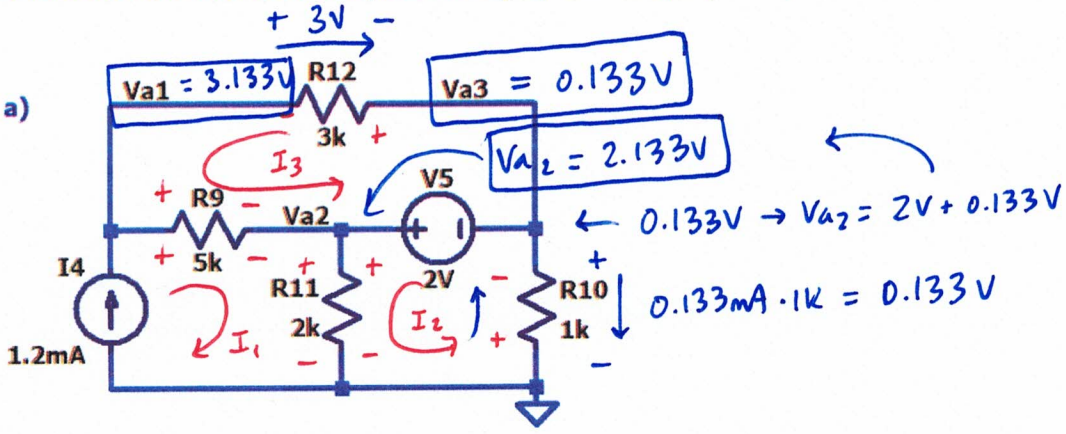
HW8: Problem 2 Solutions

2A. MESH

2. Determine the voltages and currents in the circuits given below using **both mesh analysis and superposition**. Verify your work using LTspice. No need to verify twice, but there should be hand calculations for both mesh analysis and superposition. (10 points)

$Va_1 = 3.133V$ ✓
$Va_2 = 2.133V$ ✓
$Va_3 = 0.133V$ ✓

a)



① $I_1 = 1.2mA$

② $+2V - (I_1 + I_2) \cdot 2K - I_2 \cdot 1K = 0$

$2V - (1.2mA + I_2) \cdot 2K - I_2 \cdot 1K = 0$

$2V - 2.4V - I_2 \cdot 3K = 0$

$\frac{-0.4V}{3K} = \frac{I_2 \cdot 3K}{3K} \rightarrow$ $I_2 = -0.133mA$

③ $-I_3 \cdot 3K - (I_1 + I_3) \cdot 5K - 2V = 0$

$-I_3 \cdot 3K - (1.2mA + I_3) \cdot 5K - 2V = 0$

$-I_3 \cdot 3K - 6V - I_3 \cdot 5K - 2V = 0$

$\frac{-I_3 \cdot 8K}{-8K} = \frac{8V}{-8K}$

$I_3 = -1mA$

$Va_2 = (I_1 + I_2) \cdot 2K$

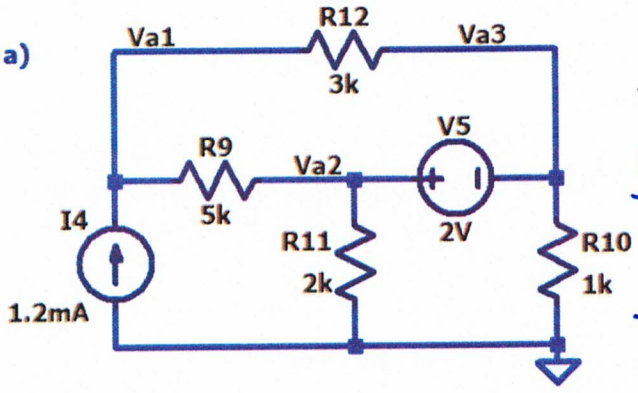
$Va_2 = (1.2mA + (-0.133mA)) \cdot 2K$

$Va_2 = 2.133V$

2A. SUPERPOSITION

$Va_1 = 3.133V$ ✓
$Va_2 = 2.133V$ ✓
$Va_3 = 0.133V$ ✓

a)

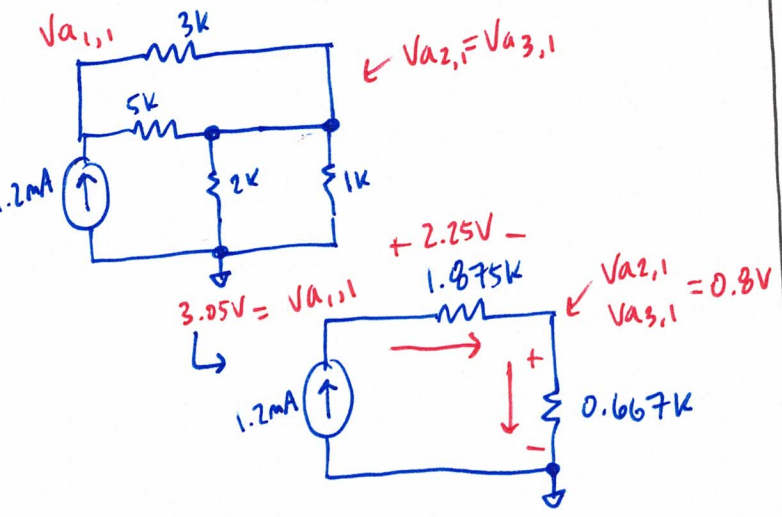


$Va_1 = Va_{1,1} + Va_{1,2}$ ✓
$Va_1 = 3.05V + 0.08V = 3.13V$

$Va_2 = Va_{2,1} + Va_{2,2}$ ✓
$Va_2 = 0.8V + 1.33V = 2.13V$

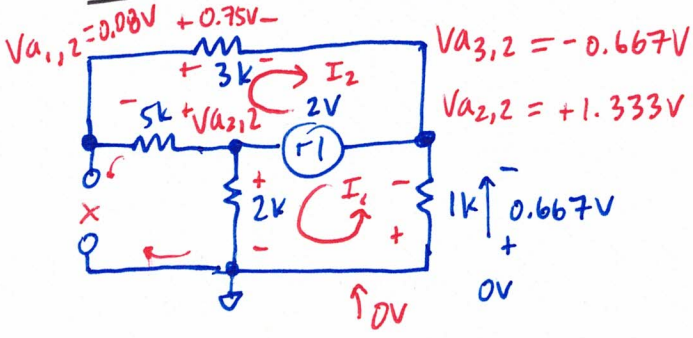
$Va_3 = Va_{3,1} + Va_{3,2}$ ✓
$Va_3 = 0.8V + (-0.667V) = 0.133V$

PART 1: No voltage source



$Va_{1,1} = 3.05V$
$Va_{2,1} = 0.8V$
$Va_{3,1} = 0.8V$

PART 2: No current source



① $+2V - I_1 \cdot 2k - I_1 \cdot 1k = 0$

$\frac{2V}{3k} = \frac{I_1 \cdot 3k}{3k} \rightarrow I_1 = 0.667mA$

② $+2V - I_2 \cdot 5k - I_2 \cdot 3k = 0$

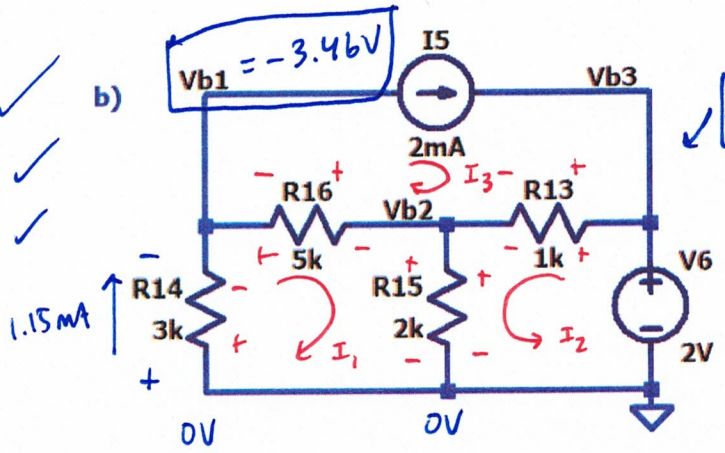
$\frac{2V}{8k} = \frac{I_2 \cdot 8k}{8k} \rightarrow I_2 = 0.25mA$

$Va_{1,2} = 0.08V$
$Va_{2,2} = 1.33V$
$Va_{3,2} = -0.667V$

2B. MESH

$Vb_1 = -3.46V$ ✓
$Vb_2 = 0.77V$ ✓
$Vb_3 = 2V$ ✓

b)



① $I_3 = 2mA$ ✓

$I_1 = -1.5(-0.77mA)$ ✓

$I_1 = 1.15mA$

② $+2V - (I_2 + I_3)\cdot 1K - (I_1 + I_2)\cdot 2K = 0$

$+2V - (I_2 + 2mA)\cdot 1K - I_1 \cdot 2K - I_2 \cdot 2K = 0$

$2V - I_2 \cdot 1K - 2V - I_1 \cdot 2K - I_2 \cdot 2K = 0$

$- I_2 \cdot 1K - I_2 \cdot 2K - I_1 \cdot 2K = 0$

$\frac{-I_2 \cdot 3K}{2K} = \frac{I_1 \cdot 2K}{2K} \rightarrow I_1 = -1.5 \cdot I_2$ ②

③ $-I_1 \cdot 3K - (I_1 - I_3)\cdot 5K - (I_1 + I_2)\cdot 2K = 0$

$-I_1 \cdot 3K - (I_1 - 2mA)\cdot 5K - (I_1 + I_2)\cdot 2K = 0$

$-I_1 \cdot 3K - I_1 \cdot 5K + 10V - I_1 \cdot 2K - I_2 \cdot 2K = 0$

$-I_1 \cdot 10K - I_2 \cdot 2K + 10V = 0$ ③

$-(-1.5 \cdot I_2 \cdot 10K) - I_2 \cdot 2K + 10V = 0$

$15K \cdot I_2 - 2K I_2 + 10V = 0$ ✓

$\frac{13K \cdot I_2}{13K} = \frac{-10V}{13K} \rightarrow I_2 = -0.77mA$

$Vb_2 = (I_1 + I_2)\cdot 2K$

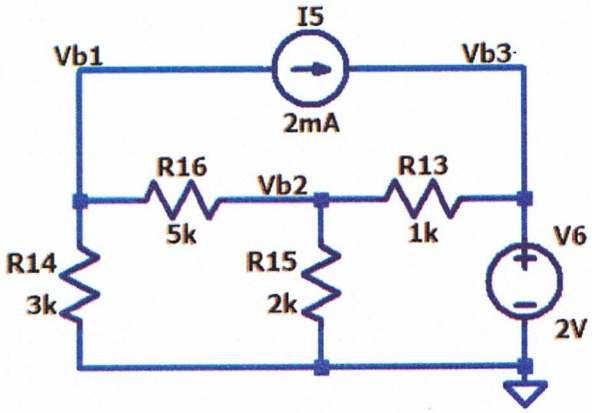
$Vb_2 = (1.15mA - 0.77mA)\cdot 2K$

$Vb_2 = 0.77V$

2B. SUPERPOSITION

$Vb_1 = -3.46V$ ✓
$Vb_2 = 0.77V$ ✓
$Vb_3 = 2V$ ✓

b)

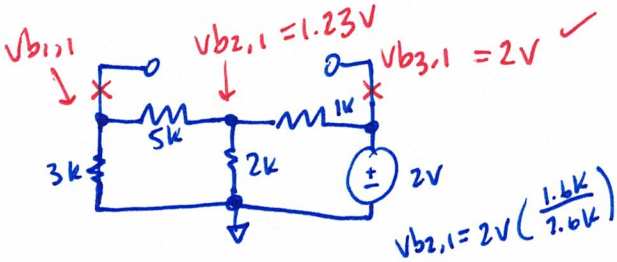


$Vb_1 = Vb_{1,1} + Vb_{1,2}$ ✓
$Vb_1 = 0.46V + (-3.13)V$ $= -3.47V$

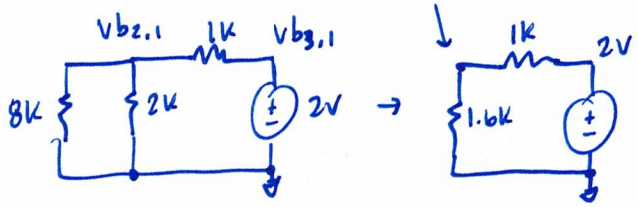
$Vb_2 = Vb_{2,1} + Vb_{2,2}$ ✓
$Vb_2 = 1.23V + (-0.46V)$ $= 0.77V$

$Vb_3 = Vb_{3,1} + Vb_{3,2}$
$Vb_3 = 2V + 0V$ $= 2V$ ✓

PART 1. No current source



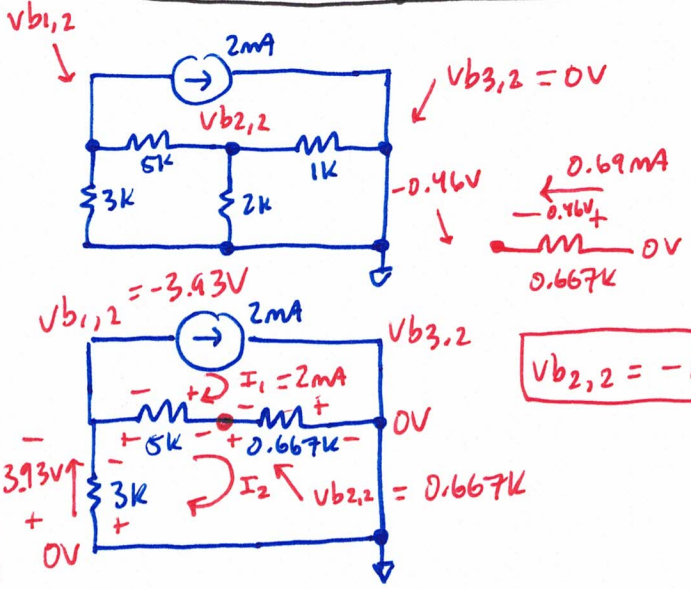
$Vb_{2,1} = 2V\left(\frac{1.6K}{7.6K}\right) = 1.23V$ ✓



$Vb_{1,1} = 1.23V\left(\frac{3K}{8K}\right) = 0.46V$ ✓

$Vb_{1,1} = 0.46V$
$Vb_{2,1} = 1.23V$
$Vb_{3,1} = 2V$

PART 2. No voltage source



$Vb_{2,2} = -0.46V$

$\rightarrow -I_2 \cdot 3K - (I_2 - 2mA) \cdot 5K - (I_2 - 2mA) \cdot 0.667K = 0$

$-I_2 \cdot 3K - (I_2 - 2mA) \cdot 5.667K = 0$

$-I_2 \cdot 3K - I_2 \cdot 5.667K + 11.33V = 0$

$-I_2 \cdot 8.667K = -11.33V$

$\frac{-I_2 \cdot 8.667K}{-8.667K} = \frac{-11.33V}{-8.667K} = 1.31mA = I_2$

$Vb_{1,2} = -3.93V$
$Vb_{2,2} = -0.46V$
$Vb_{3,2} = 0V$