An interactive impulse response extraction system

Harry W. Li,[a] Michael J. Dallabetta, and Jake Baker
Department of Electrical Engineering, Moscow, Idaho 83843

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A cross-correlation method of measuring the impulse response of linear systems is presented. In theory, by using this method, the impulse response of any linear system can be obtained while the system is being used under normal operation with no significant distortion of the normal system output. A cross-correlation system was modeled and constructed using discrete components and simulation and experimental results are presented. Unfortunately, these results show that the extracted impulse response is severely distorted by the normal system input and the system noise. However, by averaging and interleaving methods, the impulse response of any linear system can be extracted with a high degree of accuracy. © *1995 American Institute of Physics.*

I. INTRODUCTION

There are many methods of obtaining the small signal dynamic response of linear systems which include impulse and step response tests, sinusoidal oscillation tests, frequency domain methods, and the cross-correlation method which is presented in this article. The cross-correlation method is of particular interest because the impulse response of a linear system can be obtained while the system is being used under normal operation with no significant distortion of the normal system output. The impulse response is measured by applying a noise level disturbance, similar to a poisson wave, at the system input and cross correlating this disturbance with the system output. The advantage of being able to measure a system characteristic without interfering with the system's normal and continuous operation has many possibilities of practical and effective application.

This method was developed in the early sixties to measure the impulse response of reactor systems.[1] However, at that time, its practical application was not possible because of difficulties with the computational techniques needed. More recently, this method has been implemented but never on a single chip.[2,3] A preliminary investigation of an analog system has been presented[4] and it was shown that the extracted impulse response is severely distorted by both the normal system input and the system noise. The methods presented in this article (Secs. III and IV) will remedy these problems.

II. THE INTERACTIVE IMPULSE RESPONSE EXTRACTION SYSTEM

The basic architecture of the interactive impulse response extraction system (INTIRE) is shown in Fig. 1, along with an arbitrary linear system under test with the impulse response $h(t)$. The system under test has two inputs; the normal system input, $f_i(t)$, (which includes additive noise), and a disturbance signal, $f_{n1}(t)$, provided by the INTIRE system. The impulse response, $h(t)$, is obtained by correlating the system output, $f_0(t)$, with another disturbance signal $f_{n2}(t)$. It can be shown that the correlator output is

$$\phi_{on}(\tau)=\int_0^\infty h(\lambda)[\phi_{ni}(\tau-\lambda)+\phi_{nn}(\tau-\lambda)]d\lambda, \tag{1}$$

where the term $\phi_{ni}(\tau-\lambda)$ is the cross correlation of $f_{n2}(t)$ and $f_i(t)$, and $\phi_{nn}(\tau)$ is the cross correlation of $f_{n1}(t)$ and $f_{n2}(t)$. If $\phi_{nn}(\tau)$ is an impulse response, and $f_{n2}(t)$ is independent of normal system input, then

$$\phi_{nn}(\tau-\lambda)=\delta(\tau-\lambda), \tag{2}$$

$$\phi_{ni}(\tau-\lambda)=\mu_{n2}\mu i, \tag{3}$$

where μ_{n2} is the mean of $f_{n2}(t)$ and μ_i is the mean of the normal system output $f_0(t)$. If either of these means are zero, than Eq. (1) reduces to

$$\begin{aligned}\phi_{on}(\tau)&=\int_0^\infty h(\lambda)\phi_{nn}(\tau-\lambda)d\lambda\\&=\int_0^\infty h(\lambda)\delta(\tau-\lambda)d\lambda\\&=h(\tau),\end{aligned} \tag{4}$$

which is equal to the impulse response of the system under test.

Clearly, the disturbance signals provided by the INTIRE system have some very unique properties. These disturbance signals can be easily realized using maximal length shift register sequences (m-sequences). Details on the hardware needed to generate these signals is well documented.[4–6]

An example of a pair of m-sequence signals is shown in Fig. 2. Note the following properties: A pulse of width T_p occurs once every Δt s ($f_{n1}(t)$ only), the signals are periodic with period $T=N\Delta t$ where N is the m-sequence length used to generate the signal. From Fig. 2 it is clear that the cross correlation of the disturbance signals approximate an impulse response over one period and this approximation can be made arbitrarily accurate simply by reducing the pulse width T_p. Since the disturbance signals in Eq. (4) are periodic, the correlation time of the INTIRE system can be finite and Eq. (4) is approximated by

$$\hat{\phi}_{on}=\int_0^T f_0(t+\tau)f_n(t)dt=\int_0^T h(\lambda)\phi_{nn}(\tau-\lambda)d\lambda. \tag{5}$$

[a]E-mail: hli@uidaho.edu

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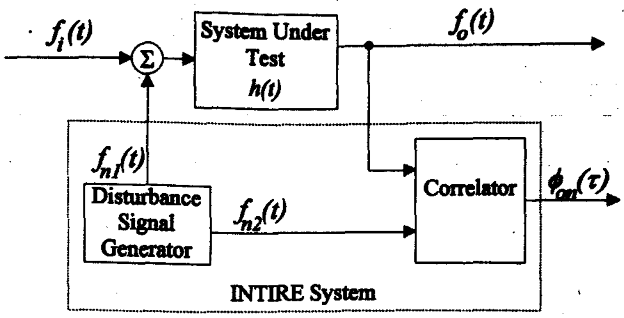

FIG. 1. Determination of a system impulse response using the cross-correlation method.

The integration time of Eq. (5) must be sufficiently long to allow the impulse response to the system under test to decay to a negligible value.

One consequence of using these disturbance signals is that the extracted impulse response is scaled by the term CT_p. That is

$$\phi_{on}(\tau)=\int_0^\infty h(\lambda)\phi_{nn}(\tau-\lambda)d\lambda\cong\frac{N+1}{2}T_p^2V_2V_1h(\tau). \tag{6}$$

In general, realizing the correlation function can be difficult. However, for this system, the correlator is simplified considerably since one of the signals being correlated, $f_{n2}(t)$, is periodic and binary. A schematic of the correlator and the necessary control signals is shown in Fig. 3. The output of the system under test, $f_0(t)$, is passed through a very selective high-pass filter to remove any dc component so that Eq. (3) reduces to zero. The output of this filter is than correlated with the binary m-sequence signal, $f_n(t)$, shown in Fig. 3(b) via the spherical wave SW-1 switches. The SW-2 switches, controlled by the pulse signal, convert $f_n(t)$ to $f_{n2}(t)$.

Finally, the SW-3 switches are used to enable the correlator for exactly one period of the disturbance signal as required by the approximation in Eq. (5). At the end of the integration, the values of the integrator outputs are proportional to $h(\tau_i)$; the value of the impulse response of the system is evaluated at τ_1. The integrator outputs can be thought of as discrete time samples of the impulse response $h(t)$. If there are a sufficient number of samples, all characteristics of $h(t)$ are preserved. For convenience, these discrete time samples are multiplexed out to a single pin.

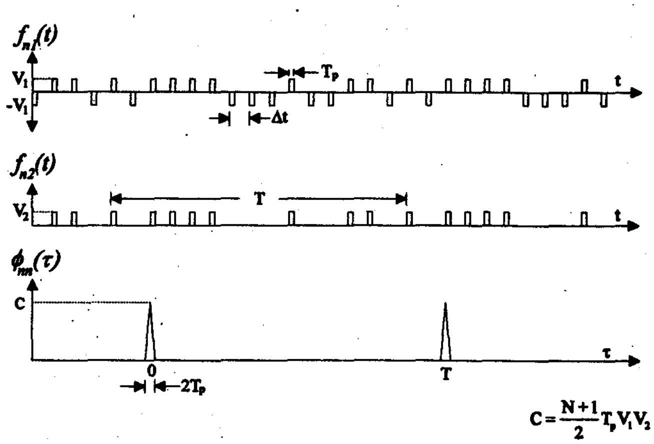

FIG. 2. The cross correlation of two unique m-sequence random impulse trains.

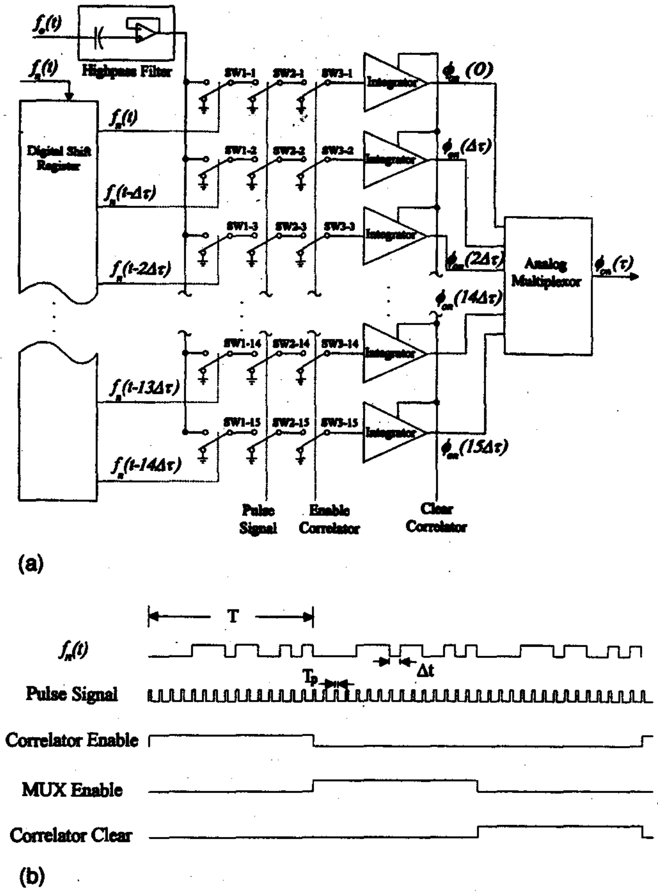

FIG. 3. (a) A schematic of the INTIRE system analog correlator, (b) input and control signals used in the correlator.

III. DISTORTION OF THE EXTRACTED IMPULSE RESPONSE

An INTIRE system was designed, simulated, and constructed using discrete components and experimental and simulation results were obtained. In each case, the impulse response of a simple second order resistance–capacitance (RC) filter was extracted.

These experiments verified the cross-correlation method and it was found that the impulse response of the system could be accurately extracted if the input disturbance signal-to-noise ratio (SNR) is sufficiently large (The input distur-

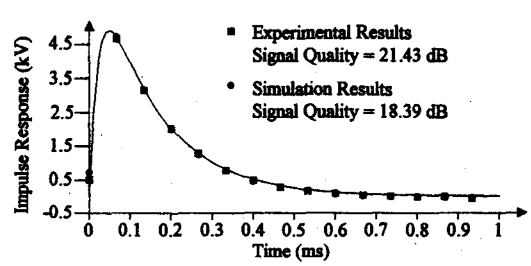

FIG. 4. The extracted impulse response with a disturbance SNR of 36.2 dB.

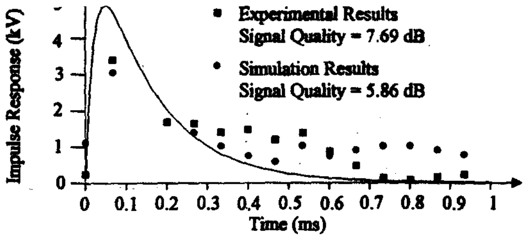

FIG. 5. The extracted impulse response with a disturbance SNR of 2.1 dB.

bance SNR is the ratio of the disturbance signal power over the normal system input power). An example of an extracted impulse response is shown in Fig. 4.

Both the experimental and simulation results were very accurate as demonstrated by the signal quality given by

$$\text{Signal Quality}=\frac{\sum_{i=0}^{14} h^2(i\Delta t)}{\sum_{i=0}^{14}[h(i\Delta t)-\phi_{on}(i\Delta t)]^2}, \tag{7}$$

which is analogous to a signal-to-noise ratio. The problem with the extracted impulse response in Fig. 4 is that the disturbance SNR is 36.2 dB. Clearly, the normal system output will be severely distorted. This is unacceptable since the goal of the INTIRE system is to extract the impulse response while the system is on-line without distorting the normal system output.

If the disturbance signal power is reduced, the extracted impulse response is severely distorted, as shown in Fig. 5. In this case, the disturbance SNR is around 2.1 dB, which is still quite high. For the INITRE system to be effective, the input disturbance SNR may be as low as -30–-40 dB. Clearly, the existing design of the INTIRE system will be inadequate for on-line test.

To understand the distortion effects of the normal system input, the impulse response was extracted for several disturbance signal powers. The results are shown in Fig. 6. From Fig. 6, it is clear that for disturbance SNRs below 20 dB, the normal system input severely distorts the extracted impulse response. Above 20 dB, the error in the extracted signal is primarily due to the disturbance signal parameters (i.e., T, Δt, and T_p).

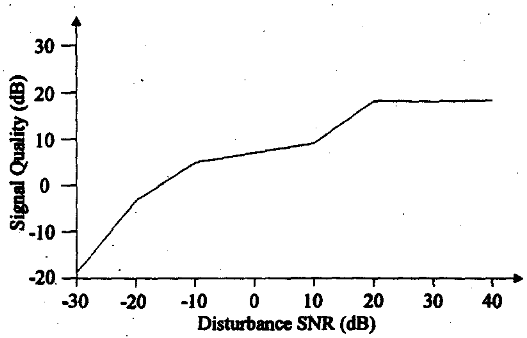

FIG. 6. Signal quality of the impulse response as a function of the disturbance SNR.

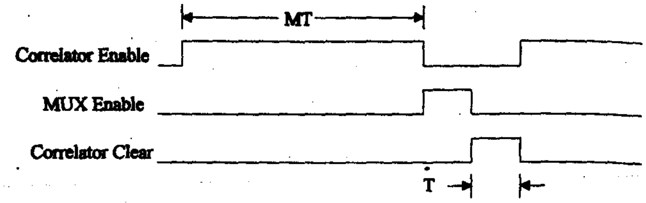

FIG. 7. The modified control signals used in the correlator.

IV. AVERAGING TECHNIQUES TO REDUCE THE DISTORTION

Up to this point, it was assumed that the normal system input would have no affect on the extracted impulse response. This assumption is based on the fact that the normal system input and the disturbance signals provided by the INTIRE system are statistically independent. In fact, these signals are statistically independent and their cross correlation

$$\phi_{ni}(\tau)=\lim_{T\to\infty}\int_{-T}^{T} f_{n2}(t)f_i(t+\tau)dt \tag{8}$$

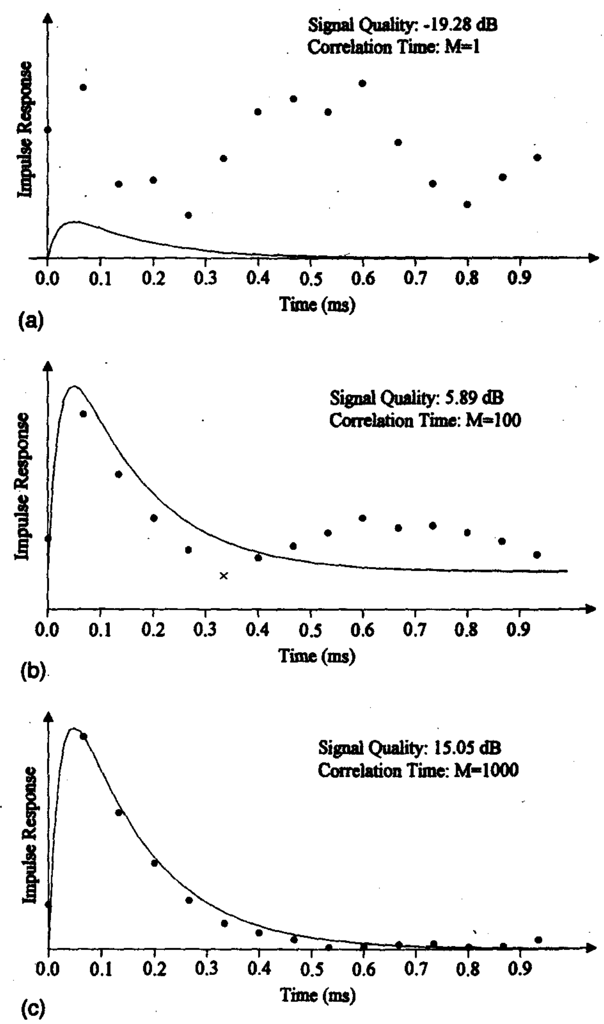

FIG. 8. (a) The impulse response with the Disturbance SNR=−30 dB, correlation time=T, (b) the impulse response with the Disturbance SNR =−30 dB, correlation time=$100T$, and (c) the impulse response with the Disturbance SNR=−30 dB, correlation time=$1000T$.

does in fact equal zero in the limit as $T \rightarrow \infty$. However, in the INTIRE system, the correlation function is approximated by Eq. (5). That is, the integration time is finite and is equal to exactly one period of the disturbance signal. As a result, the approximate cross correlation of $f_{n2}(t)$ and $f_i(t)$ is not equal to zero.

To improve the noise immunity of the INTIRE system, the integration time of the correlation function must be increased. This is accomplished by modifying the control signals used in the correlator as shown in Fig. 7.

In this case, the correlation time is increased and is equal multiple periods of the disturbance signal period, T. As a result, the noise immunity of the INTIRE system is improved and the correlator output is equal to

$$\phi_{on}(\tau)=\frac{N+1}{2} MT_p^2 V_1 V_2 h(\tau) + \int_0^{MT} h(\lambda)\phi_{nt}(\tau-\lambda)d\lambda. \quad (9)$$

The improvement in the extracted impulse response is twofold. First, the "signal" portion of the extracted impulse response in Eq. (9) is increased by a factor of M, thus increasing the signal quality. Second, the approximate cross correlation of the normal system input and the disturbance signal, $\phi_{ni}(t)$, approaches the ideal cross correlation of zero.

This method of increasing the correlation time was implemented in the Saber model. The simulation results are shown in Fig. 8. In all three cases, the input SNR is equal to -30 dB. In Fig. 8(a), the correlation time is equal to one period of the m-sequence $(M=1)$. As expected, the extracted impulse response is severely distorted. In Fig. 8(b), the correlation time is equal to 100 periods of the m-sequence and there is a considerable improvement in the extracted impulse response as noted by the signal quality. In Fig. 8(c), the correlation time is equal to 1000 periods of the m-sequence and the extracted impulse response is very accurate. For the m-sequence used in this case, increasing the correlation time further will not significantly improve the quality of the extracted impulse response as errors caused by the impulse response approximation (Fig. 2) will begin to dominate.

It should be noted that the simulations shown in Fig. 8 are behavioral and that some of the nonideal features of the correlator were neglected. Specifically, the leakage of the integrators has been neglected due to the large integration time. However, the results in Fig. 8 do demonstrate the averaging concept used in the INTIRE system. To actually implement the INTIRE system with averaging, it may be necessary to employ digital methods in the correlator.

ACKNOWLEDGMENTS

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[1] J. D. Balcomb, H. B. Demuth, and E. P. Gyftopoulos, Nucl. Sci. Eng. **11**, 159 (1961).
[2] D. A. Barr and S. J. Laverty, IEE Proc. G **138**, 160 (1991).
[3] C. Yap, MS thesis, University of Tulsa, 1987.
[4] H. W. Li, M. J. Dallabetta, and H. B. Demuth, Proc. IEEE **5**, 65 (1994).
[5] F. J. MacWilliams and N. J. A. Sloane, IEEE Proc. **64**, 1715 (1976).
[6] S. W. Golomb, *Digital Communications with Space Applications* (Prentice-Hall, Englewood Cliffs, NJ, 1964).