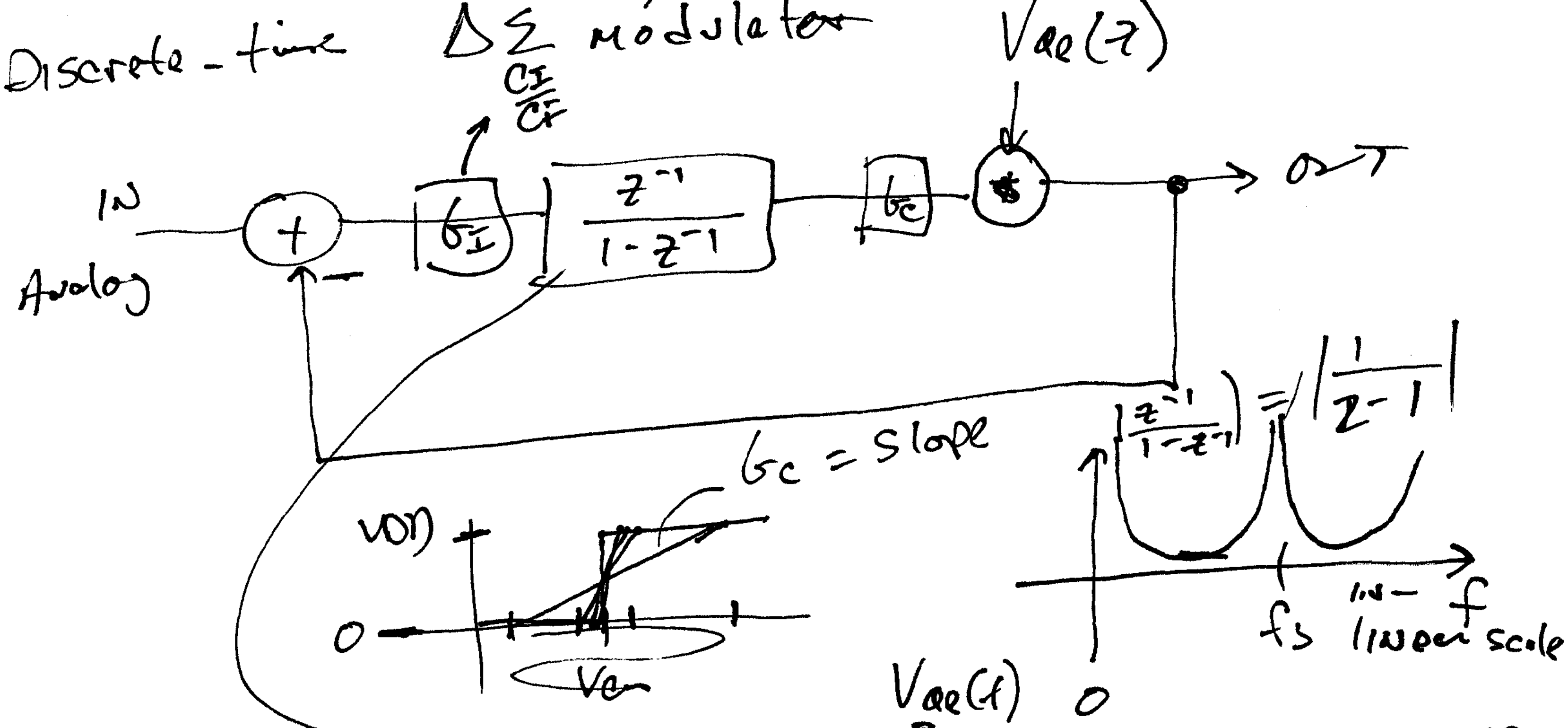
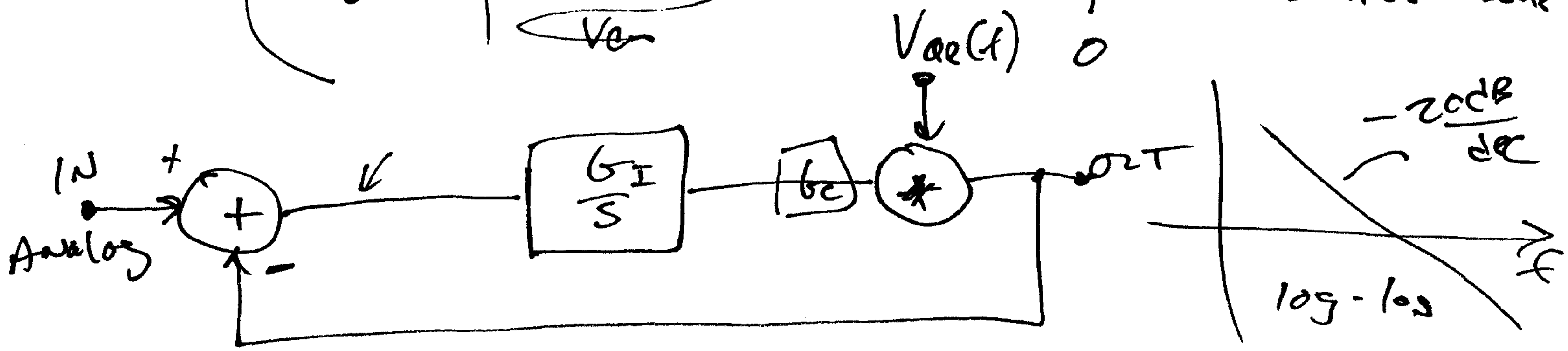
Lecture 19 4/6/09

Discrete-time ΔΣ modulator

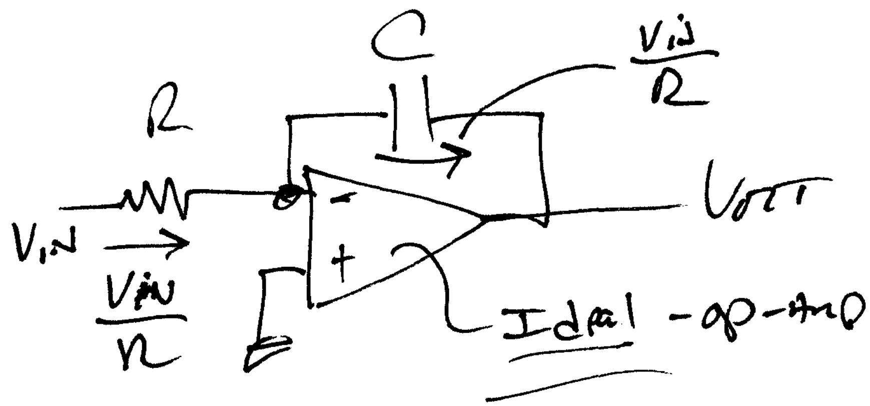




$$\left(V_{IN} - V_{OUT}\right)\frac{G_I \cdot G_C}{s} = V_{OUT}$$

$$V_{IN} \cdot \frac{G_I \cdot G_C}{s} = V_{OUT}\left(1 + \frac{G_I G_C}{s}\right)$$

$$V_{OUT} = \frac{V_{IN}}{\underbrace{\frac{s}{G_I G_C}}_{=1} + 1} = \frac{V_{IN}}{s + 1}$$



$$V_{OUT} = -\frac{V_{IN}}{R} \cdot \frac{1}{j\omega C}$$

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \left|\frac{1}{j\omega RC}\right|$$

$$G_I = \frac{1}{RC}$$

$$s = j\omega$$

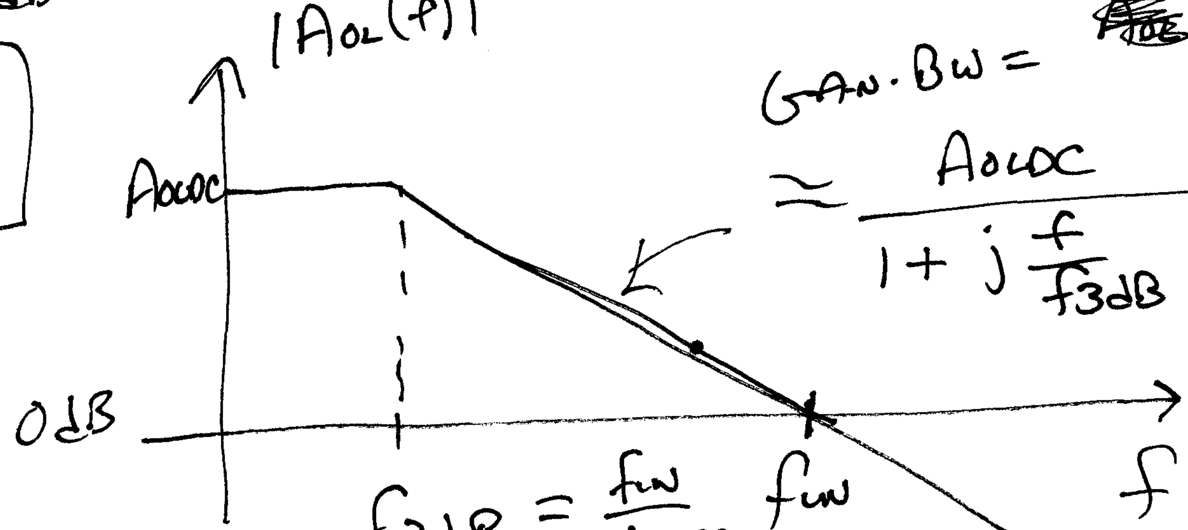
NON-ideal op-amp (integrator)

finite DC gain (open-loop)

finite BW

$$\frac{v_{OUT}}{v_+ - v_-} \approx \frac{A_{OLDC}}{j\frac{f}{f_{3dB}}}$$

$$\boxed{A_{OL}(f) \approx \frac{f_{un}}{jf}}$$

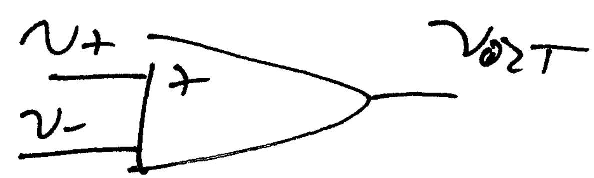


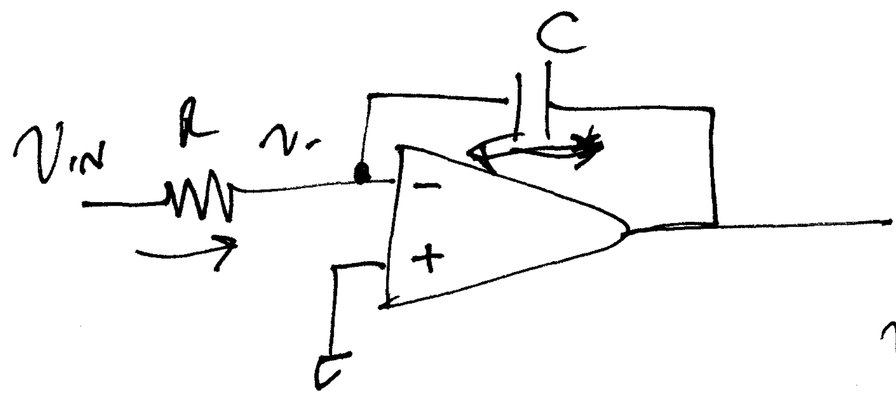
$f_{3dB} = \frac{f_{un}}{A_{OLDC}}$

GAIN · BW = ~~[illegible]~~ · f_{un}

$$\approx \frac{A_{OLDC}}{1 + j\frac{f}{f_{3dB}}} = \frac{v_{OUT}}{v_+ - v_-}$$

$$f_{3dB} \cdot A_{OLDC} = f_{un}$$





$$\frac{v_{OUT}}{v_{IN}} = \frac{f_{UN}}{j \cdot f} = A_{OL}$$

$$v_{OUT} = (\overset{0}{v_+} - v_-) A_{OL}$$

$$v_- = -\frac{v_{OUT}}{A_{OL}}$$

$$\frac{v_{IN} + \frac{v_{OUT}}{A_{OL}}}{R} + \frac{v_{OUT} + \frac{v_{OUT}}{A_{OL}}}{1/j\omega C} = 0$$

$$v_{IN} + \frac{v_{OUT}}{A_{OL}} + j\omega RC\left(v_{OUT} + \frac{v_{OUT}}{A_{OL}}\right) = 0$$

$$v_{OUT}\left(\frac{1}{A_{OL}} + j\omega RC \cdot \left(1 + \frac{1}{A_{OL}}\right)\right) = -v_{in}$$

4)

$$\frac{v_{OUT}}{v_{IN}} = \frac{-1}{\frac{1}{A_{OL}} + j\omega RC\left(1 + \frac{1}{A_{OL}}\right)}$$

$$A_{OL} \approx \frac{f_{un}}{j \cdot f}$$

$$\frac{V_{in}}{1 + j\omega RC} = \frac{-1}{\frac{jf}{f_{un}} + j\omega RC\left(1 + \frac{jf}{f_{un}}\right)}$$

$$\frac{V_{in}}{1 + j2\pi fRC}$$

$$f_{IN} < \frac{1}{2\pi RC} \qquad = \frac{-1}{\frac{jf}{f_{un}}\underbrace{\cancel{\left(1 + j\omega RC\right)}}_{\approx 1} + j\omega RC}$$

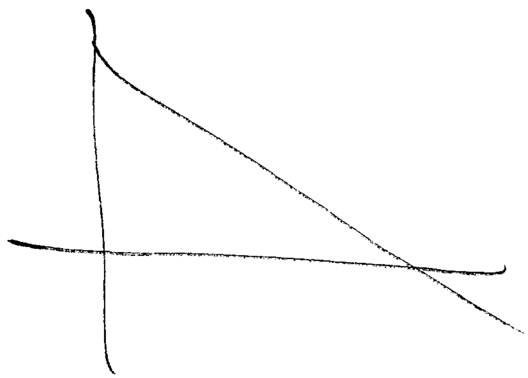
5)

$$\frac{v_{OUT}}{v_{IN}} \approx \frac{-1}{j\frac{f\cdot 2\pi}{f_{un}\cdot 2\pi} + j\omega RC}$$

$$= \frac{-1}{j\omega\left(RC + \frac{1}{2\pi f_{un}}\right)}$$

$$\left|\frac{v_{OUT}}{v_{IN}}\right| = 1 = \frac{1}{2\pi f_{un,s}\left(RC + \frac{1}{2\pi f_{un}}\right)}$$

shifted integrator unity gain freq.



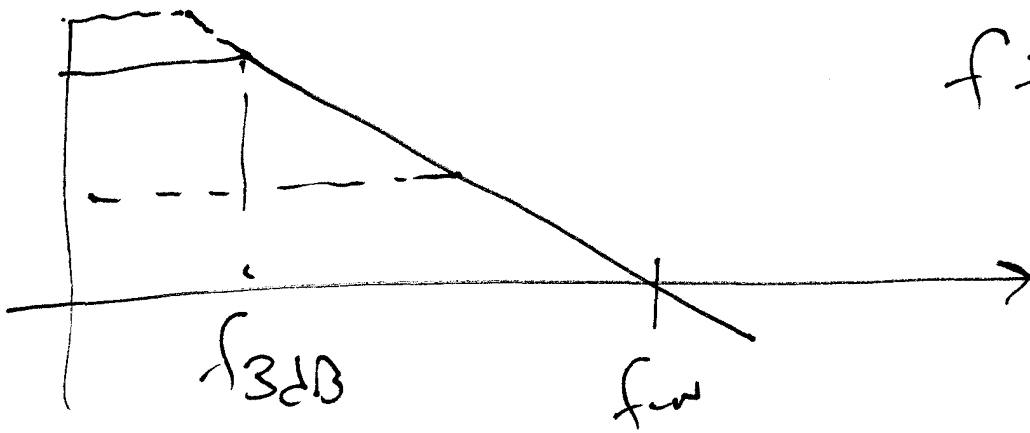
How finite BW effects ΔΣ modulator's performance

$$f_{un,s} = \frac{1}{2\pi\left(RC + \frac{1}{2\pi f_{un}}\right)}$$

$$\boxed{f_{un} > \frac{1}{2\pi RC}}$$

Op-Amp unity-gain

How DOES DC gain come in to play?



$f >> f_{3dB}$

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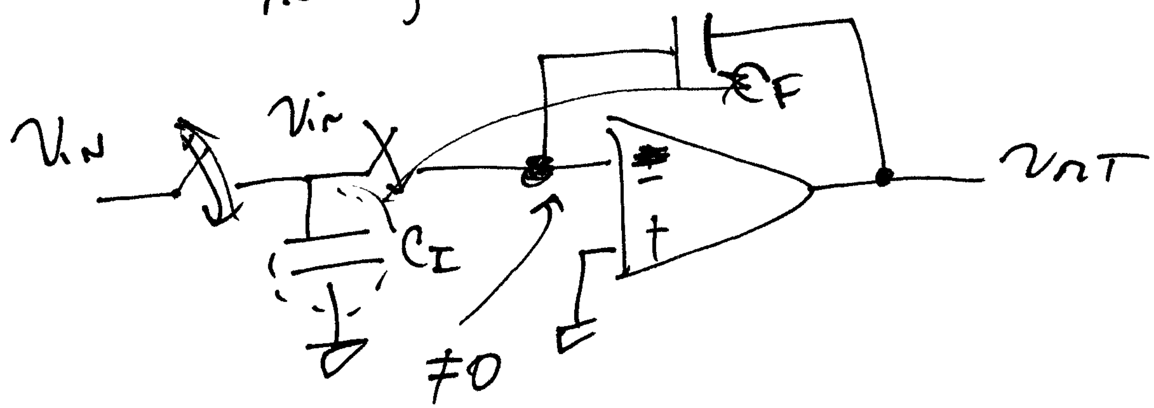
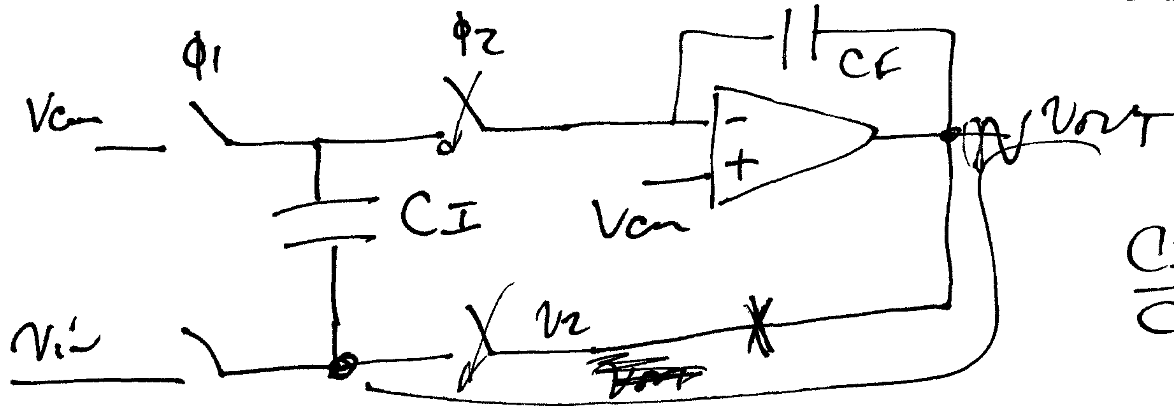
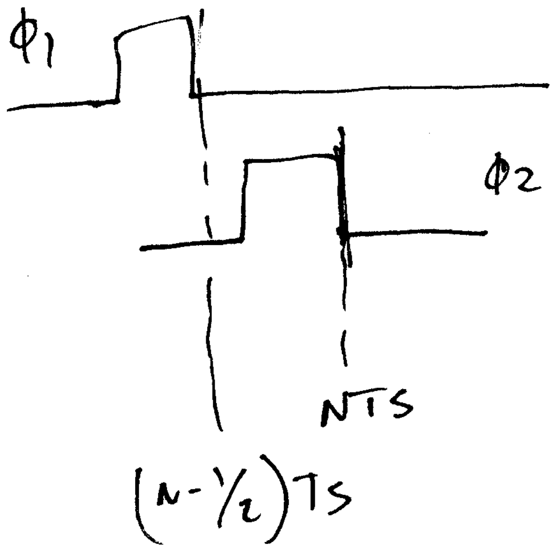
integrator leakage





Smaller swings

$\frac{C_I}{C_F}$



$$Q_1 = \left(v_{in}\left(\left(n-\tfrac{1}{2}\right)T_s\right) - V_{cm}\right) C_I$$

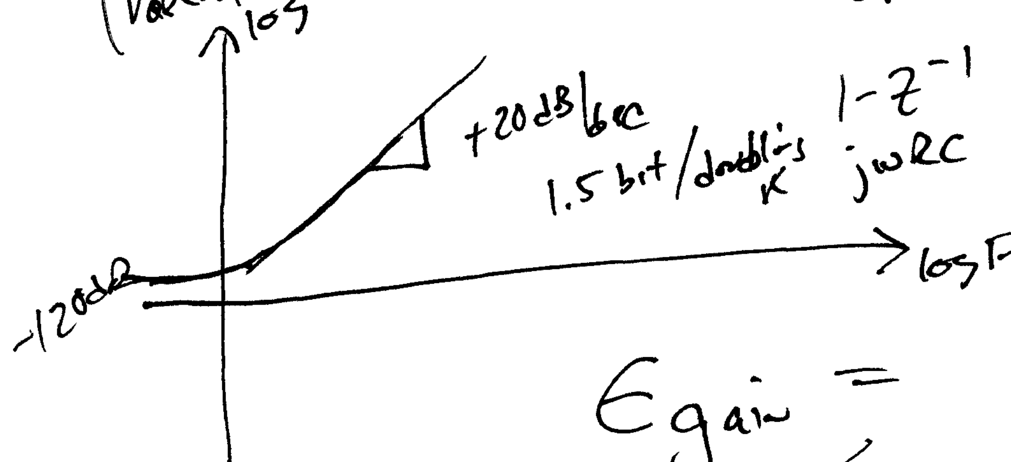
$$Q_2 = C_I \cdot \left(V_{cm} - \frac{v_{OUT}(nT_s)}{A_{OL}(f)} - v_2(nT_s)\right)$$

$$v_{OUT}(z) = \frac{C_I}{C_F} \cdot \frac{v_1 z^{-1/2} - v_2}{\left(1 + \frac{C_I}{C_F}\frac{1}{A_{OL}(f)}\right) - z^{-1}}$$

Eq. (7.32)

$$v_{nT}(z) = \frac{z^{-1}}{1+\frac{C_I}{C_F}\cdot\frac{1}{A_{OL}}}\cdot v_{in} + V_{QE}(z)\cdot\frac{1+\frac{C_I}{C_F}\frac{1}{A_{OL}}-z^{-1}}{1+\frac{C_I}{C_F}\frac{1}{A_{OL}}}$$

$|V^2_{QE}(f)|\cdot|V^2_{nT}(f)|$



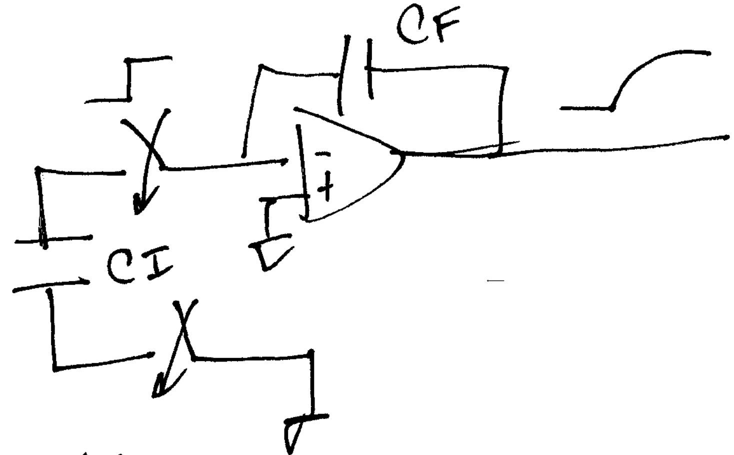
$$SNR = 6.02N + 1.76 - 5.13 + 30\log K$$

$$\underbrace{E_{gain}}_{\text{small}} = \frac{C_I}{C_F}\cdot\frac{1}{A_{OL}}$$

$K = 100$

$$V^2_{QE,RMS} = 2\cdot\frac{V^2_{LSB}}{12 f_s}\left[4(1+E_{gain})\frac{\pi^2}{f_s^2}\cdot\frac{1}{3}\left(\frac{f_s}{2K}\right)^2 + E_{gain}^2\cdot\frac{f_s}{2K}\right)$$

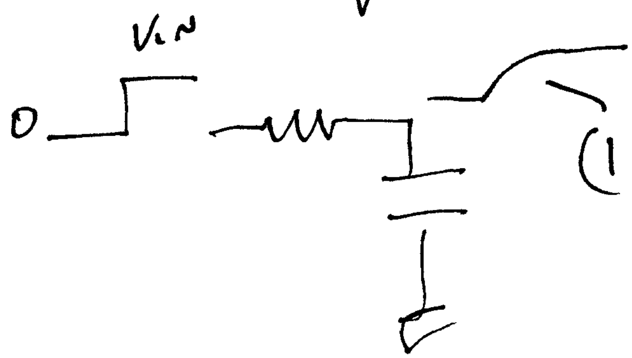
$$\boxed{A_{OL} > K}$$



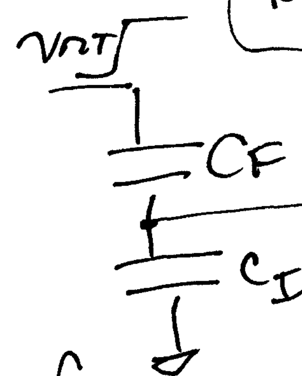
$$V_{OUT} = V_{OUTfinal}\left(1-e^{-t/\tau}\right)$$

$$\tau = \frac{1}{2\pi f_{un}}$$

$$\boxed{\beta = \frac{C_F}{C_F+C_I}}$$

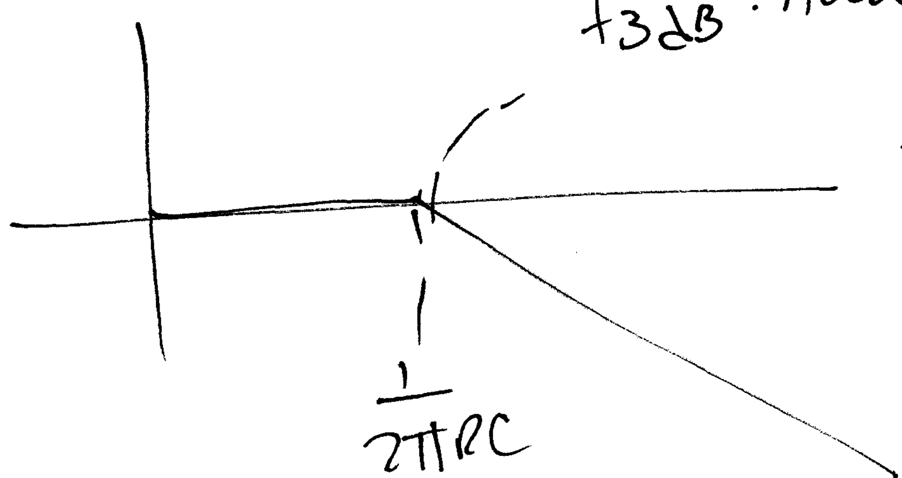


$\left(1-e^{-t/RC}\right)V_{IN}$,



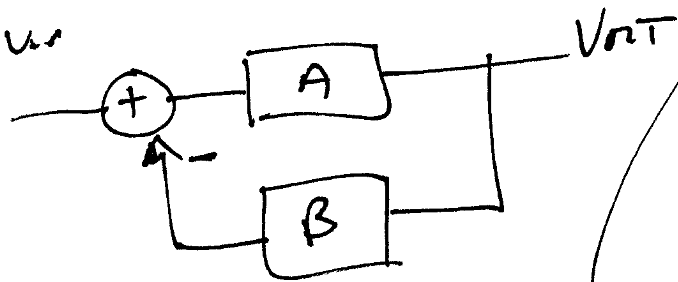
$$v_- = v_{OUT} \cdot \frac{1/j\omega C_I}{1/j\omega C_I + 1/j\omega C_F}$$

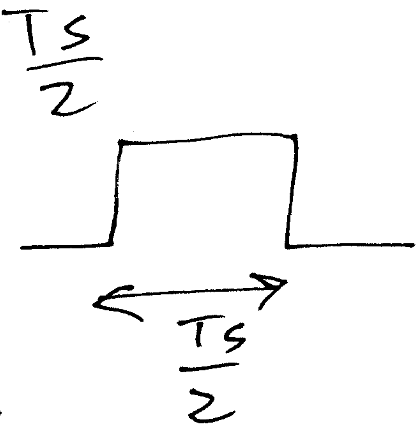
$$f_{3dB} \cdot A_{OLDC} = f_{un}$$



$$\frac{A_{OLDC}}{1 + j\frac{f}{f_{3dB}}} = \frac{1}{\frac{1}{A_{OLDC}} + j\frac{f}{f_{un}}}$$

$$V_{OUT}(z) = \frac{C_I}{C_F}\cdot\left(1-e^{-\pi\beta\cdot f_{un}/f_s}\right)\cdot\frac{V_1(z)\,z^{-1/2}-V_2(z)}{1-z^{-1}}$$





NO slewing

$$\frac{V_{OUT}}{v_{in}} = \frac{A}{1+\beta A}$$

$$= \frac{1}{\frac{1}{A}+\beta}$$