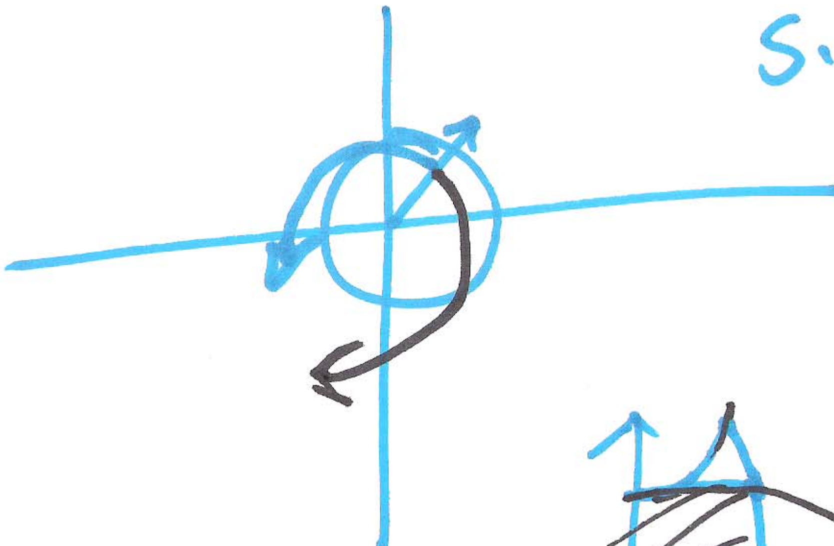
$$\mathcal{F}\{e^{j2\pi f_{in} t}\} = \int_{-\infty}^{\infty} e^{j2\pi f_{in} t} e^{-j2\pi f \cdot t} \, dt$$

$$\int_{-\infty}^{\infty} e^{j2\pi (f_{in}+f) \cdot t} \, dt = \delta(f - f_{in})$$

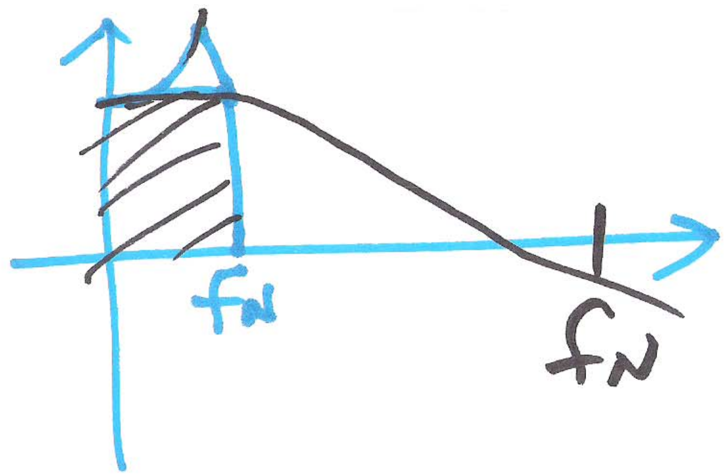
$$\mathcal{F}\left\{e^{j2\pi f_{in} t}\right\}$$

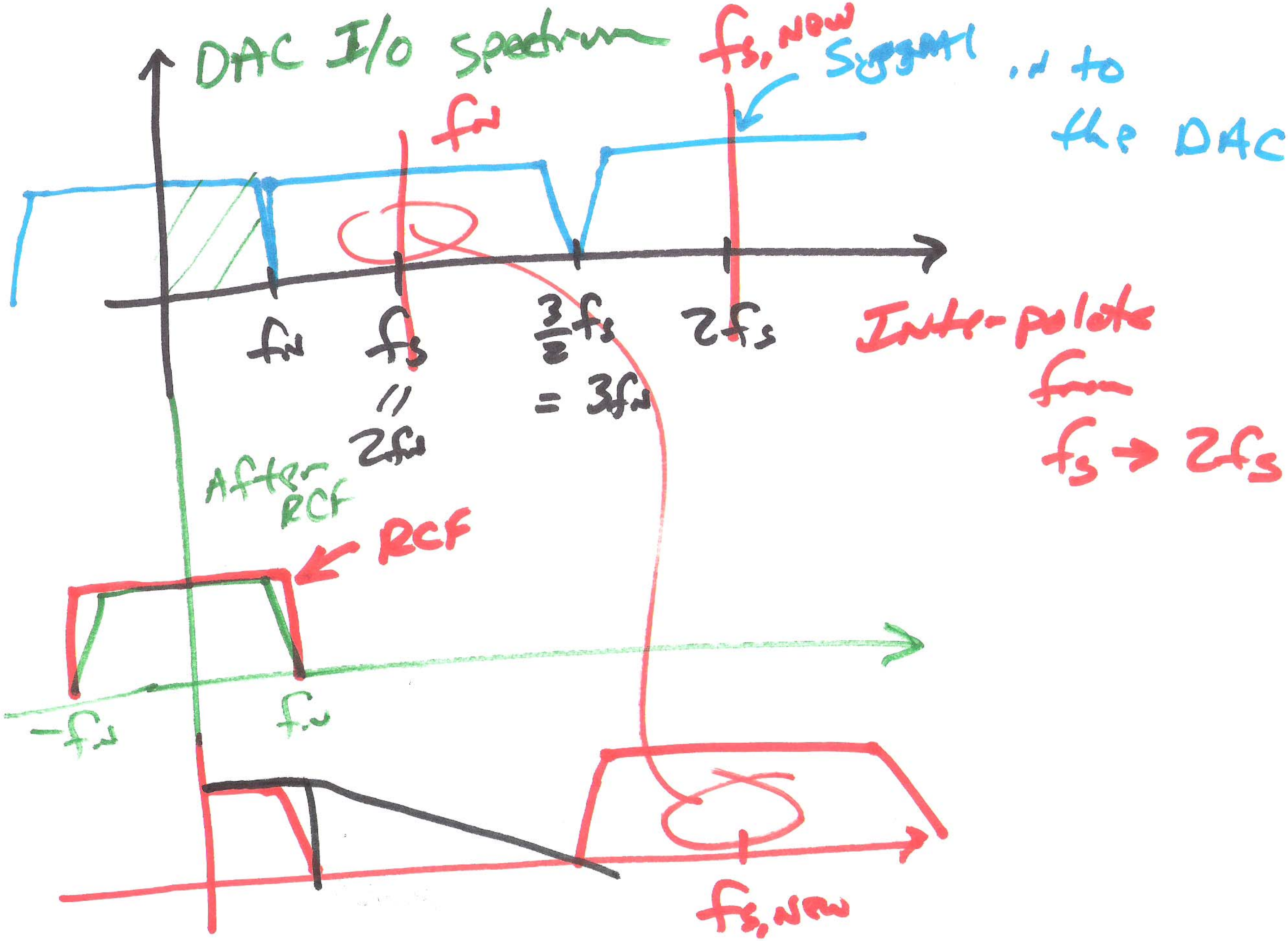



$\sin 2\pi f_{in} \cdot t$



$\sin 2\pi(-f_{in})t$




DAC I/O spectrum
$f_{s,new}$
Signal in to the DAC
$f_N$
$f_N$
$f_s$
// $2f_N$
$\frac{3}{2}f_s$ $= 3f_N$
$2f_s$
Inter-polate from $f_s \rightarrow 2f_s$
After RCF
RCF
$-f_N$
$f_N$
$f_{s,new}$

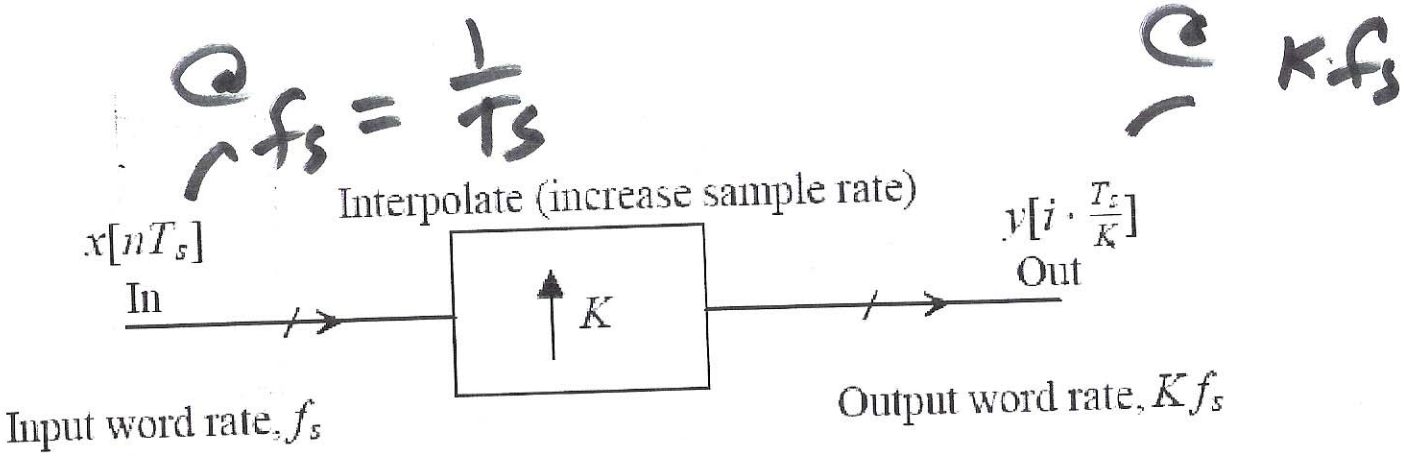



**Figure 2.24** Block diagram of an interpolation block.

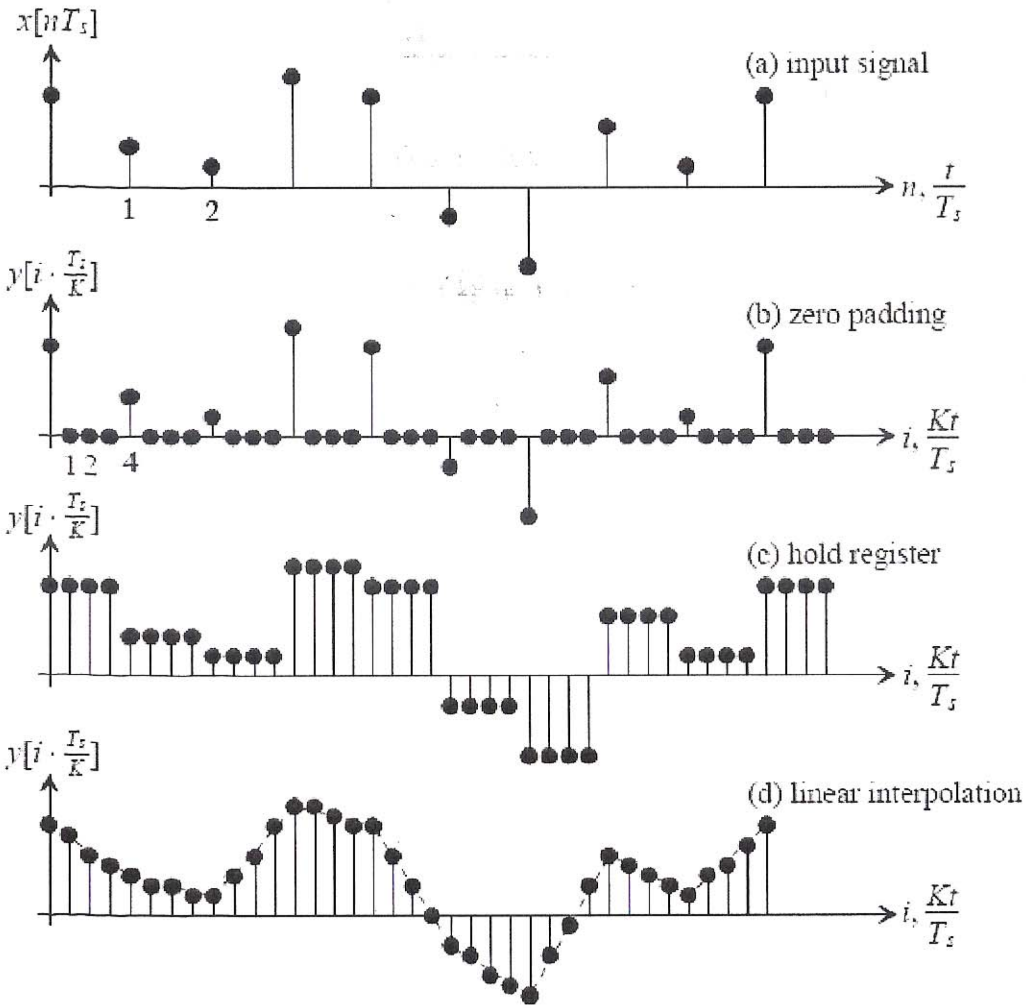



**Figure 2.25** Types of interpolation.



5)

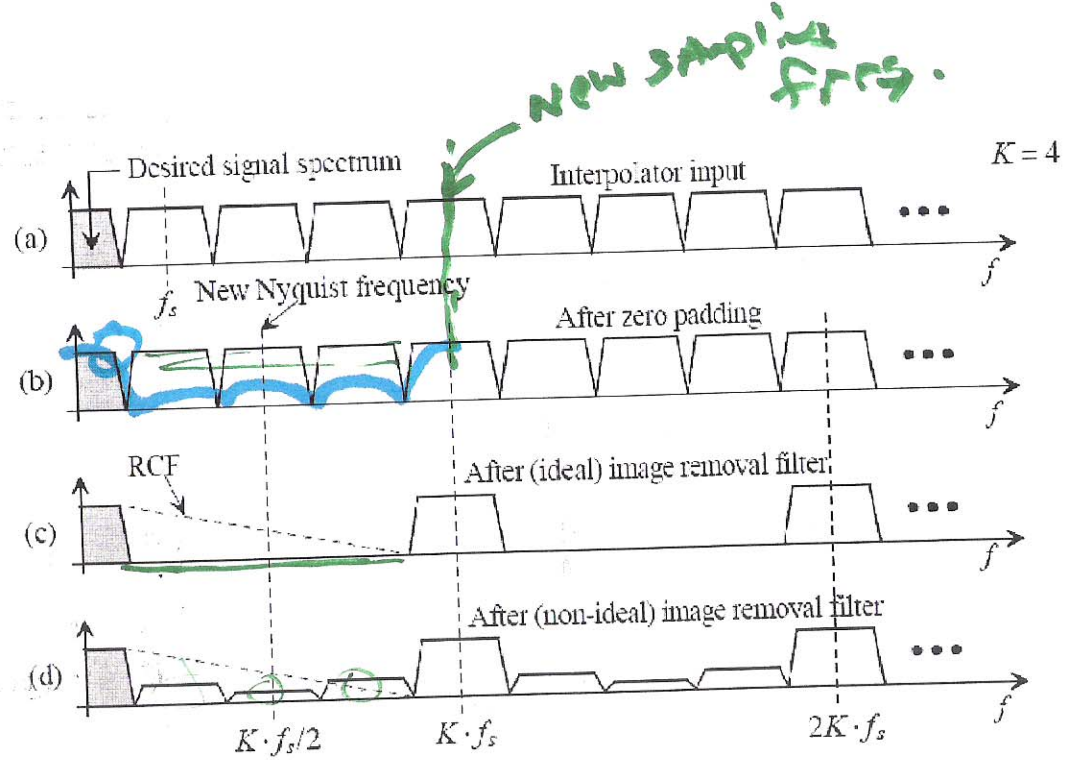
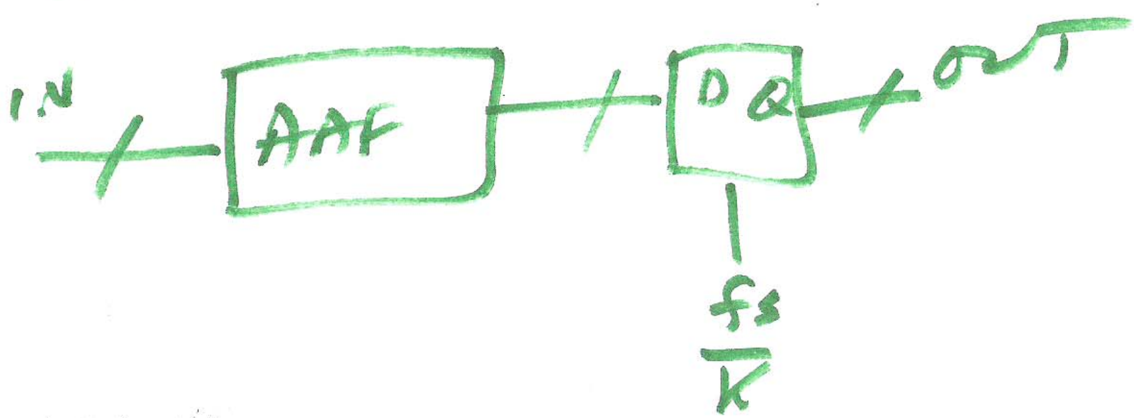



**Figure 2.27** Example spectrums when zero padding interpolation is employed.

Decimator






6)

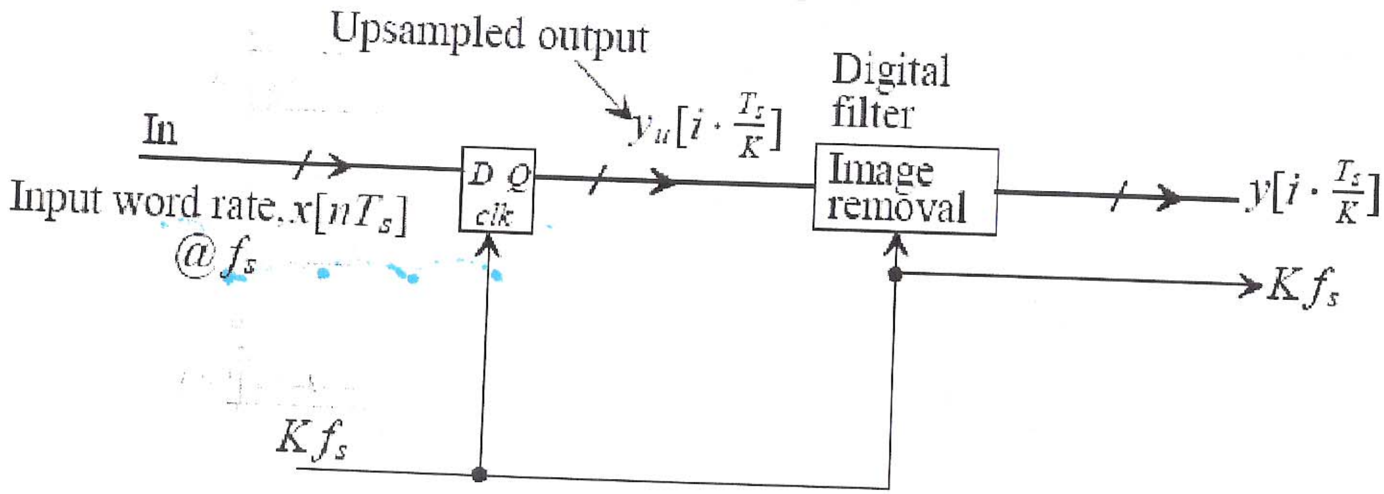
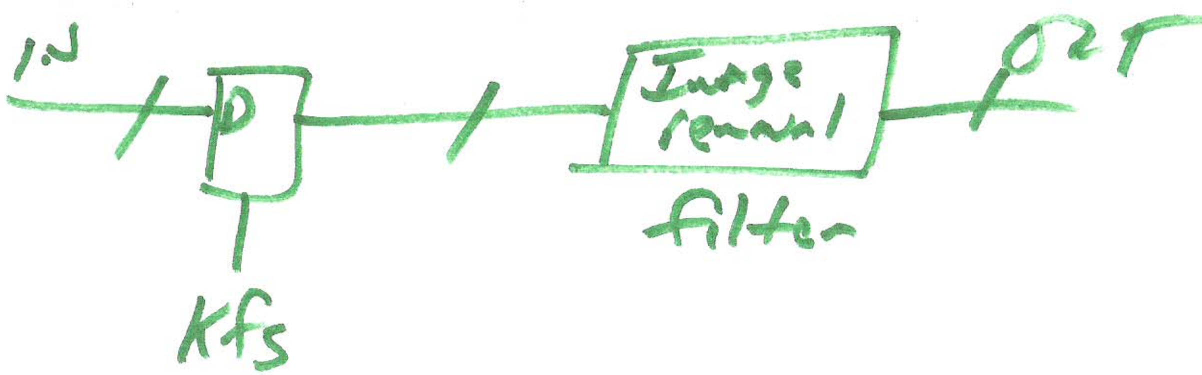



**Figure 2.28** An interpolation block using a hold register, see spectrums in Fig. 2.32.




7)

$$y\left[i\cdot\frac{T_s}{K}\right] = \sum_{i=K\cdot N}^{K(N+1)-1} \frac{1}{K} x\left[i\cdot\frac{T_s}{K}\right] = x[NT_s]$$

$$y[NT_s] = \frac{1}{K}\left[x\left[KN\cdot\frac{T_s}{K}\right] + x\left[(KN+1)\frac{T_s}{K}\right] + x\left[K(N+2)\frac{T_s}{K}\right]\right.$$

$N=0,\ K=4$

$$K\,y[NT_s] = x[0] + x[1] + x[2] + x[3]$$

$$K\,y(z) = X[z]\left[1 + z^{-1} + z^{-2} + z^{-3}\right]$$

$$H(z) = \frac{y(z)}{X(z)} = \frac{1}{K}\frac{(1+z^{-1}+z^{-2}+z^{-3})\cdot(1-z^{-1})}{(1-z^{-1})}$$

$$1 - z^{-4} \Rightarrow H(f)_{sinc}$$

$$H(z) = \frac{1-z^{-4}}{1-z^{-1}}$$

$$H(z) = \frac{Y_u(z)}{X(z)} = \frac{1}{K}\left[1 + z^{1/K} + z^{2/K} + \dots z^{(K-1)/K}\right]$$

$(1 - z^{1/K})$

Multiply through by $(1 - z^{1/K})$

$$\frac{1}{K}\left[(1 - z^{1/K}) + z^{1/K}(1 - z^{1/K}) + z^{2/K}(1 - z^{1/K} \dots\right.$$

$$H(z) = \frac{1}{K}\frac{1 - z}{1 - z^{1/K}}$$

$$e^{j2\pi f/f_s} = f \cdot T_s$$

$$H_u(f) = \frac{1}{K}\,\frac{1 - e^{j2\pi f/f_s}}{1 - e^{j2\pi \frac{f}{Kf_s}}} \rightarrow f \cdot \frac{T_s}{K}$$

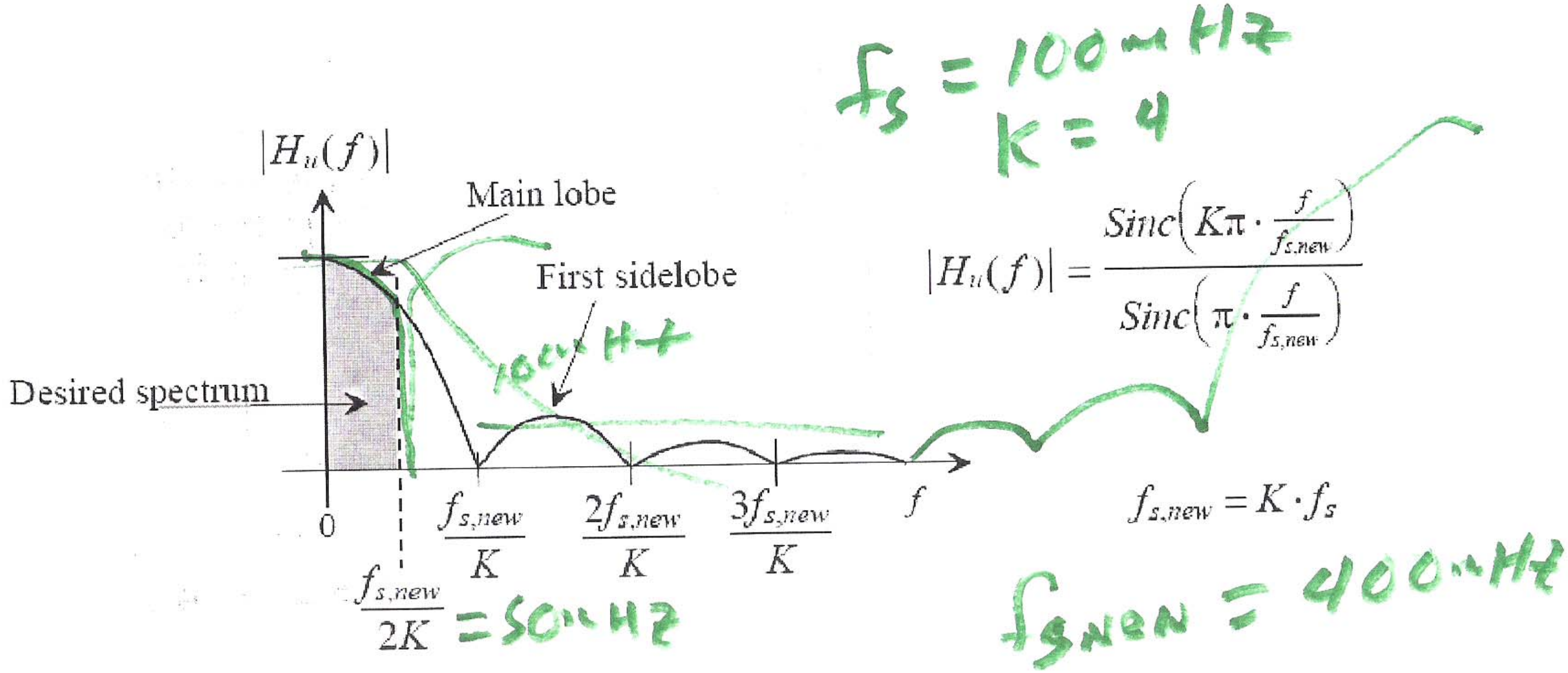
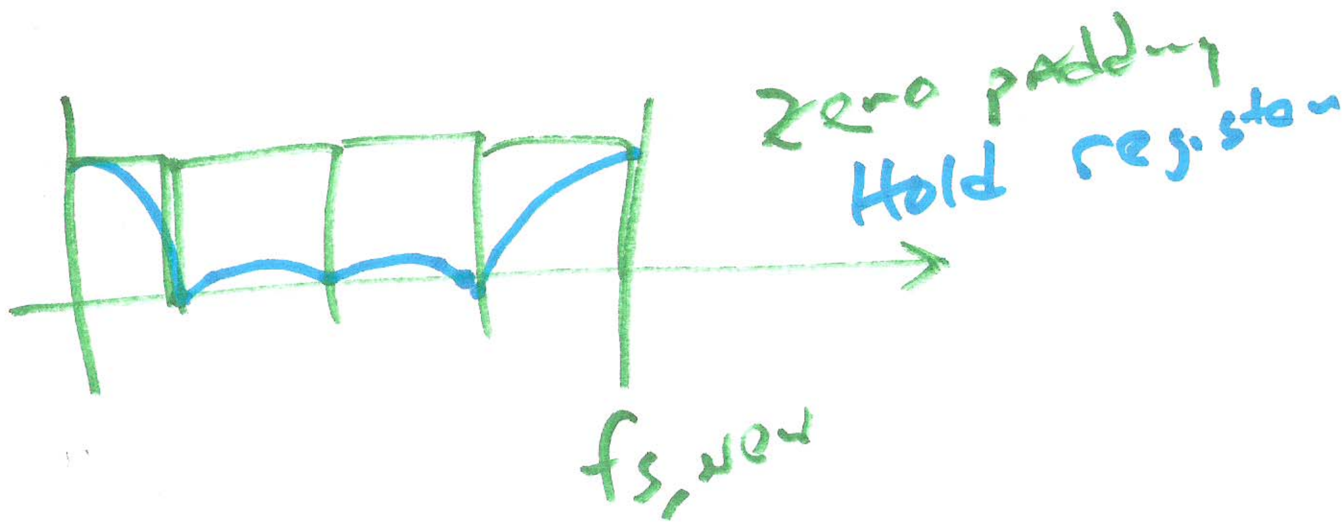



**Figure 2.29** Frequency response interpolated data sees using a hold register.



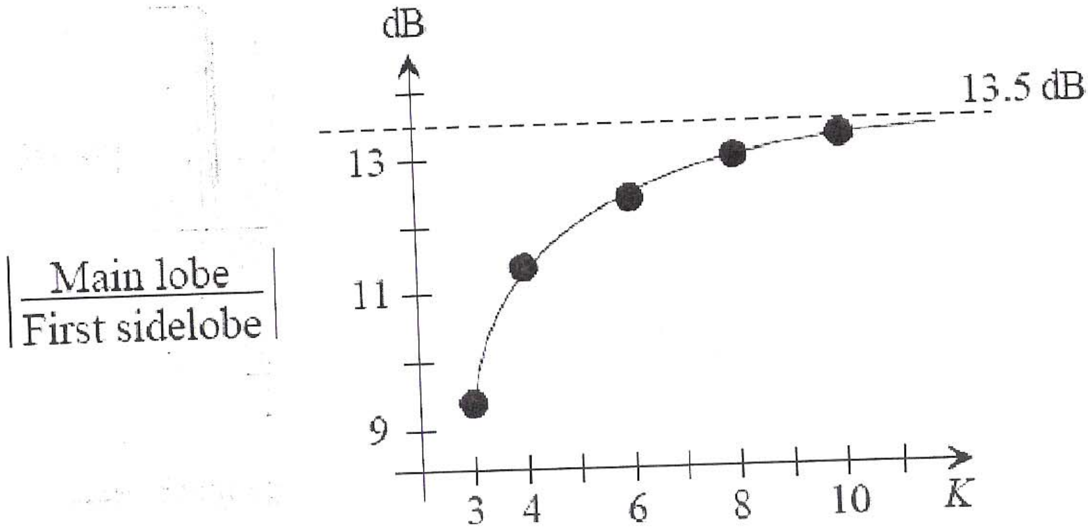



**Figure 2.30** Attenuation versus K.

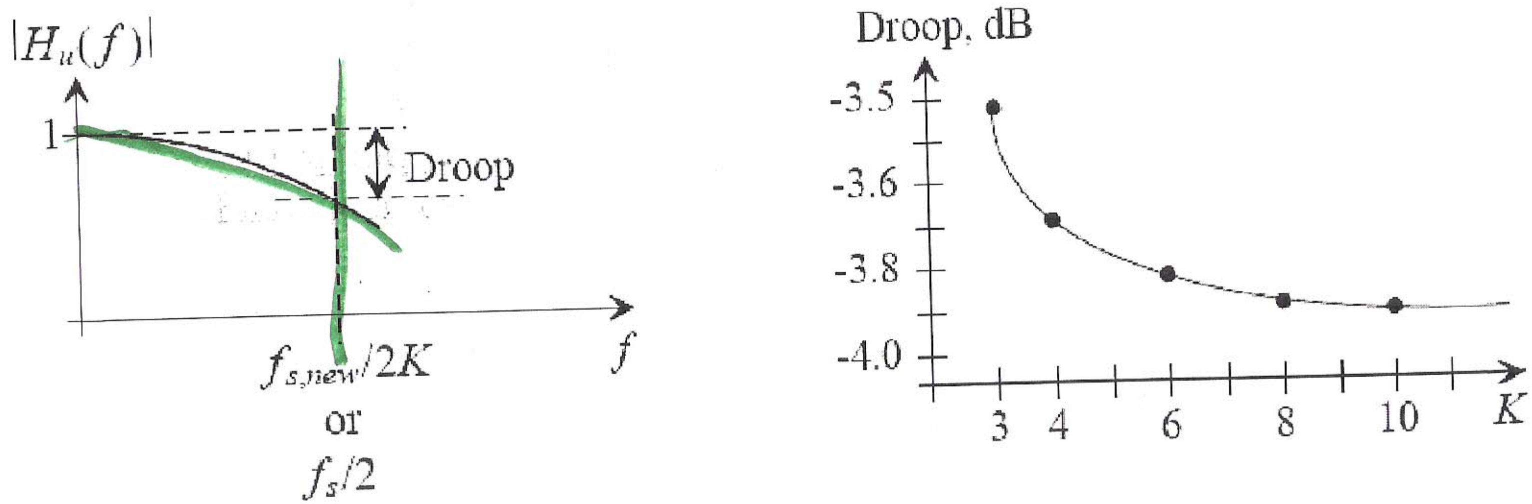



**Figure 2.31** Droop at edge of signal bandwidth.



12)

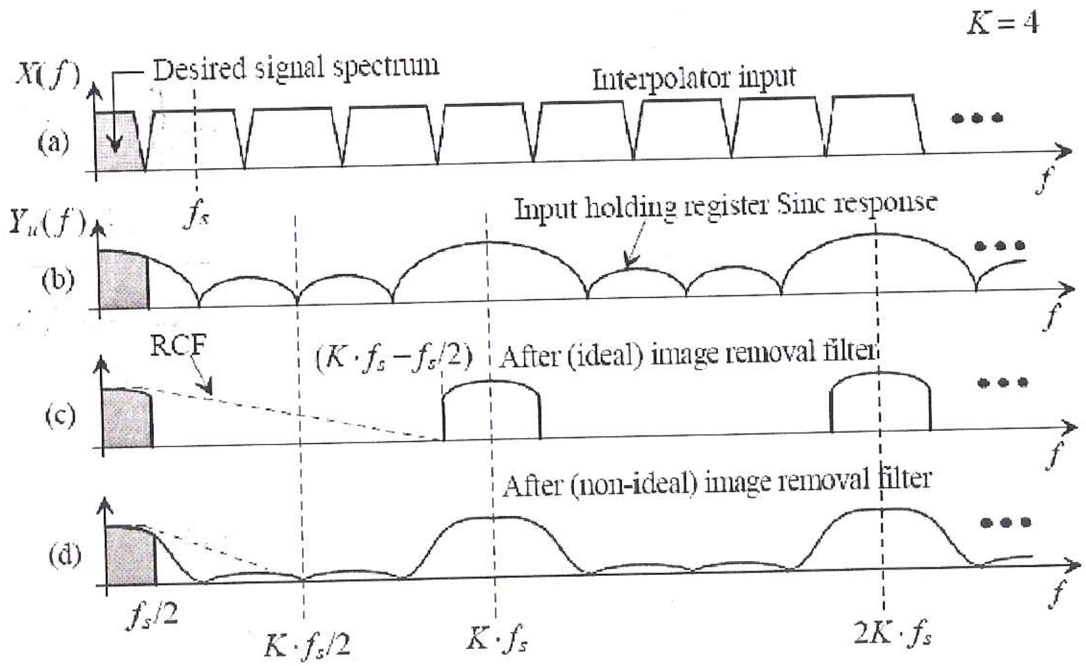



**Figure 2.32** Example spectrums when interpolation using a hold register is employed.