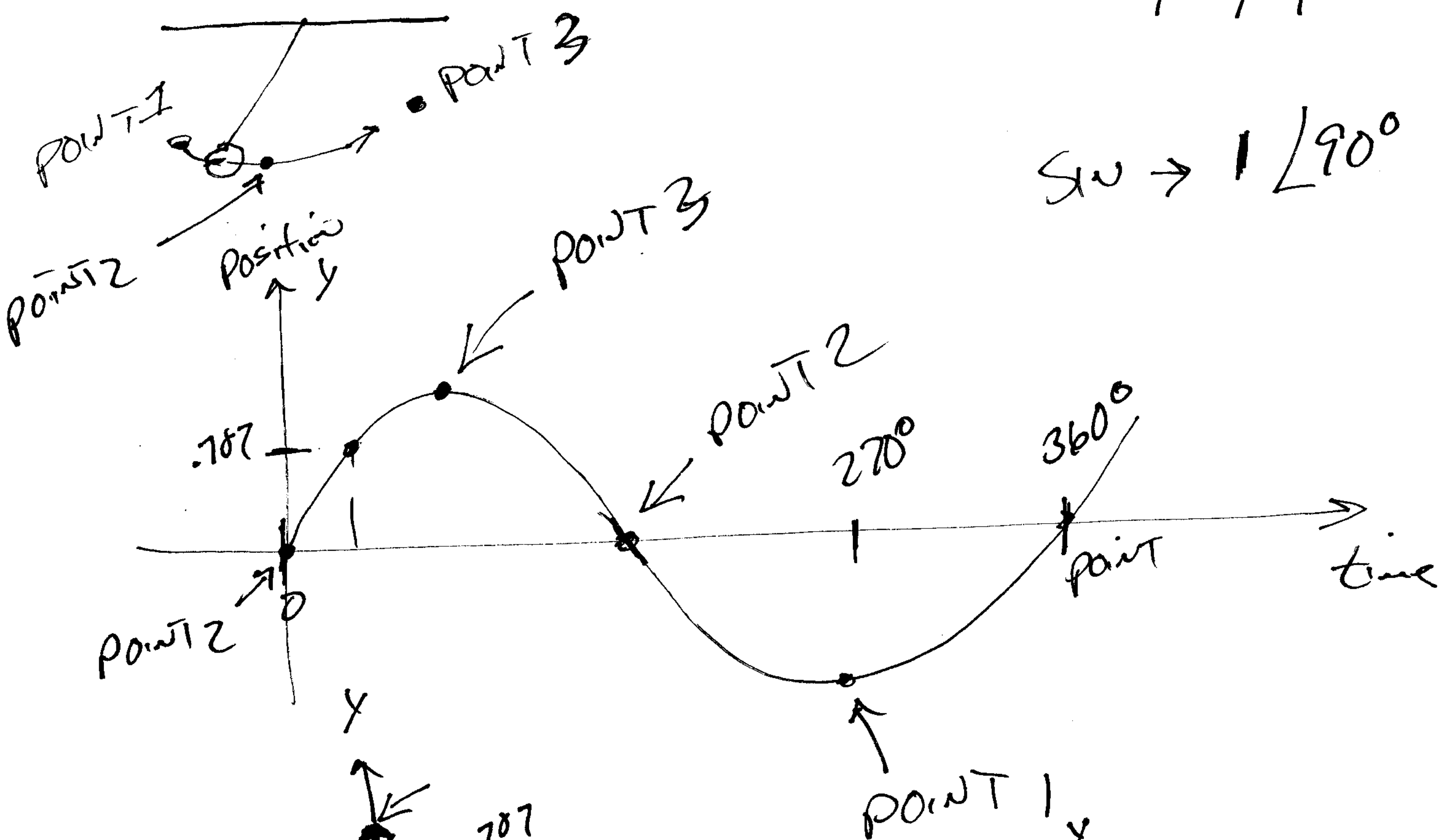
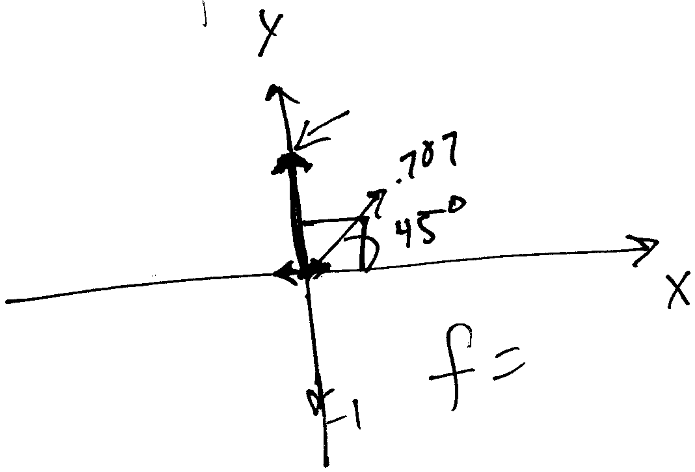
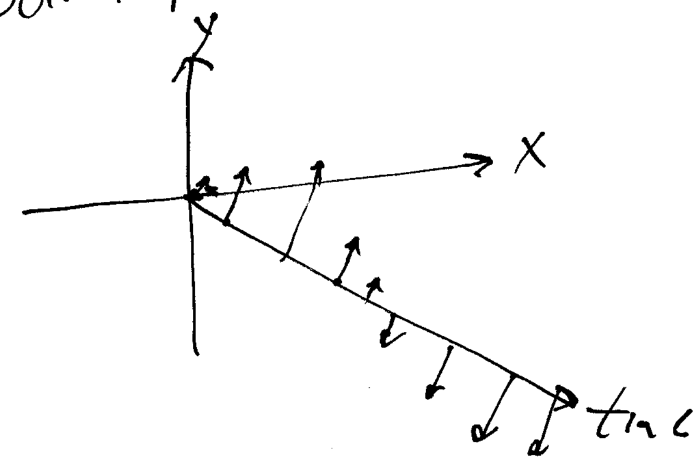
1/21/09



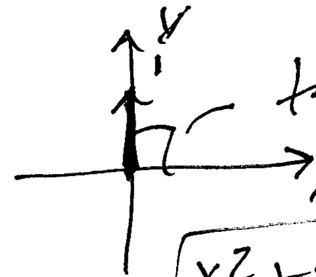
$\sin \to 1\angle 90°$



$f =$



$\sin 2\pi f_0 \cdot t \Rightarrow 1\angle 90^\circ \qquad x = 0 \,,\; y = 1$

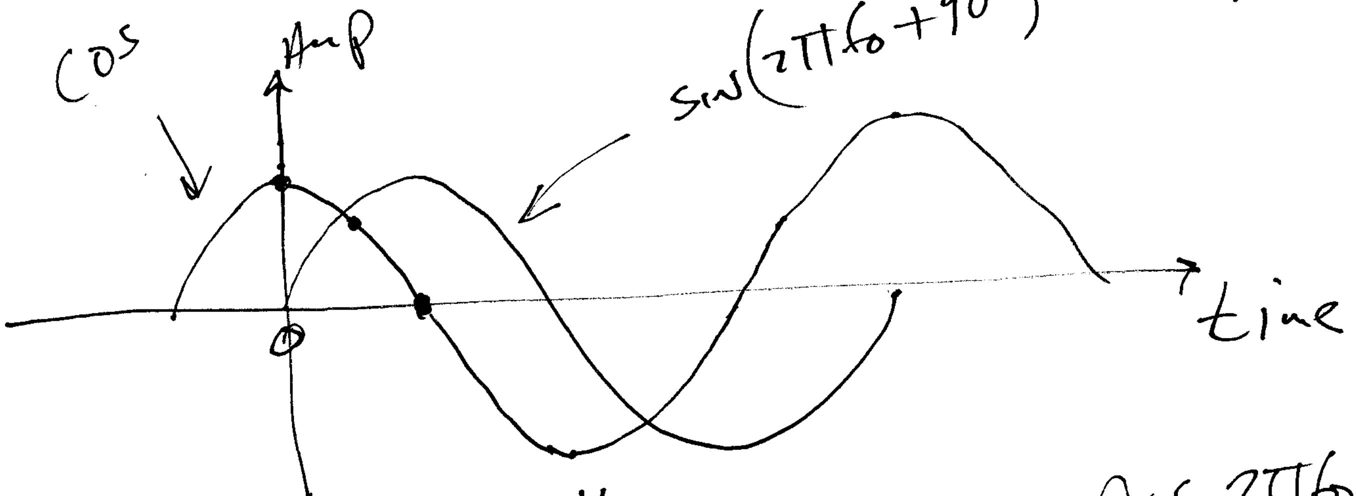


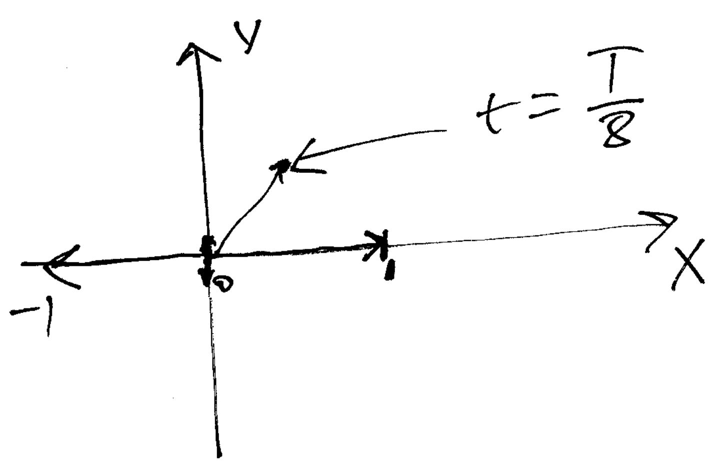
$\tan^{-1}\dfrac{y}{x} = \tan^{-1}\dfrac{1}{0}$

$= 90^\circ$

$\sqrt{x^2+y^2} = 1$

$\sin(2\pi f_0 + 90^\circ) = \cos(2\pi f_0)$





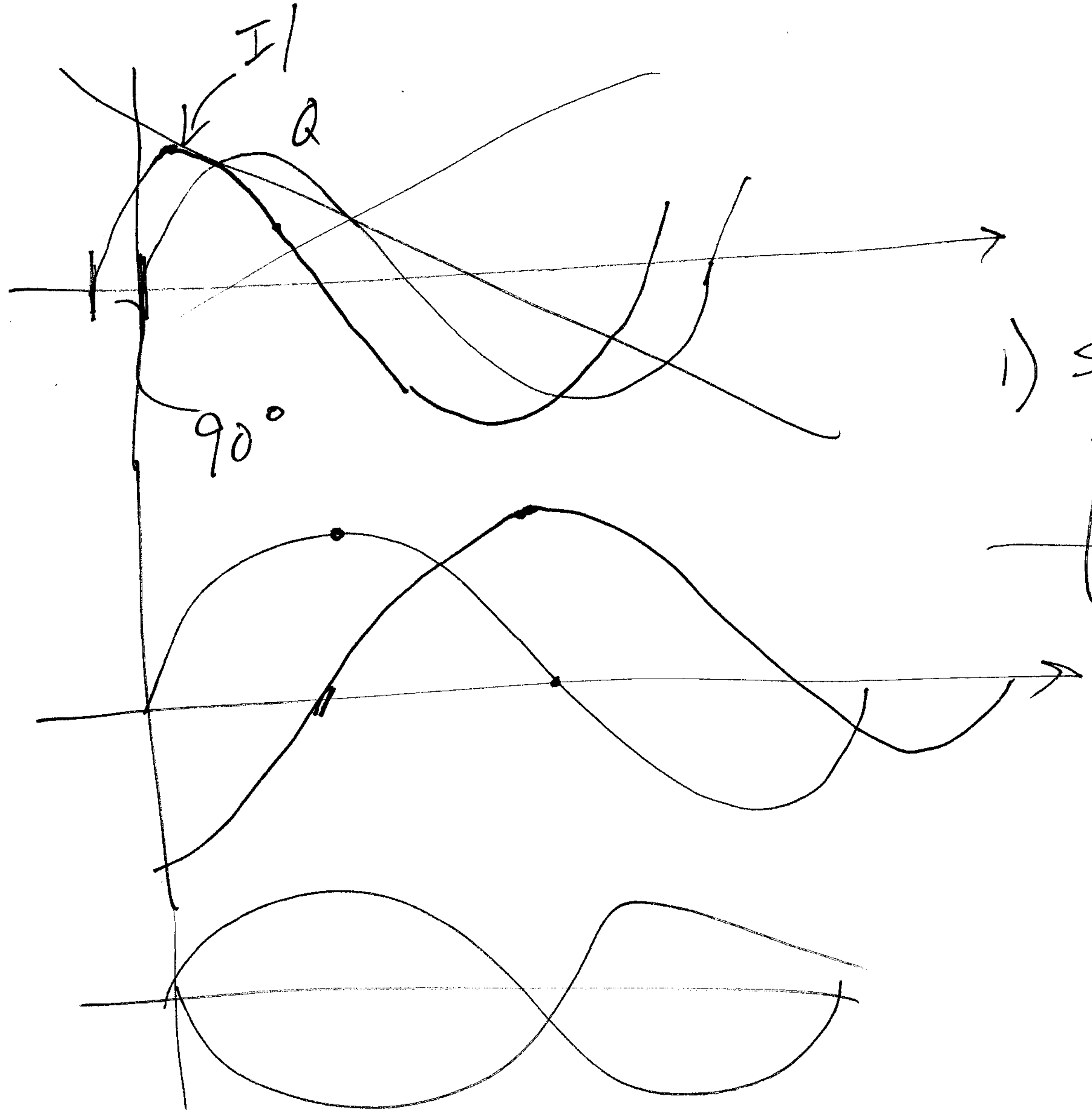
$\cos 2\pi f_0 \cdot t$

$1\angle 0^\circ$

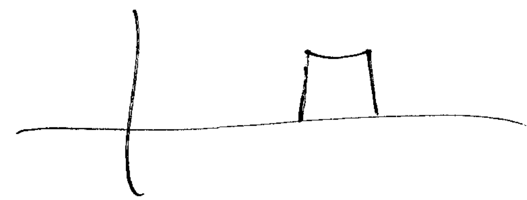
2)

I/Q → Quadrature Amp. MOD.

QAM



1) SAME BW



3)

Why use complex plane?

Shifting signals in time

or multiplying signals together

Taylor series

$$e^K = 1 + K + \frac{K^2}{2!} + \frac{K^3}{3!} + \cdots$$

$$\cos K = 1 - \frac{K^2}{2!} + \frac{K^4}{3!} - \frac{K^6}{6!} + \cdots$$

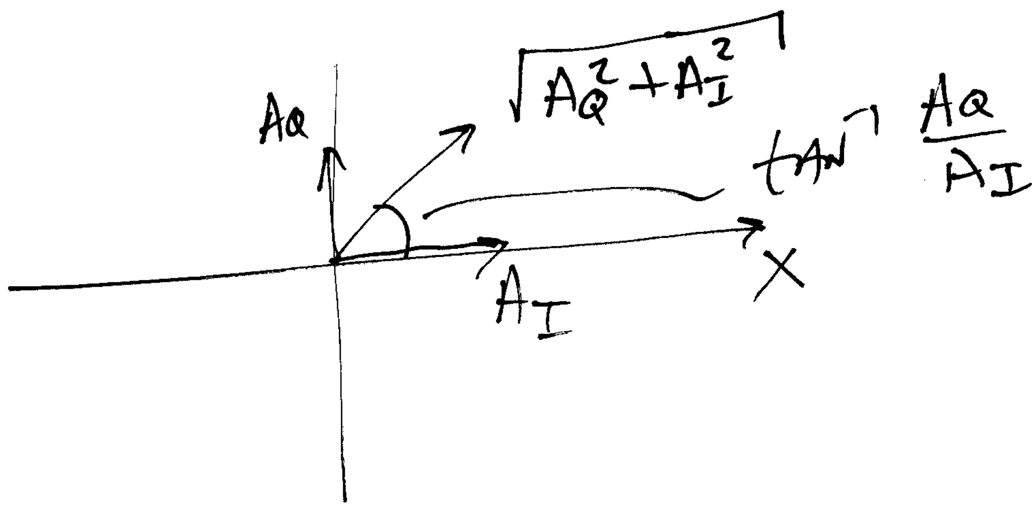
$$\sin K = K - \frac{K^3}{3!} + \frac{K^5}{5!} + \frac{K^7}{7!} \text{ etc.}$$

$$e^K \neq \cos K + \sin K = 1 + K - \frac{K^2}{2!} - \frac{K^3}{3!} + \frac{K^4}{4!} + \frac{K^5}{5!} + \cdots$$

$$e^{jk} = 1 + jk + \frac{-k^2}{2!} - \frac{jk^3}{3!}$$

$$j = \sqrt{-1}$$

$$e^{jk} = \underbrace{A_I \cos k}_{X} + j\,\underbrace{\overset{A_Q}{\sin k}}_{Y}$$



$$\frac{t_d}{T_0}\cdot 2\pi = 2\pi f_0 \cdot t_d$$

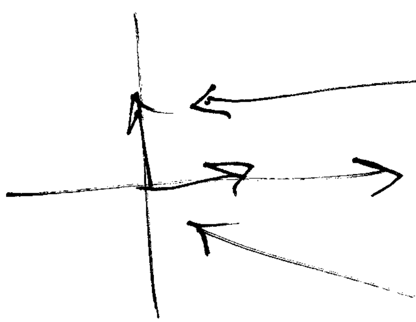
$x(t) = A_I \cos 2\pi f_0 t$ → delay → $x(t-t_d)$

$$x(t-t_d) = A_I \cos\Big(2\pi f_0 \cdot t - \underbrace{2\pi \cdot \frac{t_d}{T_0}}_{\theta}\Big)$$

$$\tan^{-1}\frac{\sin(\ )}{\cos(\ )}$$

$$= \mathrm{Re}\{A_I e^{j\cdot(2\pi f_0 t - \theta)}\}$$

$$A_I \cdot e^{j2\pi f_0 \cdot t} \cdot e^{-j2\pi f_0 \cdot t_d}$$



$$\cos(2\pi f_0 \cdot t_d) + j\sin(2\pi f_0 \cdot t_d)$$

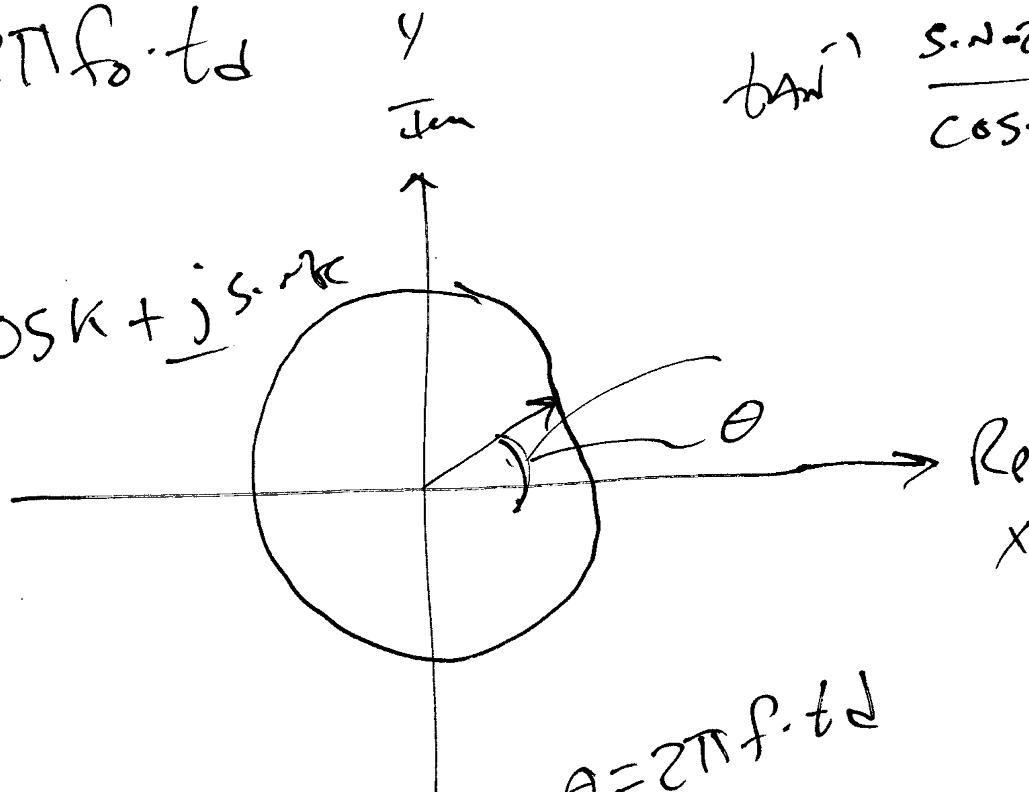
$$\left|e^{-j2\pi f_0 t_d}\right| = 1 \qquad \angle e^{-j2\pi f_0 t_d} = -2\pi f_0 \cdot t_d$$

b)

$e^{-j2\pi f_0 \cdot t_d}$

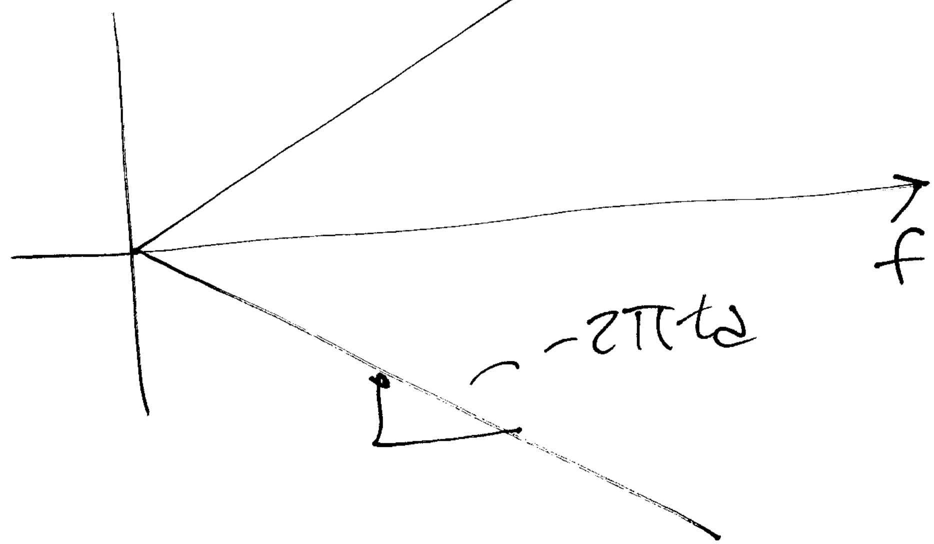
$\tan^{-1} \frac{\sin -2\pi f_0 t_d}{\cos -2\pi f_0 t_d} = -2\pi f_0 \cdot t_d$

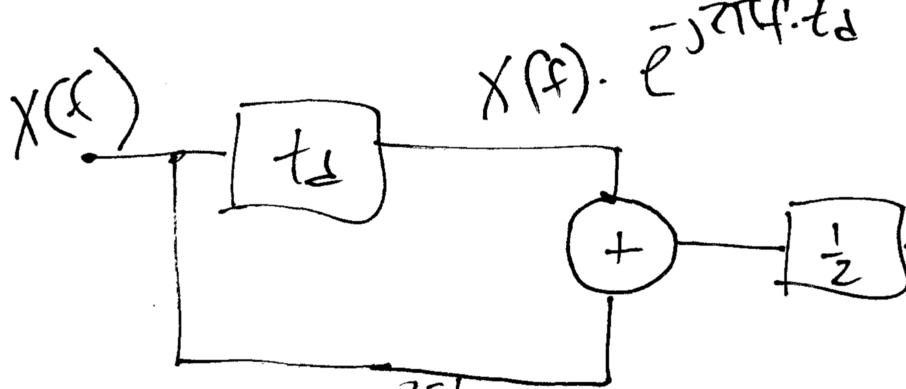
$e^{jk} = \cos k + j \sin k$



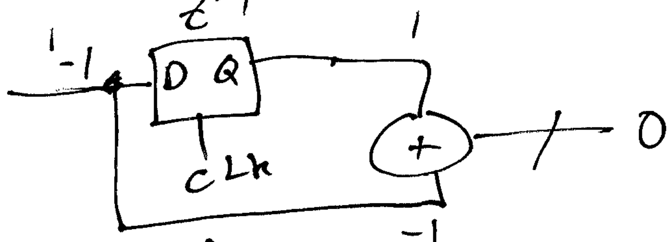
$z = e^{j2\pi f \cdot t_d}$

$\theta = 2\pi f \cdot t_d$





$$Y(f) = \frac{1}{2} \cdot X(f)\left(1 + e^{-j2\pi f t_d}\right)$$



$$H(f) = \frac{Y(f)}{X(f)} = \frac{1}{2}\left(1 + e^{-j2\pi f t_d}\right)$$

$$= \frac{1}{2}\left(\underbrace{1 + \cos 2\pi f t_d}_{\text{Real}} + j\,\underbrace{\sin(-2\pi f t_d)}_{\text{Im.}}\right)$$

$$|H(f)| = \frac{1}{2}\sqrt{(1 + \cos 2\pi f t_d)^2 + \sin^2(+2\pi f t_d)}$$