CHAPTER 2

Power Computations

2.1 INTRODUCTION

Power computations are essential in analyzing and designing power electronics circuits. Basic power concepts are reviewed in this chapter, with particular emphasis on power calculations for circuits with nonsinusoidal voltages and currents. Extra treatment is given to some special cases that are encountered frequently in power electronics. Power computations using the circuit simulation program PSpice are demonstrated.

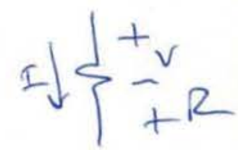
2.2 POWER AND ENERGY

Instantaneous Power

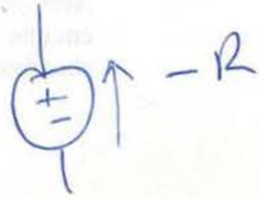
The instantaneous power for any device is computed from the voltage across it and the current in it. *Instantaneous power* is

$$p(t) = v(t)i(t) \tag{2-1}$$

This relationship is valid for any device or circuit. Instantaneous power is generally a time-varying quantity. If the passive sign convention illustrated in Fig. 2-1*a* is observed, the device is absorbing power if $p(t)$ is positive at a specified value of time t. The device is supplying power if $p(t)$ is negative. Sources frequently have an assumed current direction consistent with supplying power. With the convention of Fig. 2-1*b*, a positive $p(t)$ indicates the source is supplying power.



give battery example



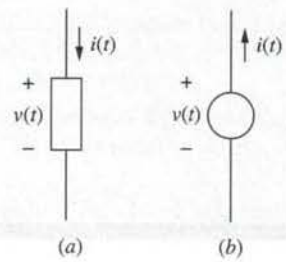


Figure 2-1 (*a*) Passive sign convention: $p(t) > 0$ indicates power is being absorbed; (*b*) $p(t) > 0$ indicates power is being supplied by the source.

Energy

Energy, or work, is the integral of instantaneous power. Observing the passive sign convention, energy absorbed by a component in the time interval from t_1 to t_2 is

$$W = \int_{t_1}^{t_2} p(t)\, dt \tag{2-2}$$

If $v(t)$ is in volts and $i(t)$ is in amperes, power has units of watts and energy has units of joules.

Average Power

Periodic voltage and current functions produce a periodic instantaneous power function. Average power is the time average of $p(t)$ over one or more periods. *Average power P* is computed from

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\, dt = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\, dt \tag{2-3}$$

where T is the period of the power waveform. Combining Eqs. (2-3) and (2-2), power is also computed from energy per period.

$$P = \frac{W}{T} \tag{2-4}$$

Average power is sometimes called *real power* or *active power*, especially in ac circuits. The term *power* usually means average power. The total average power absorbed in a circuit equals the total average power supplied.

EXAMPLE 2-1

Power and Energy

Voltage and current, consistent with the passive sign convention, for a device are shown in Fig. 2-2*a* and *b*. (*a*) Determine the instantaneous power $p(t)$ absorbed by the device. (*b*) Determine the energy absorbed by the device in one period. (*c*) Determine the average power absorbed by the device.

■ Solution

(*a*) The instantaneous power is computed from Eq. (2-1). The voltage and current are expressed as

$$v(t) = \begin{cases} 20\text{ V} & 0 < t < 10\text{ ms} \\ 0 & 10\text{ ms} < t < 20\text{ ms} \end{cases}$$

$$i(t) = \begin{cases} 20\text{ V} & 0 < t < 6\text{ ms} \\ -15\text{ A} & 6\text{ ms} < t < 20\text{ ms} \end{cases}$$

Instantaneous power, shown in Fig. 2-2*c*, is the product of voltage and current and is expressed as

$$p(t) = \begin{cases} 400\text{ W} & 0 < t < 6\text{ ms} \\ -300\text{ W} & 6\text{ ms} < t < 10\text{ ms} \\ 0 & 10\text{ ms} < t < 20\text{ ms} \end{cases}$$

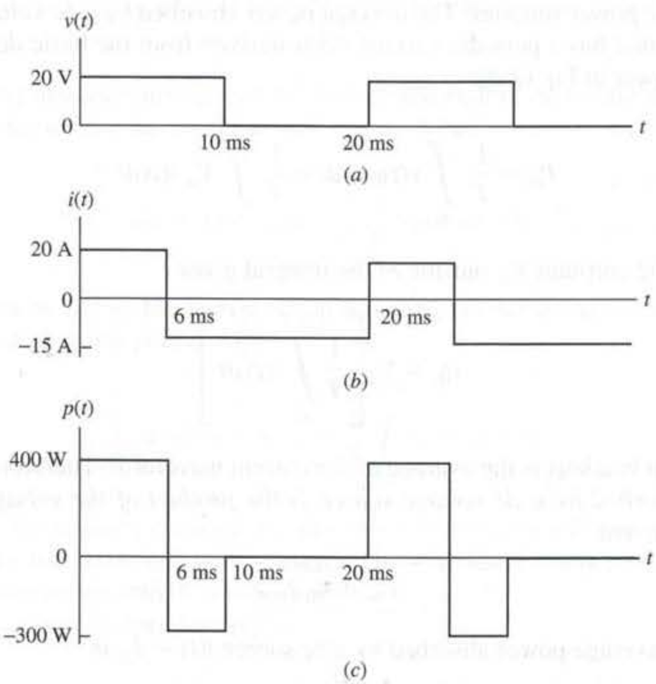


Figure 2-2 Voltage, current, and instantaneous power for Example 2-1.

(*b*) Energy absorbed by the device in one period is determined from Eq. (2-2).

$$W = \int_0^T p(t)\,dt = \int_0^{0.006} 400\,dt + \int_{0.006}^{0.010} -300\,dt + \int_{0.010}^{0.020} 0\,dt = 2.4 - 1.2 = 1.2\text{ J}$$

(*c*) Average power is determined from Eq. (2-3).

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{0.020}\left(\int_0^{0.006} 400\,dt + \int_{0.006}^{0.010} -300\,dt + \int_{0.010}^{0.020} 0\,dt\right)$$

$$= \frac{2.4 - 1.2 - 0}{0.020} = 60\text{ W}$$

Average power could also be computed from Eq. (2-4) by using the energy per period from part (*b*).

$$P = \frac{W}{T} = \frac{1.2\text{ J}}{0.020\text{ s}} = 60\text{ W}$$

A special case that is frequently encountered in power electronics is the power absorbed or supplied by a dc source. Applications include battery-charging circuits and dc power supplies. The average power absorbed by a dc voltage source $v(t) = V_{dc}$ that has a periodic current $i(t)$ is derived from the basic definition of average power in Eq. (2-3):

$$P_{dc} = \frac{1}{T}\int_{t_0}^{t_0+T} v(t)i(t)\,dt = \frac{1}{T}\int_{t_0}^{t_0+T} V_{dc}\, i(t)\,dt$$

Bringing the constant V_{dc} outside of the integral gives

$$P_{dc} = V_{dc}\left[\frac{1}{T}\int_{t_0}^{t_0+T} i(t)\,dt\right]$$

The term in brackets is the average of the current waveform. Therefore, *average power absorbed by a dc voltage source is the product of the voltage and the average current.*

$$\boxed{P_{dc} = V_{dc} I_{avg}} \tag{2-5}$$

Similarly, average power absorbed by a dc source $i(t) = I_{dc}$ is

$$P_{dc} = I_{dc} V_{avg} \tag{2-6}$$