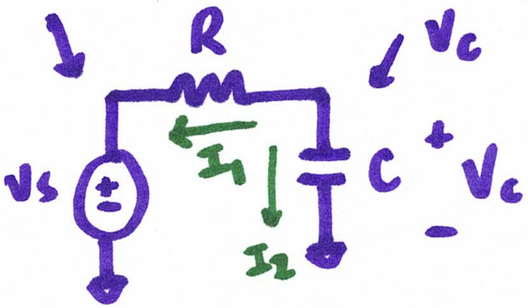
"FIFE" Equation Derivation

$V_s = V_{cf}$



$$\cancel{-I_1} = I_2$$
$$+I_1 = I_2 + I_1$$
$$\underline{0 = I_2 + I_1}$$

$$I_1 = \frac{V_c - V_s}{R}$$
$$I_2 = C \cdot \frac{dV_c}{dt}$$

$$\underbrace{\frac{V_c - V_s}{R}}_{I_1} + \underbrace{C\frac{dV_c}{dt}}_{I_2} = 0$$

$$-\left(\frac{1}{C}\right)\frac{V_c - V_s}{R} = -C\frac{dV_c}{dt}\left(\frac{-1}{C}\right)$$

$$-1\left(\frac{V_c - V_s}{RC}\right) = \frac{dV_c}{dt}$$

$$-\frac{1}{RC} = \frac{dV_c}{dt}\cdot\left(\frac{1}{V_c - V_s}\right)$$

$$\int_0^t -\frac{1}{RC}\cdot dt = \int_{V_{ci}}^{V_c}\left(\frac{1}{V_c - V_s}\right)dV_c$$

$$-\frac{1}{RC}\cdot(t - 0) = -\frac{t}{RC} = \ln|V_c - V_s|\,\Big|_{V_{ci}}^{V_c}$$

$$-\frac{t}{RC} = \ln|V_c - V_s| - \ln|V_{ci} - V_s|$$

$$e^{-\frac{t}{RC}} = \cancel{e^{\ln}}\left|\frac{V_c - V_s}{V_{ci} - V_s}\right|$$

$$e^{-t/RC} = \frac{V_c - V_s}{V_{ci} - V_s}$$

$\tau = R\cdot C$

$$V_s + (V_{ci} - V_s)e^{-t/RC} = V_c$$

$$\boxed{V_c(t) = V_{cf} + (V_{ci} - V_{cf})e^{-t/\tau}}$$