Concise Thermal to Electrical Parameters Extraction of Thermoelectric Generator for Spice Modeling

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***Abstract*—This paper presents a comprehensive method of extracting thermal to electrical parameters that appear in a thermoelectric module (TEM). A step-by-step procedure was developed and followed to succinctly extract the parameters needed to model the TEM. These parameters include the Seebeck coefficient, electrical conductivity, thermal resistance, and thermal conductivity from a datasheet. Data extracted from the manufacturer datasheet, material properties, and geometries were successfully utilized to compute the thermal capacities and resistances necessary to perform the thermal to electrical conversion steps required for simulation accuracy. In addition, temperature variations of the intrinsic internal parameters were accounted for in this process. The aforementioned extracted parameters are compatible with the LTspice circuit simulator and other related software. The steps that it takes to simulate any thermo-electrical system with the LTspice simulator are not only found to be of two kinds, but are also thoroughly explained in this paper.**

***Index Terms*—Thermoelectric module (*TEM*), Thermo-electrical equivalence, Parameter extraction, Spice, Device properties.**

I. Introduction

Solar energy is undoubtedly one of the cleanest and most abundant renewable resources present on Earth [1]. This fact inarguably places the Sun, the primary source of energy, at the center of all surface phenomena and living beings on Earth. This tremendous amount of energy in conjunction with the global atmospheric greenhouse effect, provide the means for the prosperity of the immensely diverse forms of life found on the planet Earth [2].

Many researchers have been concentrating their efforts towards photovoltaic (PV) energy. Solar energy, however, is a broader concept than energy delivered from PV. For example, thermoelectric generators (TEG) can also use solar energy, in the form of heat, for the generation of electrical power. It's possible for TEGs to directly convert as much solar energy into electrical energy as their fellow PVs do. This conversion of energy from the Sun to electricity is done via the Seebeck effect, named for the discovery made in 1821 by the German physicist, Thomas J. Seebeck.

TEGs are versatile heat controlled solid-state pn junction devices that have no moving parts and emit zero green house gases (GHG) into the environment during the course of their operation [3]. In addition, they can be used for cooling, heating or energy generation. TEGs are mainly used for power production in remote locations or small-scale generation where reliability and durability in the supply are required [4]. The heat sources could be derived from the Sun, a radioisotope or waste heat from any conventional power plant [5].

The main objective of this paper is twofold: 1) develop a step by step methodology for how to model a TEM using the LTspice simulator and 2) extraction of parameters from both datasheet and device properties and geometries. The actual thermal to electrical analogy model in LTspice is presented in [6].

II. TEG Basic Principles

As reported in [7] and [9], several physical phenomena take place in a thermoelectric device. In this paper we will briefly mentioned the most relevant of them that are of particular significance in this research.

Convection and Radiation - heat transfers through three thermally possible transport mechanisms. The predominant mechanisms in a TEM are convection and radiation [10].

Joule heating - another name for this thermo-electrical phenomenon is resistive heating. It is known to be the process by which the passage of a current through a conductor emits heat. The amount of heat, Q, released due to Joule heating is proportional to the square of the current flow times the resistance ($R = \rho \cdot l/A$) as seen in Eq. (1).

$$Q = I^2 \cdot R \qquad (1)$$

Where ρ, Q, I, l, and A are the resistivity of the material (Ω.m), the Joule heating (Watts), the current (Amperes), the length of the material (meters, m), and the surface area (m^2) respectively.

Peltier cooling/heating – this particular effect is of great interest when the TEM is run as a cooler. The Peltier effect supplies power to the module with a resultant cooling of one side and heating of the other.

Finally, the *Seebeck power generation* - as detailed in [6], occurs when two dissimilar metals are looped together. They develop an emf when the two junctions are kept at different temperature.

The *Thompson effect* is explicitly neglected in this work due to its smaller contribution in terms of cooling as reported in [11].

III. EXTRACTION OF PARAMETERS FROM DATASHEET

The parameters were extracted from the manufacturer's datasheet and utilized in conjunction with the internal parasitic components to derive the proposed model [6]. The following values were provided by Custom Thermoelectric: the maximum power, P_{max} = 21.6 *W*; the maximum voltage, V_{max} = 7.2 *V*; the maximum current, I_{max} = 3 *A*; Thermal Conductivity, k = 2.18W/*m·K*, and the optimal efficiency of the module, η = 6.5%.

The electrical resistance of the module, R_{Elect}

$$R_{Elect} = \frac{(V_{max})^2}{P_{max}} = 2.4\ \Omega \quad (2)$$

The Seebeck coefficient, α

$$\alpha = 2 \cdot \frac{Vmax}{Th - Tc} = 0.0534 \text{ V/K} \quad (3)$$

The thermal resistance of the module, R_{Ther}

$$R_{Ther} = 2 \cdot \Delta T \cdot \eta \cdot (2 - \eta) \cdot \frac{R_{Ther}}{(\Delta T - \eta \cdot Ta)^2 \cdot \alpha^2} = \frac{0.6365K}{W} \quad (4)$$

The resistance of the thermal insulation, R_{Insul} as reported in [11] can be calculated as follows based on the experimental data gotten after steady state conditions had been reached. These steady state conditions were reached after 32 minutes of experiment [6]. The temperature values recorded are: hot side, T_H = 324.89K and cold side, T_C = 311.46*K*.

$$R_{Insul} = \frac{\alpha^2 \cdot R_{Ther}{}^2 \cdot (Tc + Th + 2.273)^2}{(T_C - T_H)^2} \cdot \frac{(Tc - Troom)}{(2R_{Elect} + \alpha^2 \cdot R_{Ther} \cdot (Tc + Th + 2.273))} = 5.9K/W \quad (5)$$

Where, T_{room} represents the normal temperature of a living room.

IV. TEM MODELING STEPS

Modeling a TEM system with SPICE as performed in [6] could be ambiguous as it hasn't been clearly explained up to this point. The following are the steps that it takes to simulate any thermo-electrical system with an electronic SPICE simulator such as LTSpice.

- Identify the physical components
- Calculate their *Biot* number as explained thoroughly in [12]; its value determines the approach to adopt in the analysis. Should that value be much less than unity, the lumped capacitance method is solely recommended for accuracy in the results. If it isn't much less than, then some sort of numerical discretization method should be considered.
- Calculate the thermal resistances and capacitances of all the mechanical parts (TEM, Aluminum plates, an insulation chamber, etc.) [6] by splitting them into smaller parts. The nature of the material and other properties such as density and volume should be accounted for in calculation.
- A further step in the design is to not only define, but also draw the electrical parts that will encompass all the parasitic elements such as *R*, *L*, and *C*. The latter point is not important at this stage in the research, but will be given a higher priority in future work where the goal is to utilize a DC-DC converter for accurate results.
- Express the electrical equivalence of the thermal parameters. Table I lists the most commonly used thermal to electrical analogies as presented in [13].

TABLE I. THERMAL TO ELECTRICAL EQUIVALENCE

Thermal	Electrical
°*C*/Watt	Ohm (Resistor)
Joules/°*C*	Farad (Capacitor)
Watt	Ampere (Current Source)
° *C*	Volt (Voltage Source)
Ambient Temperature	GND (0V)

- Then connect these analogy blocks in series and/or parallel in order to reconstruct the actual system.
- Finally, the TEM is ready to be modeled in SPICE.

V. PARAMETERS COMPUTATIONS

Some of the material properties that turned out to be helpful in this study were mentioned in [16]. Table II tabulates the most relevant values. The TE modules were specified by the manufacturer to be bismuth telluride (Bi_2Te_3) and that the ceramic substrates or faces are made of alumina (Al_2O_3).

TABLE II: TEM PROPERTIES

Material	$\rho[kg/m^3]$;	c [*J/kg · K*];	κ [W/*m · K*]
Aluminum	2770	875	177
Alumina	3570	837	35.3
Bi_2Te_3	7530	544	1.5

Where, ρ is density, c is specific heat, and κ is conductivity.

A. Aluminum plates ($56mm \cdot 56mm \cdot 12.7mm$)

1) *Thermal Resistance, R_{AL}.*

$$R_{AL} = \frac{1}{\kappa \cdot (A/l)} = 2.29 \cdot 10^{-2} [K/W] \quad (6)$$

Where κ = 177*W/m* is the thermal conductivity, *A* is the surface area, and l is the length of the plate.

2) *Thermal Capacity, C_{AL}*

$$C_{AL} = \rho \cdot Cp \cdot V$$

$$= \frac{2770kg \cdot 875J \cdot (0.056)^2 \cdot (0.0127m^3)}{m^3 \cdot kg \cdot K}$$

$$= 96.53J \tag{7}$$

Where ρ is the density, C_p is the specific heat capacity, and V is the volume of the plate.

B. TEM

It is essential to first have a sound knowledge of the exact mass the TEM, its molar mass, M and its molar heat capacity for one to be able to reproduce the heat capacity of the module in SPICE. Hence, the mass was simply determined by a sensitive electronic balance and compared to the one given on the datasheet. The C_p of a material can be determined as follows:

$$Cp = \frac{C_{mol}}{M} \; [\frac{J}{kg} \cdot K] \tag{8}$$

Where C_{mol} is the molar heat capacity in $J/mol{\cdot}K$, and M represents the molar mass in g/mol.

The molar heat capacity of Bi_2Te_3 was found in [14] to be 126.19 [$J/mol{\cdot}K$] at normal temperature, i.e. 298.15 K. In a like manner and for simplicity in the approach, the molar mass, M of the aforementioned material, was accessed at [17] site. It was found there to be 800.760 [g/mol]. Additionally, the mass of the semiconductor devices can be obtained through a mere subtraction knowing the entire mass (m_T = $4.8.10^{-2}kg$) of the TEM and that of the ceramic plates as can be seen from Eq. (11) below. Hence, the following values can be easily computed:

1) *Mass of the ceramic plates, m_{cer}*

$$m_{cer} = \rho \cdot V\; [kg] = \frac{3570kg}{m^3} \cdot (0.056m)^2 \cdot (0.002m)$$

$$= 2.239 \cdot 10^{-2} kg \tag{9}$$

2) *Molar heat capacity of the ceramic plates, C_{cer}*

$$C_{cer} = \rho \cdot Cp \cdot V\; [J/K]$$

$$= \frac{3570kg \cdot 837W \cdot (6.272) \cdot (10^{-6}m^3)}{m^3 \cdot m \cdot K}$$

$$= 18.74J/K \tag{10}$$

3) *The mass of Bi_2Te_3, m_{Bi2Te3}*

$$m_{Bi2Te3} = m_T - m_{cer}\; [kg]$$

$$= (4.8 - 2.239) \cdot 10^{-2} kg$$

$$= 2.561 \cdot 10^{-2} kg \tag{11}$$

4) *Molar heat capacity of Bi_2Te_3, C_{Bi2Te3}*

$$C_{Bi2Te3} = \frac{C_{mol}}{M} \cdot m_{Bi2Te3}\; [J/K]$$

$$= \frac{126.16J \cdot mol}{800.76g \cdot mol \cdot K} \cdot 25.61g = \frac{4.036J}{K} \tag{12}$$

5) *The overall heat capacity of the TEM, C_{TEM}*

The heat capacity of the module can be computed based on the knowledge of the two previous heat capacities already determined in Eqs. (10) and (12).

$$C_{TEM} = C_{cer} + C_{Bi2Te3} \cong 23J/K \tag{13}$$

In order to model the actual behavior of the TEM, its internal parameter variations are taking in account in the present model based on the work done in [15]. Hence, the internal parasitic inductances and capacitances values for both TEG and TEC are already available in the latter reference. Thus, they are used in the present work, because the current setup [6] is not meant to characterize the internal parameters or their variations with temperature.

Priority was given in this study to the most dominant physical phenomena that take place in a TEM. Therefore, among all the mechanisms, *Seebeck*, *Peltier*, and *Joule effects* were given the most attention. The TEM was first run as a heat pump or TEC. A TEC energy flow is solely based on *Joules effect* as seen in the following energy balance equations.

The heat flow at the absorbing (*cold*) side is given by Eq.(14).

$$Qc - \alpha \cdot T_C \cdot I + \tfrac{1}{2} I^2 \cdot R_{Int} + \kappa \cdot \Delta T = 0 \tag{14}$$

The heat flow at the emitting (*hot*) side is given by Eq.15.

$$Qh - \alpha \cdot T_H \cdot I - \tfrac{1}{2} I^2 \cdot R_{Int} + \kappa \cdot \Delta T = 0 \tag{15}$$

The overall energy balance equation is represented by Eq. 16.

$$Qh - Qc - P_{Elect} = 0 \tag{16}$$

Hence, the electrical power consumed by the device necessary to remove the heat from one side relative to the other is as written in Eq. (17).

$$P_{Elect} = Qh - Qc = \alpha \cdot \Delta T \cdot I + R_{Int} \cdot I^2 \tag{17}$$

Where, T_H and T_C represent the hot and cold side temperatures respectively; R_{Int} is the internal resistance of the TEM; α, κ, and ΔT as mentioned above are the Seebeck coefficient, the thermal conductivity, and the differential temperature (T_H - T_C) respectively.

According to the study performed in [15], the SPICE model behavior of the TEC will be unrealistic if the internal parasitic capacitance is neglected as it was done in [11]. In addition to that, α, κ, and R_{Int} variations with temperature must be accounted for toward the same objective. This study is meant to accommodate all the aforementioned issues that might affect the realistic SPICE implementation of the TEM [6].

VI. ANALYSIS

Parameter extraction was solely based on three things: 1) manufacturer's reported data, 2) device properties, and 3) device geometries.

The method to develop an accurate model was successful as can be seen through the involvement in the computations of the various masses and capacitances of both the ceramic substrate as well as the Bi_2Te_3 semiconductor material (Eq. 9-13) required for the modeling. Also, the Fig. 3 (Spice Model) and Fig. 6 (Comparative results) in our accompanying paper [6] portray the accuracy in the parametric extraction and the subsequent results. A calibration measure for this research, with respect to parameters extraction prior to the actual analogy, can be directly linked to the closeness on how simulated and experimental curves matched in Fig. 6 [6]. The minor offset errors could come from the internal parasitic components' effects.

VII. CONCLUSION

An experimental setup was designed and built to characterize and study the performance of a commercial TEM. One of the main objectives of this paper was to develop prior steps necessary to an LTspice TEM modeling scheme through thermal to electrical equivalence strategies. Also, this research sought to extract TEM parameters from two inter-related sources: manufacturer datasheet and device geometries cut to fit our needs. Actual data extracted from the manufacturer datasheet, material properties, and geometries were successfully utilized to compute the thermal capacities and resistances necessary to perform the thermal to electrical conversion steps required for the simulation to be as accurate as possible. Very good agreement between the theoretical and experimental results was achieved with the model developed in this paper. The minor offset errors are likely the result of internal parasitic effects.

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