EE361

The Generalized Fourier Series

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$B\alpha = \begin{bmatrix} \langle f \cdot \beta_0 \rangle \\ \langle f, \beta_1 \rangle \\ \vdots \\ \langle f \cdot \beta_{N-1} \rangle \end{bmatrix}$ (weight)

$f(t) =$

$f_1(t) = t^0 = 1$

$f_2(t) = t^1$

$P_{projection}(t) f_1 + f_2 \cdots f_N)$

$b_n = t^n$

$\begin{bmatrix} \langle b_0, b_0 \rangle & \langle b_0, b_1 \rangle \cdots \\ \langle b_1, b_0 \rangle & \langle b_1, b_1 \rangle \cdots \\ \vdots & \ddots \end{bmatrix}$

$Proj(t) = \underbrace{f_0(t)}_{\text{Basis signal}} \cdot \underbrace{\alpha_0}_{\text{weight}} + f_1(t) \cdot \alpha_1 \cdots f_{N-1}(t) \cdot \alpha_{N-1}$

$b_0 = t^0 = 1 \qquad \langle b_0, b_0 \rangle = \int_{t=0}^{t=1} 1 \cdot 1 \, dt = 1$

$\langle b_0, b_1 \rangle = \int_0^1 1 \cdot t^1 \, dt = \frac{t^2}{2}\Big|_0^1 = 0.5$

$\langle b_1, b_1 \rangle = \langle t^1, t^1 \rangle = \int_{t=0}^{t=1} t \cdot t \, dt = \int_0^1 t^2 \, dt = \frac{t^3}{3}\Big|_0^1 = 0.333$

$\langle b_{N-1}, b_{N-1} \rangle$

$$\begin{bmatrix} 1 & .5 & \cdots \\ .5 & .3 & \\ \vdots & & \ddots \end{bmatrix}$$

$\langle f(t) = e^t \cdot b_0 \rangle = \int_0^1 e^t \cdot t^0 \, dt$

$$C_n = e^{j2\pi n t} + C_0 = 1$$

~~$n = 1 : N-1$~~

$$f_p(t) = \underbrace{a_0}_{\text{weights}} DC + a_1\left(\frac{e^{j2\pi t} + e^{-j2\pi t}}{2}\right) + a_2\left(\frac{e^{j4\pi t} + e^{-j4\pi t}}{2}\right)$$

$$\left(\cos w = \frac{e^{jw} + e^{-jw}}{2}\right) \quad \cdots + a_{N-1}\left(\frac{e^{j2\pi(N-1)t} + e^{-j2\pi(N-1)t}}{2}\right)$$