

Final Exam – Wednesday, May 11
EE220 – Circuits I
Spring 2022

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To get full credit:

- Show your work.
- Put a box around each of your answers.
- Make sure to **follow all instructions**.
- **Good Luck!**

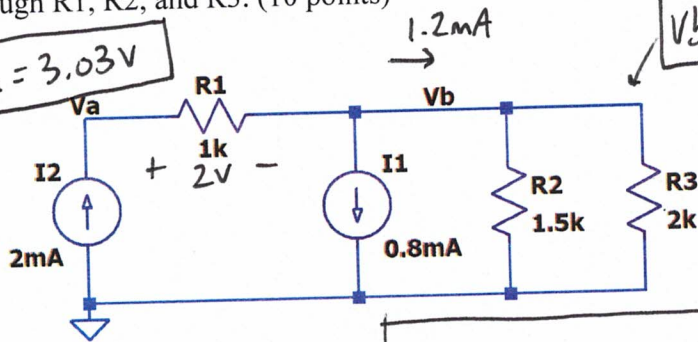
Problem	Points Earned	Points Possible
1	✓	10
2	✓	5
3	✓	10
4	✓	15
5	✓	15
6	✓	10
7	✓	10
8	✓	10
9	✓	5
10	✓	10
Total:	100	100

1. Determine the voltages (V_a , V_b) labeled in the circuit below, as well as the currents flowing through R_1 , R_2 , and R_3 . (10 points)

$$I(R_1) = 2\text{mA}$$

$$V_a = 3.03\text{V}$$

$$V_b = 2\text{k} \cdot 0.514\text{mA} = 1.03\text{V}$$

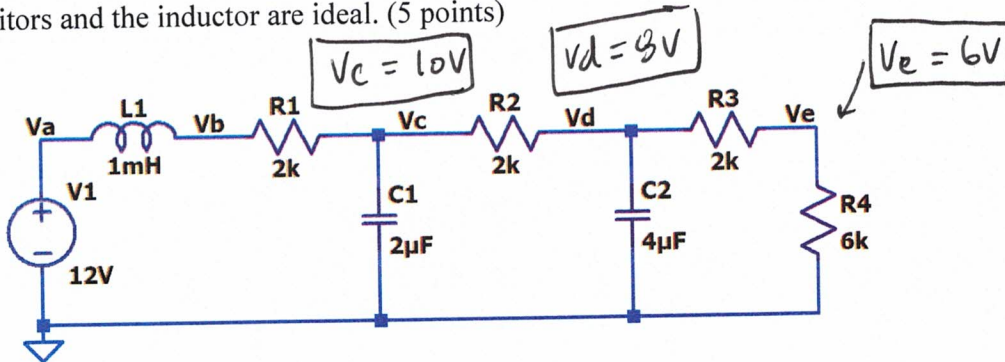


$$I(R_2) = 1.2\text{mA} \left(\frac{2\text{k}}{3.5\text{k}} \right) = 0.686\text{mA}$$

$$I(R_3) = 1.2\text{mA} \left(\frac{1.5\text{k}}{3.5\text{k}} \right) = 0.514\text{mA}$$

2. Determine the voltages (V_a , V_b , V_c , V_d , V_e) labeled in the circuit below, as well as the currents flowing through R_1 , R_2 , R_3 , and R_4 . Note that V_1 is a DC voltage source. Assume the circuit has been sitting here in steady state for a long time. Assume also that the capacitors and the inductor are ideal. (5 points)

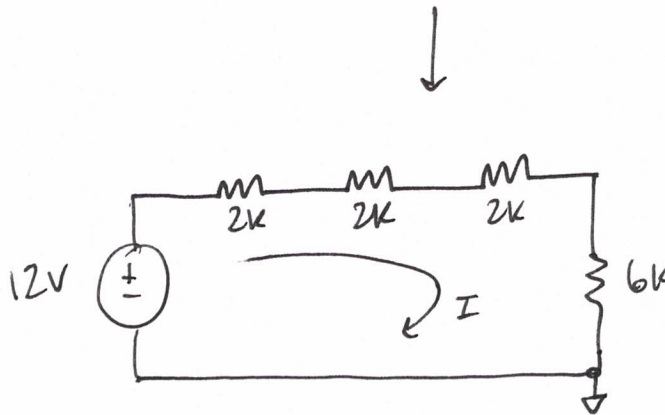
$$V_a = V_b = 12V$$



$$V_c = 10V$$

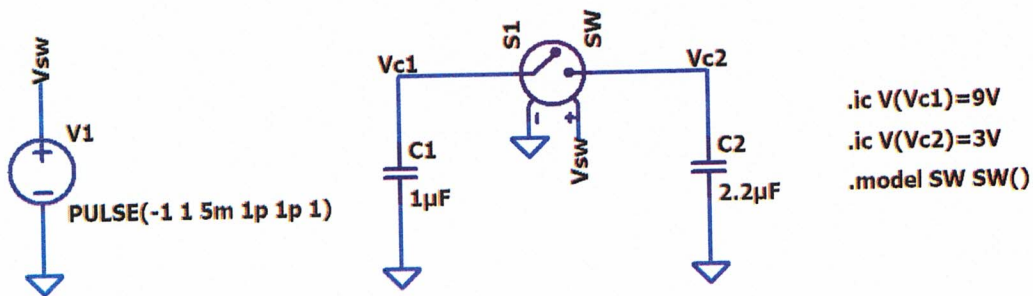
$$V_d = 9V$$

$$V_e = 6V$$



$$I = I(R_1) = I(R_2) = I(R_3) = I(R_4) = \frac{12V}{12k} = 1mA$$

3. The capacitors shown in the circuit below are initially charged to the voltages specified by the given initial conditions. The switch is initially open. How much charge is initially stored on C1? How much charge is initially stored on C2? At time $t=5\text{ms}$, the switch in the circuit closes, and charge sharing occurs. What are the values of the labeled voltages (V_{c1} and V_{c2}) some time after the switch closes? Assume that both the switch and the capacitors are ideal. (10 points)



$$Q(C_1) = C_1 \cdot V_{c1} = 1\mu F \cdot 9V = 9\mu C$$

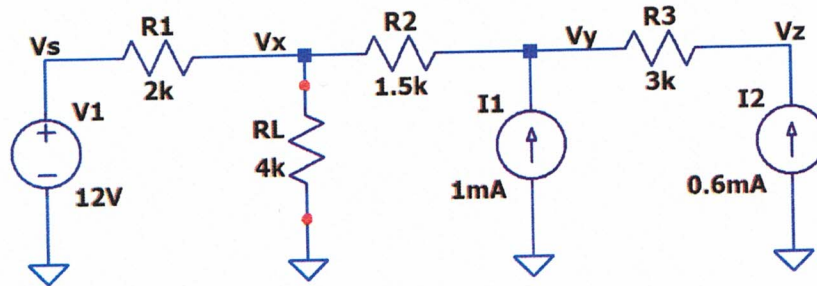
$$Q(C_2) = C_2 \cdot V_{c2} = 2.2\mu F \cdot 3V = 6.6\mu C$$

$$\therefore Q_{TOT} = 15.6\mu C \rightarrow C_{TOT} = 3.2\mu F$$

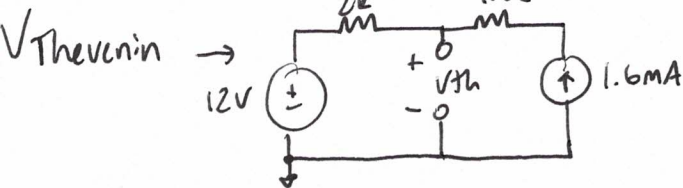
$$V_f = \frac{Q_{TOT}}{C_{TOT}} = \frac{15.6\mu C}{3.2\mu F} = 4.875V$$

$$V_{c1} = V_{c2} = V_f = 4.875V$$

4. Determine V_x labeled in the circuit below using any method you choose. Then, find the Thevenin and Norton equivalent circuits **when the load resistor (R_L) is removed**. Verify that your equivalent circuits are correct by connecting the load resistor across the terminals of your equivalent circuits and comparing the output voltage to your original calculation of V_x . (15 points)

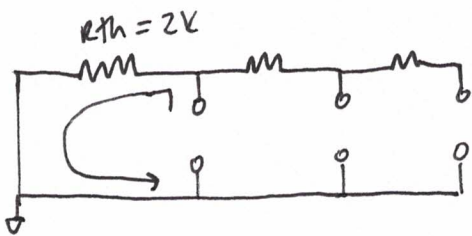


Thevenin & Norton:

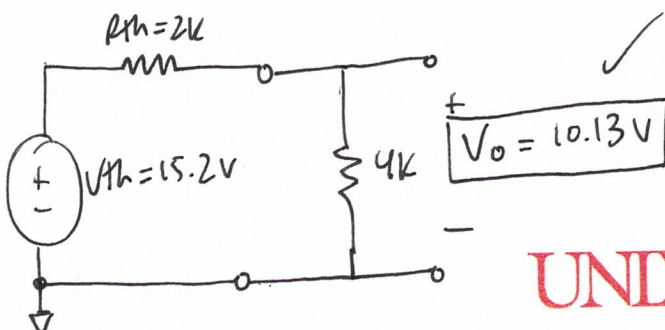


$$V_{th} = 12V + 2k(1.6mA)$$

$$V_{th} = 15.2V$$

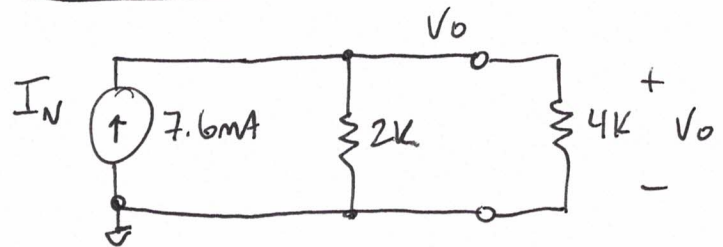


$$R_{th} = 2k$$



Norton

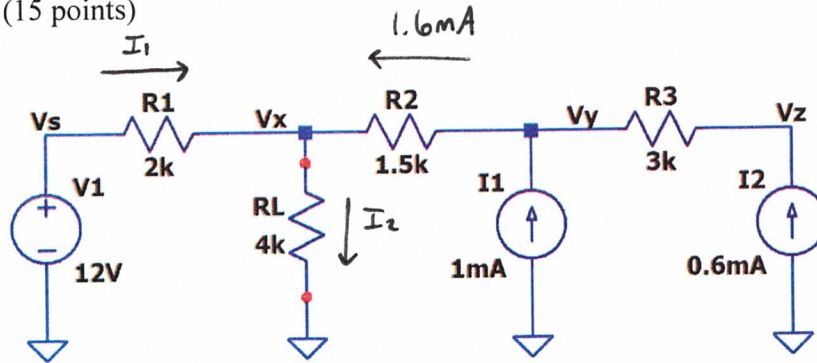
$$I_N = \frac{V_{th}}{R_{th}} = \frac{15.2V}{2k} = 7.6mA$$



$$V_0 = 7.6mA \left(\frac{2k}{6k} \right) \cdot 4k$$

$$V_0 = 10.13V$$

4. Determine V_x labeled in the circuit below using any method you choose. Then, find the Thevenin and Norton equivalent circuits **when the load resistor (R_L) is removed**. Verify that your equivalent circuits are correct by connecting the load resistor across the terminals of your equivalent circuits and comparing the output voltage to your original calculation of V_x . (15 points)



Finding V_x :

$$I_1 + 1.6\text{mA} = I_2$$

$$I_1 = \frac{12\text{V} - V_x}{2\text{k}} \quad I_2 = \frac{V_x}{4\text{k}}$$

$$\frac{12\text{V} - V_x}{2\text{k}} + 1.6\text{mA} = \frac{V_x}{4\text{k}}$$

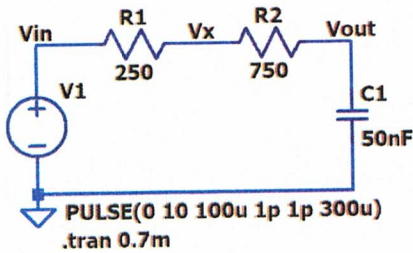
$$2(12\text{V} - V_x) + 4\text{k}(1.6\text{mA}) = V_x$$

$$6.4\text{V} + 24\text{V} - 2V_x = V_x$$

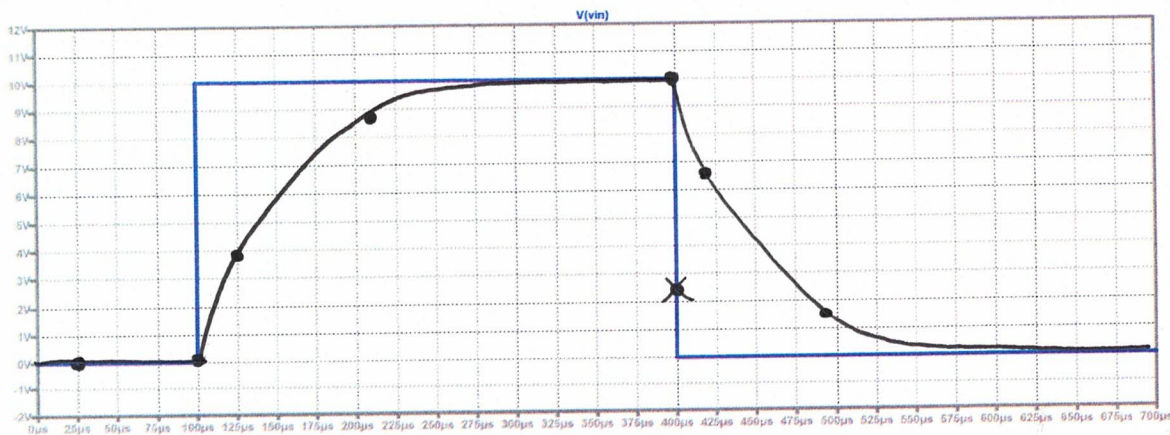
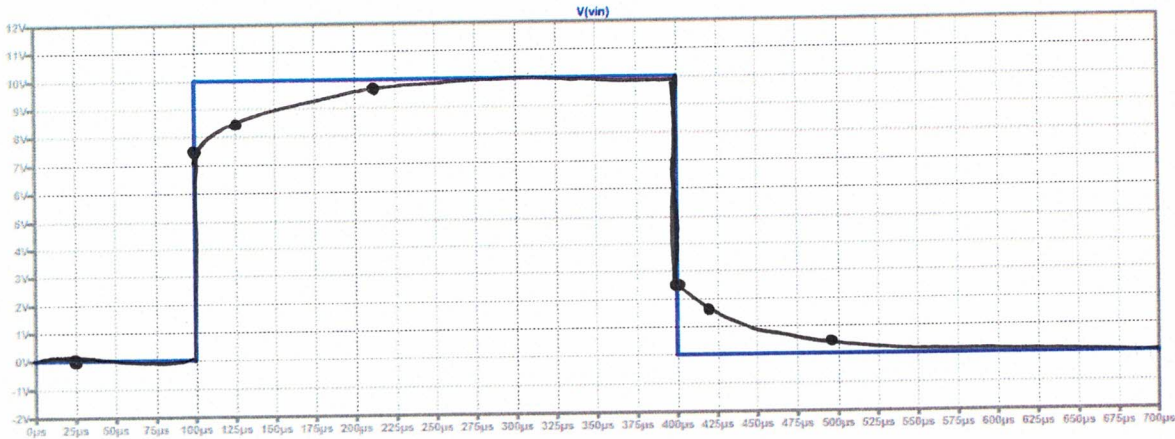
$$30.4\text{V} = 2V_x + V_x$$

$$30.4\text{V} = 3V_x \rightarrow \boxed{V_x = 10.13\text{V}}$$

5. Given the RC circuit and the input waveform $V_{in}(t)$ (plotted below in blue), sketch the waveforms for $V_x(t)$ and $V_{out}(t)$, one on each of the given plots. **Be sure to specify which waveform you are plotting, and on which plot you are plotting it, below. If it is not clear to me which one you are attempting to plot, you will lose points.** Fill in the table with voltage values for $V_x(t)$ and $V_{out}(t)$ at the specified times. (15 points)



Time (t)	$V_x(t)$	$V_{out}(t)$
25 μ s	0V	0V
125 μ s	8.48V	3.93V
210 μ s	9.72V	8.89V
420 μ s	1.68V	6.70V
490 μ s	0.41V	1.65V



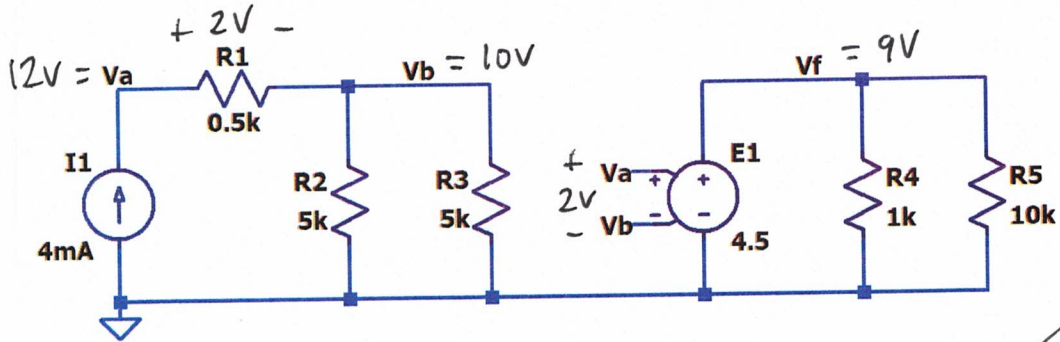
V_x rising: $V_x(t) = 10V + (7.5V - 10V)e^{-(t-0.1ms)/\tau}$
 $V_x(t) = 10V + (-2.5e^{-(t-0.1ms)/0.05ms})$

V_x falling: $V_x(t) = 0V + (2.5V - 0V)e^{-(t-0.4ms)/\tau}$
 $V_x(t) = 2.5V \cdot e^{-(t-0.4ms)/0.05ms}$

V_{out} rising: $V_{out}(t) = 10V + (0 - 10V)e^{-(t-0.1ms)/\tau}$
 $V_{out}(t) = 10V(1 - e^{-(t-0.1ms)/0.05ms})$

V_{out} falling: $V_{out}(t) = 10V(e^{-(t-0.4ms)/0.05ms})$

6. Determine each of the voltages labeled in the circuit below (V_a , V_b , V_f). Do R_3 and R_5 dissipate the same amount of power? Show your power calculations to support your answer. (10 points)

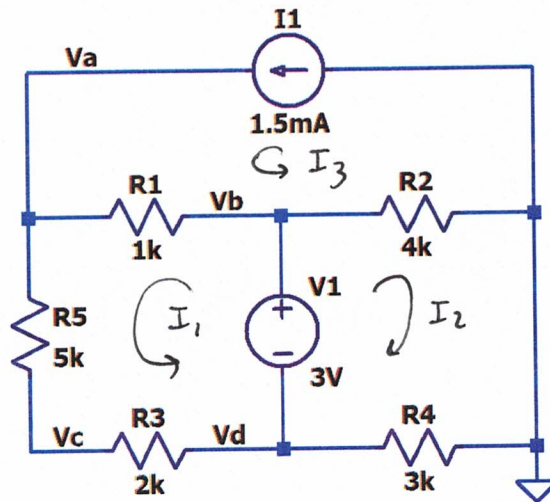


$V_a = 12V$ $V_b = 10V$ $V_f = 9V$

$P(R_5) = 9V \cdot (9V/10k) = 8.1mW$
 $P(R_3) = 10V \cdot (10V/5k) = 20mW$

} No, they don't
 dissipate the same
 amount of power.

7. Determine each of the voltages labeled in the circuit below (V_a , V_b , V_c , V_d) using mesh analysis. Do R_2 and R_3 dissipate the same amount of power? Show your power calculations to support your answer. (10 points)



$$I_3 = 1.5 \text{ mA}$$

$$\begin{aligned} \textcircled{1} \quad & 3\text{V} - (I_1 - 1.5\text{mA}) \cdot 1\text{k} - I_1(7\text{k}) = 0 \\ & 3\text{V} - I_1 \cdot 1\text{k} + 1.5\text{V} - I_1(7\text{k}) = 0 \\ & 4.5\text{V} = I_1(8\text{k}) \end{aligned}$$

$$I_1 = 0.563 \text{ mA}$$

$$\begin{aligned} \textcircled{2} \quad & 3\text{V} - (I_2 + 1.5\text{mA}) \cdot 4\text{k} - I_2 \cdot 3\text{k} = 0 \\ & 3\text{V} - I_2 \cdot 4\text{k} - 6\text{V} - I_2 \cdot 3\text{k} = 0 \end{aligned}$$

$$\begin{aligned} -3\text{V} &= I_2 \cdot 7\text{k} \\ \therefore I_2 &= -0.429 \text{ mA} \end{aligned}$$

$$V_a = (0.563\text{mA} \cdot 7\text{k}) + (0.429\text{mA} \cdot 3\text{k})$$

$$V_a = 5.22\text{V}$$

$$V_b = 3\text{V} + 1.28\text{V} = 4.28\text{V}$$

$$V_d = 0.429\text{mA} \cdot 3\text{k} = 1.28\text{V}$$

$$V_c = 5.22\text{V} - (5\text{k} \cdot 0.563\text{mA})$$

$$V_c = 2.41\text{V}$$

$$P(R_2) = ((1.5\text{mA}) - 0.429\text{mA}) \cdot 4.28\text{V} = 4.59\text{mW}$$

$$P(R_3) = (V_c - V_d) \cdot 0.563\text{mA} = 0.636\text{mW}$$

↑ Do not dissipate same power.

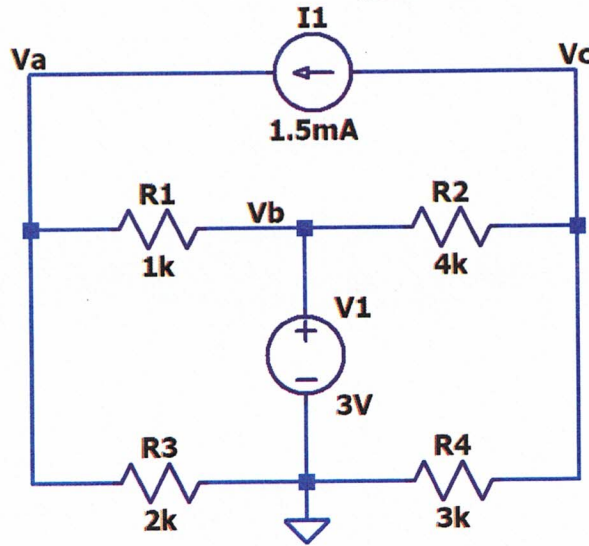
8. Determine the voltages V_a , V_b , and V_c labeled in the circuit below using superposition. (10 points)

$$V_a = V_{a1} + V_{a2}$$

$$V_a = 2V + 1V$$

$$V_a = 3V$$

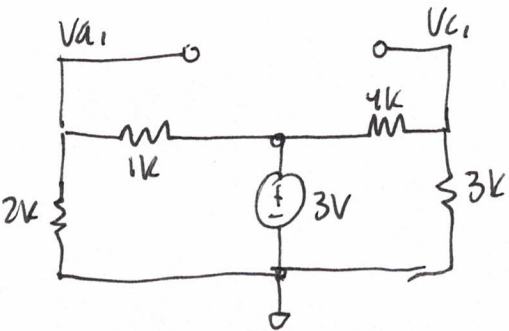
$$V_b = 3V$$



$$V_c = V_{c1} + V_{c2}$$

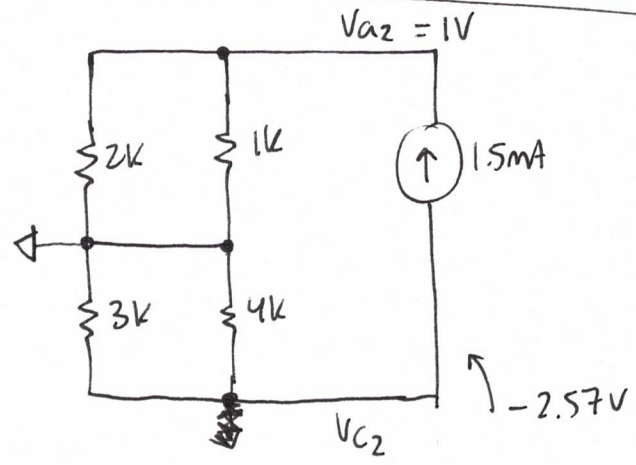
$$V_c = 1.29V + (-2.57V)$$

$$V_c = -1.28V$$



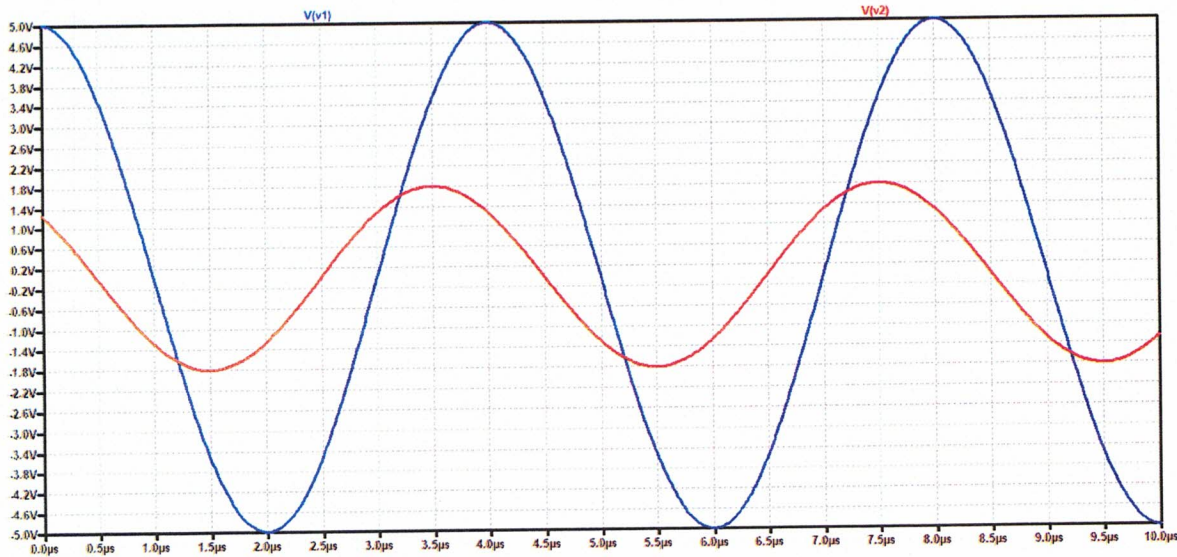
$$V_{a1} = 2V$$

$$V_{c1} = 1.29V$$



$$V_{c2} = -2.57V$$

9. Two sinusoidal waveforms are plotted below. Both waveforms have the same frequency, but their amplitudes are different, and they are not in phase. **Determine the equations for V_1 and V_2 as a function of time, t .** Write your equations in terms of a cosine function, where V_1 has a phase of 0° . Which signal is leading? Which is lagging? **Be sure to include the actual values for amplitude (V), frequency (Hz) and phase shift (degrees) in your equations.** If you leave these as variables, you will not get the points. (5 points)



V_1

$$V_A = 5V$$

$$\phi = 0^\circ$$

$$T = 4\mu s$$

$$f = 250kHz$$

$$\omega = 2\pi \cdot 250kHz$$

V_2

$$V_A = 1.8V$$

$$\phi = \frac{td}{T} \cdot 360^\circ \rightarrow td = 0.5\mu s \rightarrow \phi = \frac{0.5\mu s}{4\mu s} \cdot 360^\circ = 45^\circ$$

$$T = 4\mu s$$

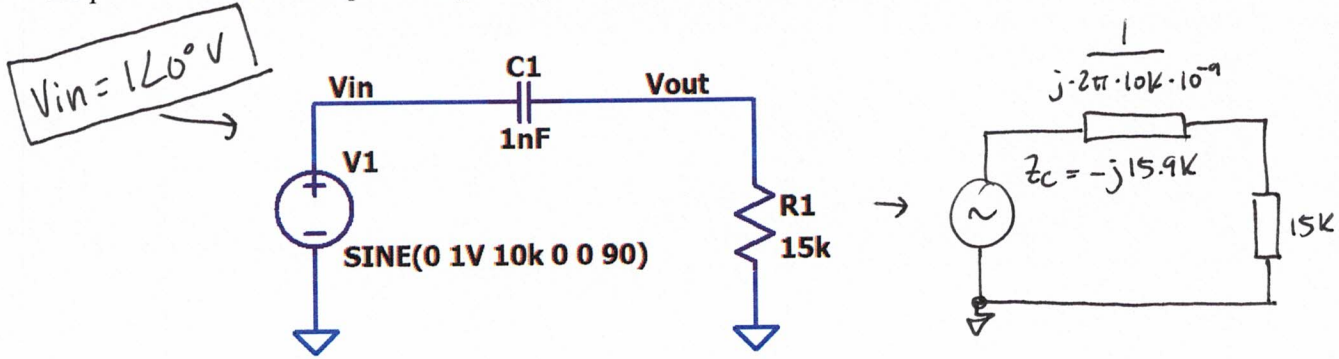
$$f = 250kHz$$

$$\omega = 2\pi \cdot 250kHz$$

$$V_1(t) = 5 \cos(2\pi \cdot 250kHz \cdot t) V$$

$$V_2(t) = 1.8 \cos[2\pi \cdot 250kHz \cdot t + 45^\circ] V$$

10. Determine the phasor representation of V_{in} , V_{out} , and I in the circuit given below. Note that V_{in} is a cosine wave with a phase shift of 0° (synonymous with a sine wave with a phase shift of 90°). You do not need to draw or sketch any of the waveforms. I only want the phasor form of the input voltage, output voltage, and current. (10 points)



$$Z_T = 15k - j15.9k$$

$$|Z_T| = 21.87k$$

$$\phi_{Z_T} = \tan^{-1}\left(\frac{-15.9k}{15k}\right) = -46.66^\circ$$

$$\rightarrow Z_T = 21.87k \angle -46.66^\circ \Omega$$

$$I = \frac{1 \angle 0^\circ V}{21.87k \angle -46.66^\circ \Omega} = 45.7 \mu A \angle 46.67^\circ = I$$

$$V_{out} = (45.7 \mu A \angle 46.67^\circ) \cdot (15k \angle 0^\circ \Omega) = 0.685 \angle 46.67^\circ V$$