EE 220: Circuits I

1.) $C \cdot V = Q$ — capacitors

2.) $I = \frac{dQ}{dt}$

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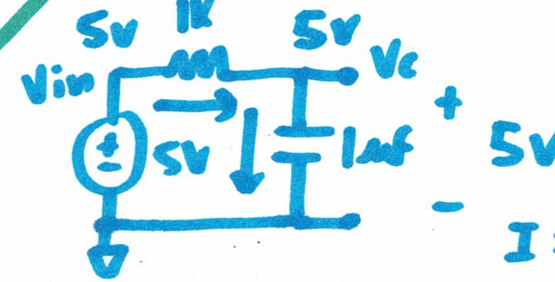
$C \cdot v(t) = q(t)$

$C \cdot \frac{dv(t)}{dt} = \frac{dq(t)}{dt}$



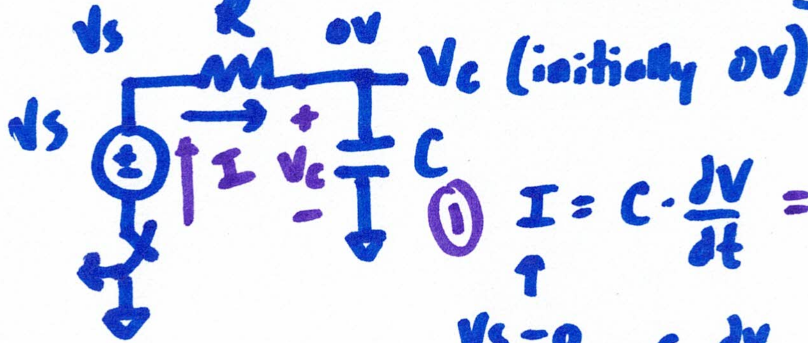
$I = \frac{V_{in} - V_c}{1K} = 0$

$I = C \cdot \frac{\dot{d}v}{dt} \Rightarrow I = 0$

$$I(t) = C \cdot \frac{dv(t)}{dt}$$

→ capacitors act like an open for DC

Charging a cap. (RC circuits)



① $I = C \cdot \frac{dV}{dt} = \frac{dq}{dt}$ ← rate of change of charge w.r.t. time

$$\frac{Vs - 0}{R} = C \cdot \frac{dV}{dt}$$

(non-zero) (non-zero)

② $C \cdot V = q$

$$V_c = \frac{q}{C}$$

$$Vs - I \cdot R - V_c = 0 \quad \text{(KVL)}$$

$$Vs - \frac{dq}{dt} \cdot R - \frac{q}{c} = 0$$

~~$Vs = \frac{dq}{dt} R + \frac{q}{c}$~~

$$Vs - \frac{q}{c} = R \cdot \frac{dq}{dt}$$

$$\frac{1}{R}\left(Vs - q/c\right) = \frac{dq}{dt}$$

→

$$\frac{1}{R}\left(Vs - \frac{q}{c}\right) dt = dq$$

$$\frac{1}{R \cdot C}\left(Vs \cdot C - q\right) dt = dq$$

$$(-1)\frac{1}{R \cdot C} dt = \frac{1}{Vs \cdot C - q} dq \, (-1)$$

$$\int_0^t -\frac{1}{RC} dt = \int_0^q \frac{1}{q - Vs \cdot C} dq$$

$$-\frac{t}{RC} = \ln\left|q - Vs \cdot C\right| \Big|_0^q \longrightarrow$$

↓

$$-\frac{t}{RC} = \ln|q - V_s \cdot C| \Big|_0^q$$

$$\ln|a| - \ln|b| = \ln\left|\frac{a}{b}\right|$$

$$-\frac{t}{RC} = \ln|q - V_s \cdot C| - \ln|0 - V_s \cdot C|$$

$$e^{-\frac{t}{RC}} = \cancel{e^{\ln}}\left|\frac{q - V_s \cdot C}{-V_s \cdot C}\right|$$

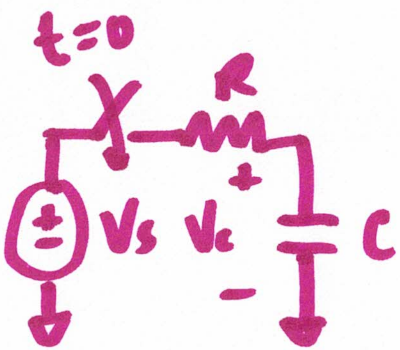
$$\cancel{C} \cdot V = \frac{q}{C}$$

$$V = \frac{q}{C}$$

$$-V_s \cdot C \quad e^{-t/RC} = \frac{q - V_s \cdot C}{\cancel{-V_s \cdot C}} \cancel{(-V_s \cdot C)}$$

$$-(V_s \cdot C)e^{-t/RC} = q - \cancel{V_s \cdot C}$$

$$+V_s \cdot C \qquad \cancel{+V_s \cdot C}$$



$$\frac{(V_s \cdot \cancel{C}) - (V_s \cdot \cancel{C})e^{-t/RC}}{\cancel{C}} = \frac{q}{C} \quad \swarrow V_c$$

constant

$$\boxed{V_c(t) = V_s(1 - e^{-t/RC})}$$

constant

charging a cap.

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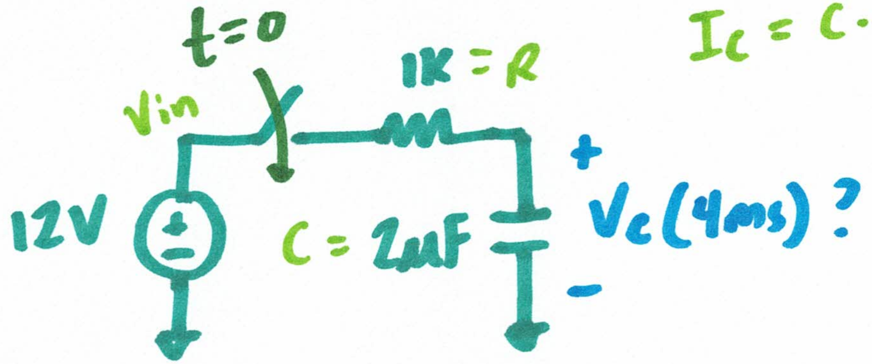
equation: charging a capacitor

$$V_c(t) = V_{in}\left(1 - e^{-t/\tau}\right)$$

$V_{in} \rightarrow V_{cf}$

$\tau = RC$



$$I_c = C \cdot \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = \frac{I_c}{C}$$

$$V_c(4ms) = 12V \cdot \left(1 - e^{-t/2ms}\right)$$

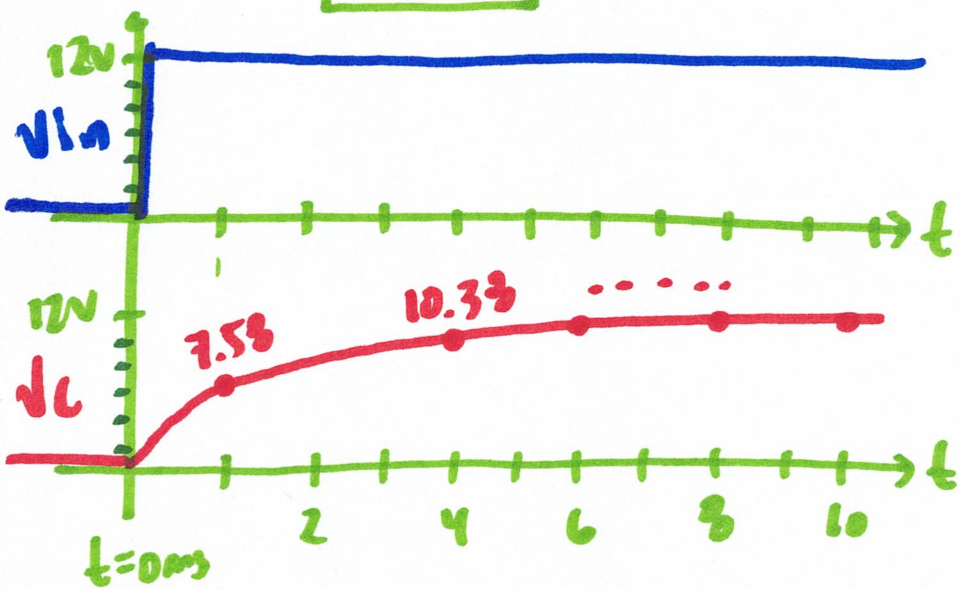
$$V_c(4ms) = 12V \cdot \left(1 - e^{-(4ms/2ms)}\right)$$

$$\boxed{V_c(4ms) = 10.376V}$$

$$\tau = 1k\Omega \cdot 2\mu F$$

$$\boxed{\tau = 2ms}$$

$$\frac{10.376V}{12V} = 0.865 \times 100\% = 86.5\%$$



$V_c(2ms)$	$= 7.58V$
$V_c(4ms)$	$= 10.38V$
$V_c(6ms)$	$= 11.40V$
$V_c(8ms)$	$= 11.78V$
$V_c(10ms)$	$= 11.92V$

discharging a capacitor: $V_c(t) = V_{c_i} \cdot e^{-t/\tau}$

$\tau = RC$

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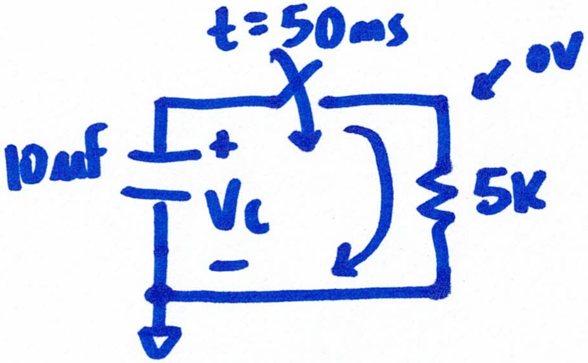

$V_{ci} = 10V$

* what is the cap. voltage at t = 150ms?

$$V_c(t) = V_{c_i} \cdot e^{-t/\tau}, \quad \tau = 5K \cdot 10uf = 50ms$$

$$V_c(t) = V_{c_i} \cdot e^{-t/50ms}$$

$$V_c(t) = V_{ci} \cdot e^{-(t-50ms)/50ms}$$

$$V_c(150ms) = 10V \cdot e^{-(150ms-50ms)/50ms}$$

$$V_c(150ms) = 1.353V$$