

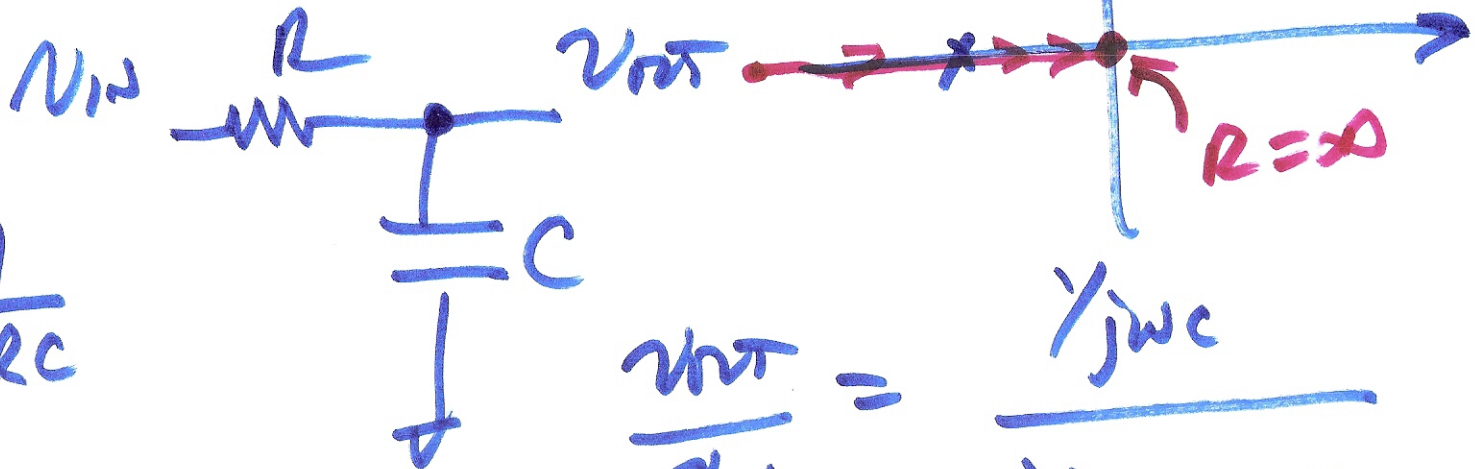
Sept. 20, 2010

### Lecture 8

Analog filters

$R=0$

$s = \sigma + j\omega$

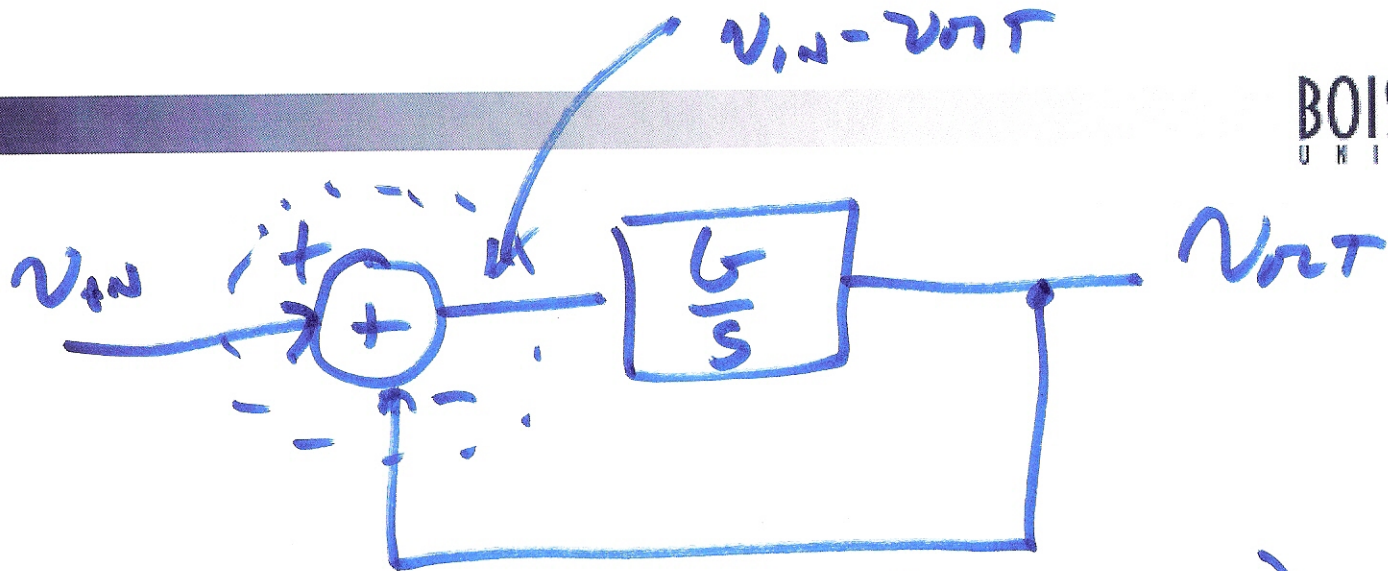


$$s_{\text{pole}} = -\frac{1}{RC}$$

$$\frac{V_{out}}{V_{in}} = \frac{1/j\omega C}{1/j\omega C + R}$$

$$\frac{1}{1+sRC} = \frac{1}{1+j\omega RC} \quad \sigma = -\frac{1}{RC}$$

1)

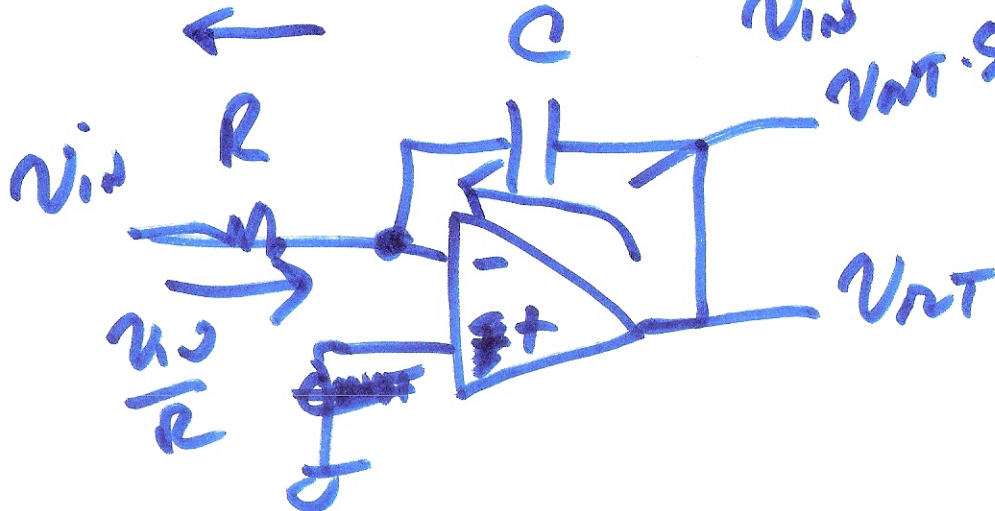


$v_{in}(-1/R)$

$$v_{out} = \frac{G}{s} (v_{in} - v_{out})$$

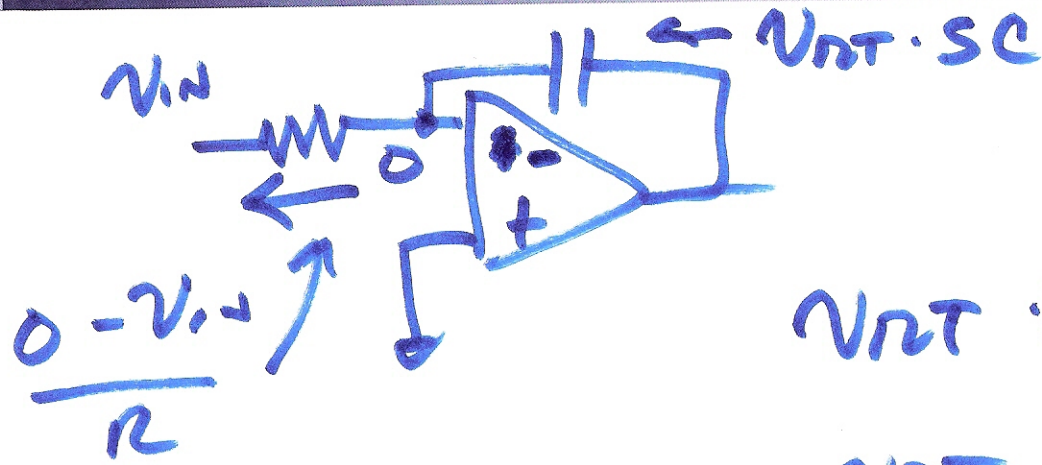
$-v_{in}/R$

$$\frac{v_{out}}{v_{in}} = \frac{1}{1 + s/RC} = \frac{1}{1 + sRC}$$



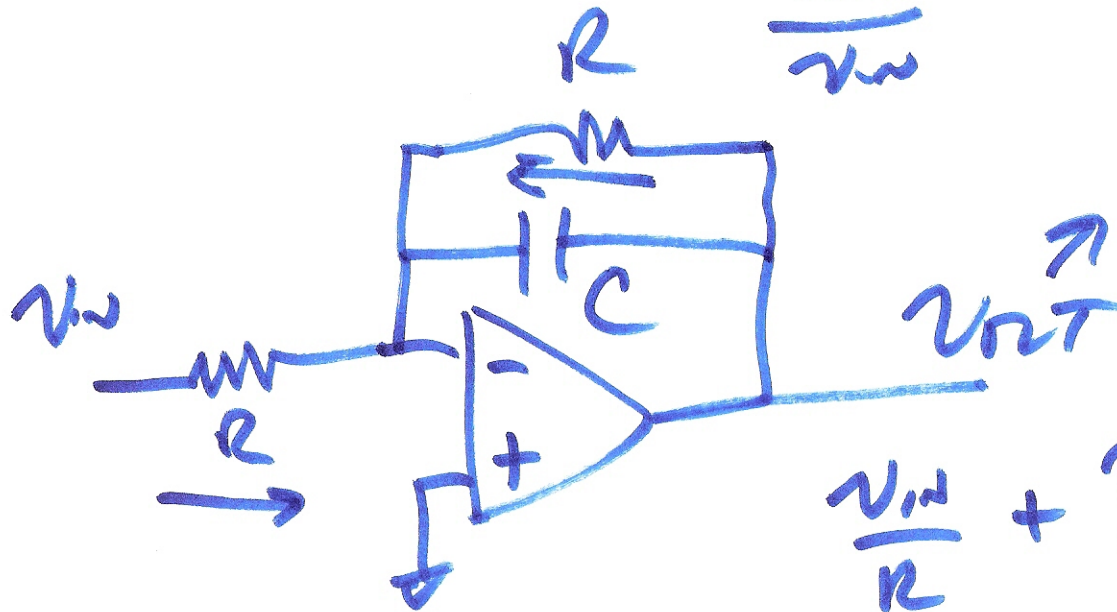
$G = \frac{1}{RC}$

2)



$$V_{out} \cdot sC = -\frac{V_{in}}{R}$$

$$\frac{V_{out}}{V_{in}} = -\frac{1}{sRC}$$

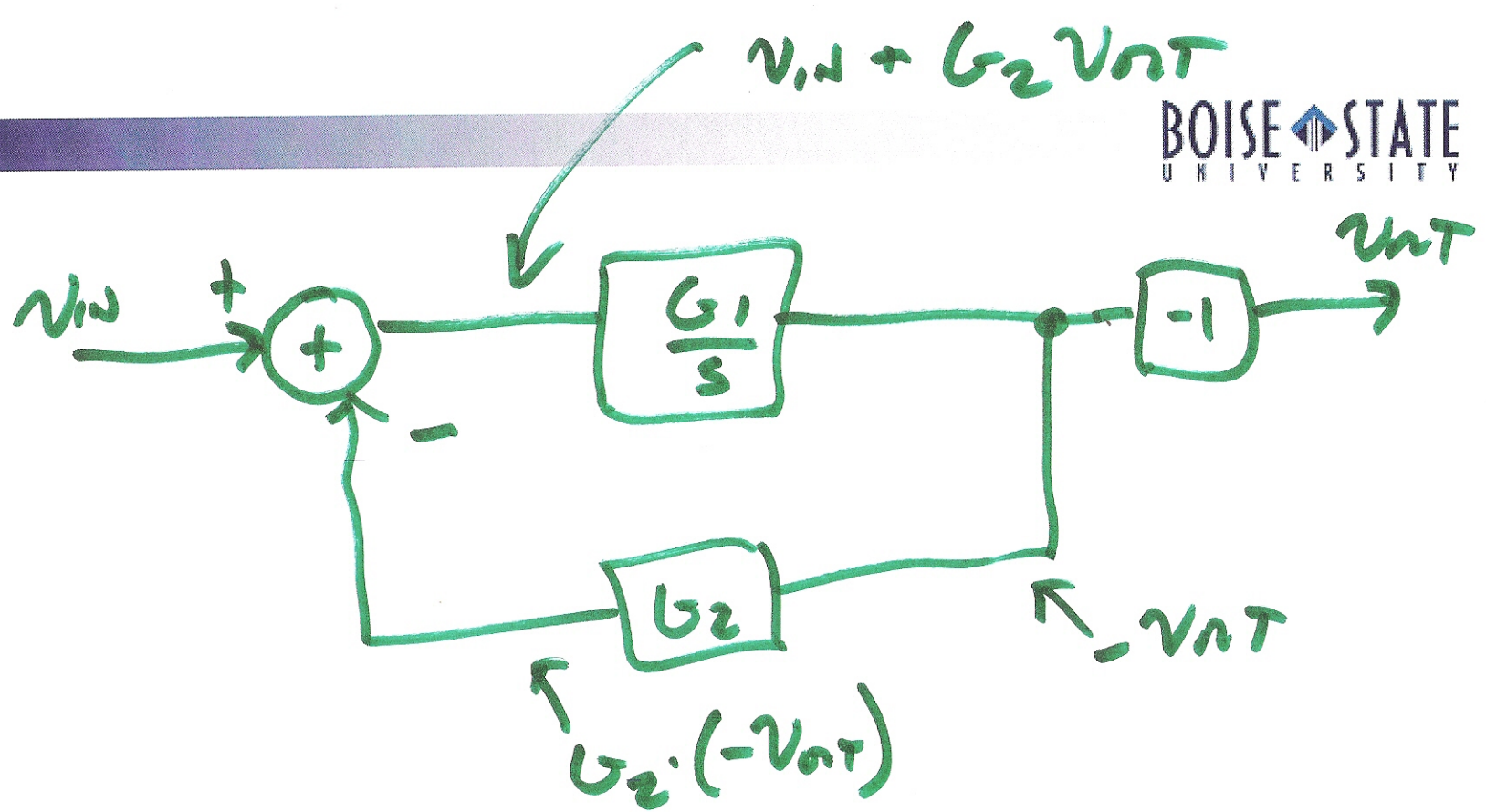


$$\frac{V_{in}}{R} + \frac{V_{out}}{R} + V_{out} sC = 0$$

$$\frac{V_{out}}{V_{in}} = \frac{-1}{1 + sRC}$$

$$-\frac{V_{in}}{R} = V_{out} \left( \frac{1}{R} + sC \right)$$

3)

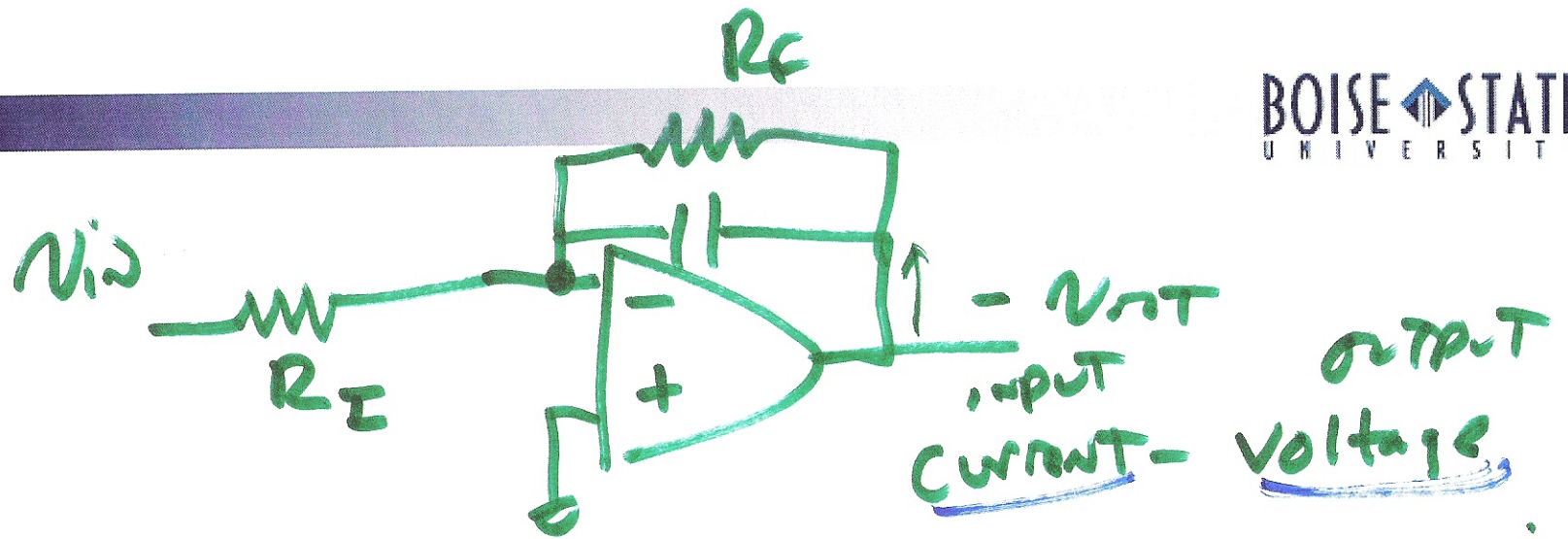


$$-V_{out} = \frac{G_1}{s} (V_{in} + G_2 V_{out})$$

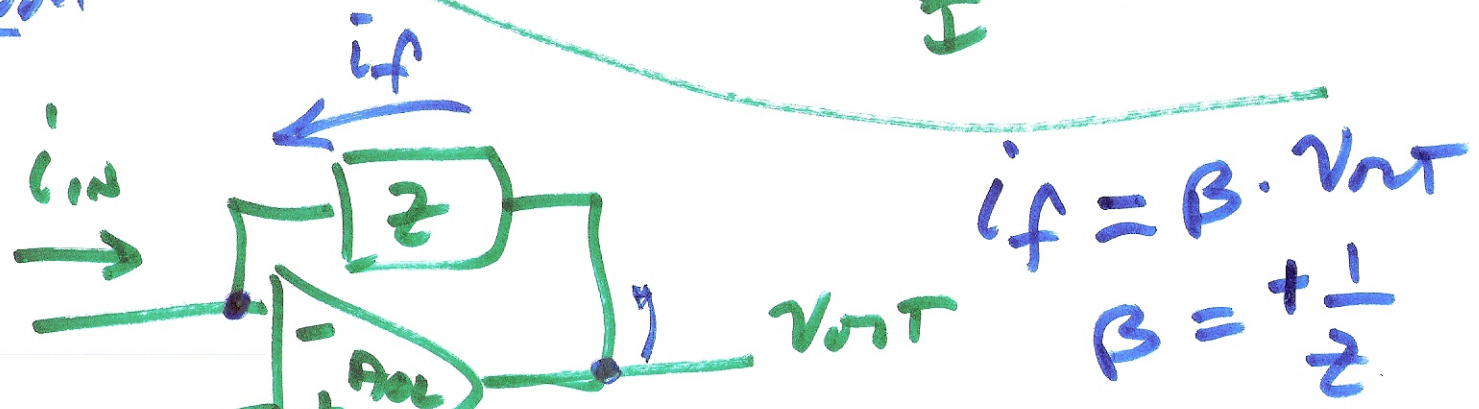
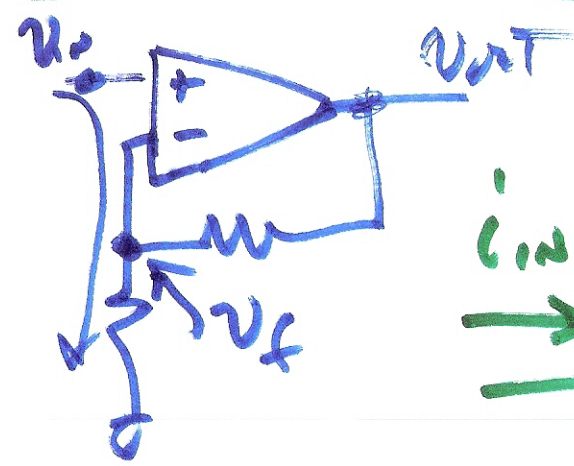
$$-V_{out} \left( 1 + \frac{G_1 G_2}{s} \right) = \frac{G_1}{s} V_{in}$$

$$\frac{-V_{out}}{V_{in}} = \frac{\cancel{G_1} \cancel{s}}{G_1 G_2 + s} = \frac{1}{G_2} \frac{1}{1 + \frac{s}{G_1 G_2}}$$

4)



$\frac{V}{I} = \text{transimped.}$

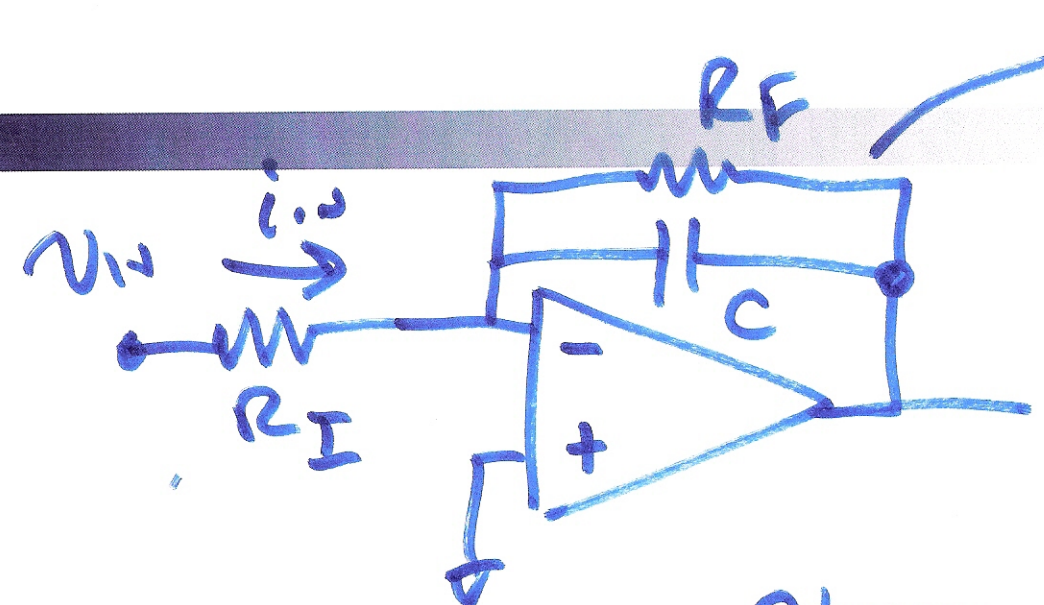


$\frac{A_{OL}}{A_{OL}\beta + 1} = A_{CL}$   
 $i_{in} + \frac{V_{out}}{Z} = 0$

$\frac{1/A_{OL}}{\beta + \beta \cdot 1/A_{OL}} \xrightarrow{A_{OL} \rightarrow \infty} = A_{CL} = \frac{1}{\beta}$

$\frac{V_{out}}{i_{in}} = -\frac{Z}{\beta}$

5)



$$\frac{R_F}{1 + sR_F C} = Z$$

$$G_1 G_2 = R_F C$$

$$\left(G_1 \frac{R_F}{R_I}\right)^{-1} = R_F C$$

$$i_{in} = \frac{v_{in}}{R_I}$$

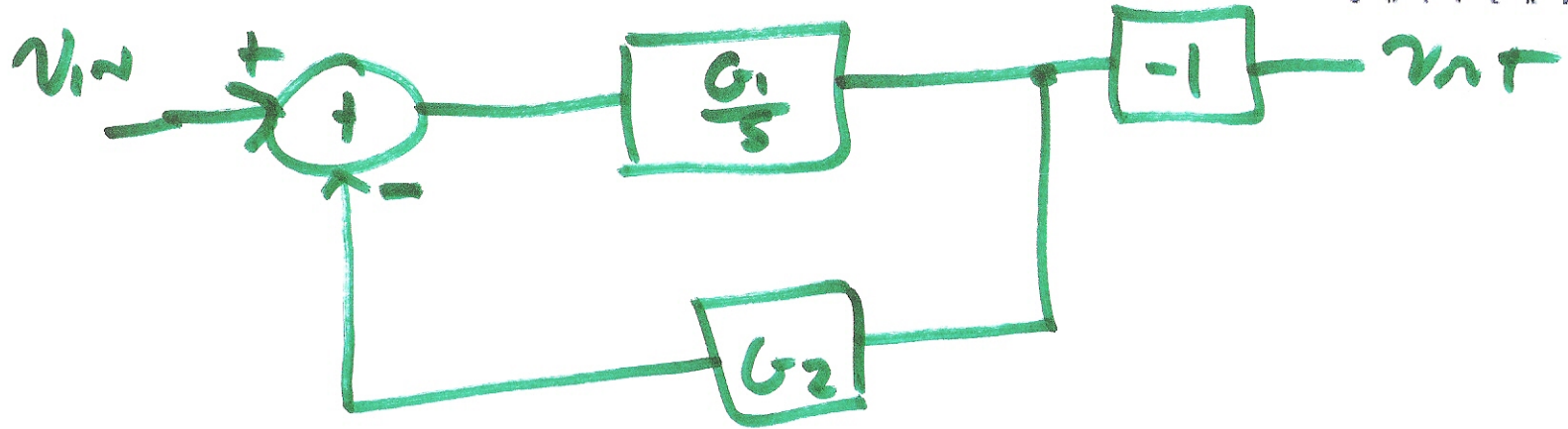
$$\frac{v_{OUT}}{v_{in}} = \frac{-R_F}{1 + R_F \cdot s \cdot C}$$

$$\frac{-\frac{R_F}{R_I}}{1 + sR_F C}$$

$$\frac{v_{OUT}}{i_{in}} = \frac{-R_F}{1 + sR_F C}$$

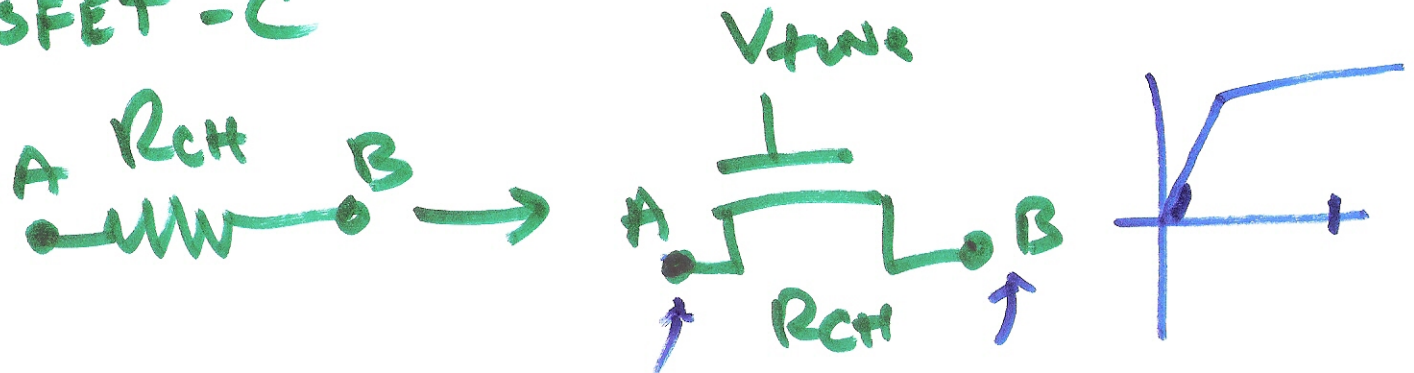
$$= \frac{-\frac{1}{G_2}}{1 + \frac{s}{G_1 G_2}} \left\{ \begin{array}{l} G_2 = \frac{R_I}{R_F} \\ G_1 = \frac{1}{R_I \cdot C} \end{array} \right.$$

6)



$$\frac{v_{out}}{v_{in}} = \frac{1}{1 + \frac{s}{G_1 G_2}}$$

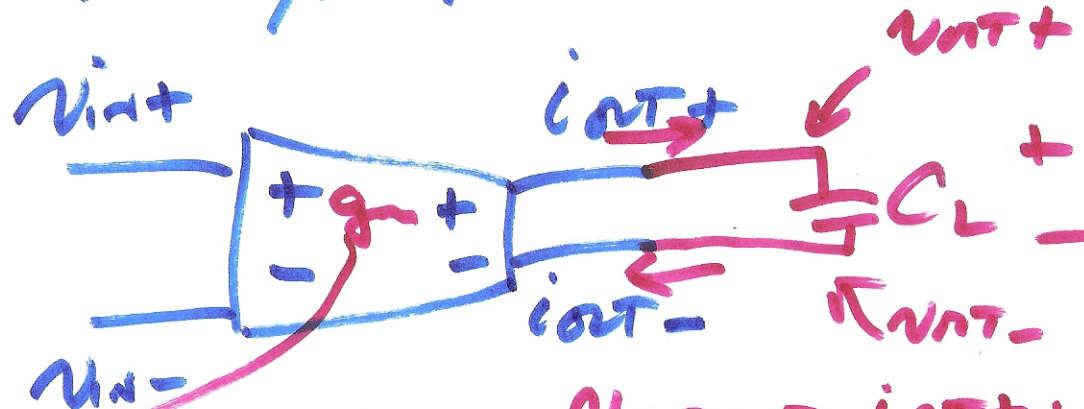
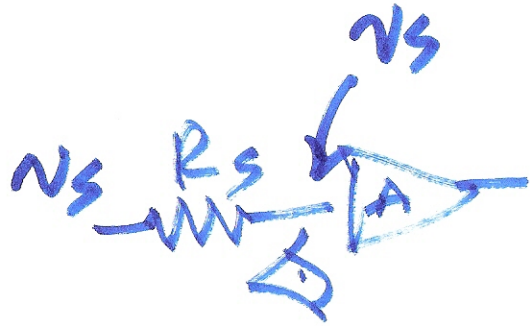
MOSFET - C



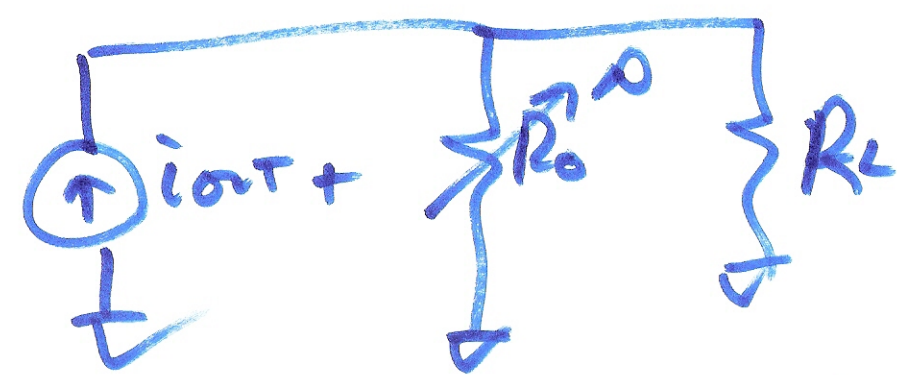
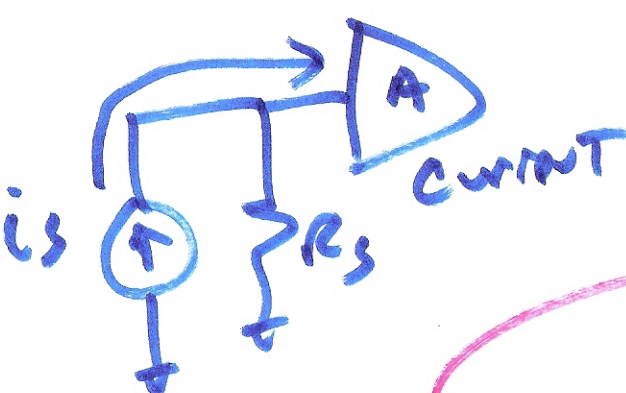
7)

# $g_m$ -C (transconductor) OTA

transconductor with only two high-impedance nodes INPUTS/OUTPUTS



$$v_{out+} - v_{out-} = i_{out+} \cdot \frac{1}{j\omega C_L} = i_{out+} \cdot \frac{1}{j\omega C_L}$$



$$g_m (v_{in+} - v_{in-}) = (i_{out+} - i_{out-})$$

$$= \frac{g_m}{2} (v_{in+} - v_{in-}) = \frac{1}{2} i_{out+}$$

8)



# Integrator using $g_m$ -C

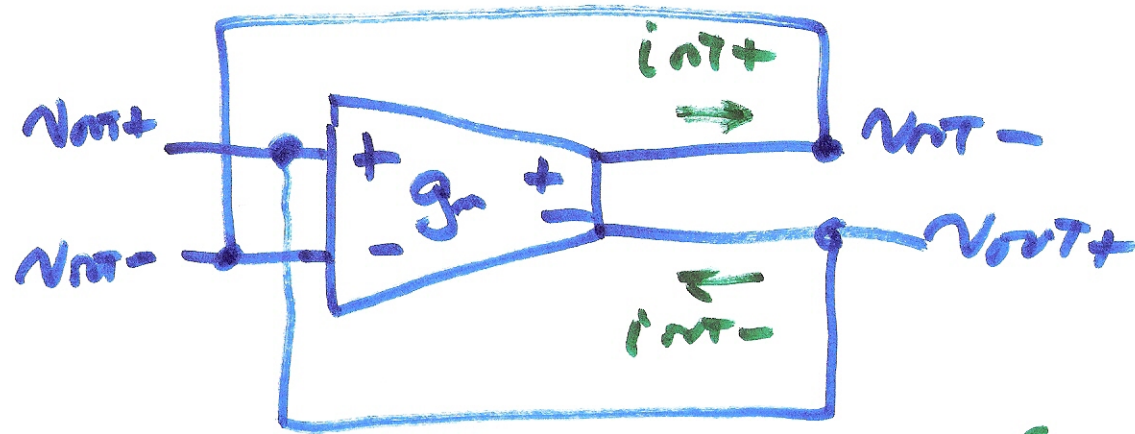


$$v_{out+} - v_{out-} = i_{out+} \cdot \frac{1}{j\omega C}$$

$$= g_m (v_{in+} - v_{in-}) \frac{1}{j\omega C}$$

$$\frac{v_{out+} - v_{out-}}{v_{in+} - v_{in-}} = \frac{1}{j\omega \cdot \frac{2}{g_m} \cdot C}$$

a)

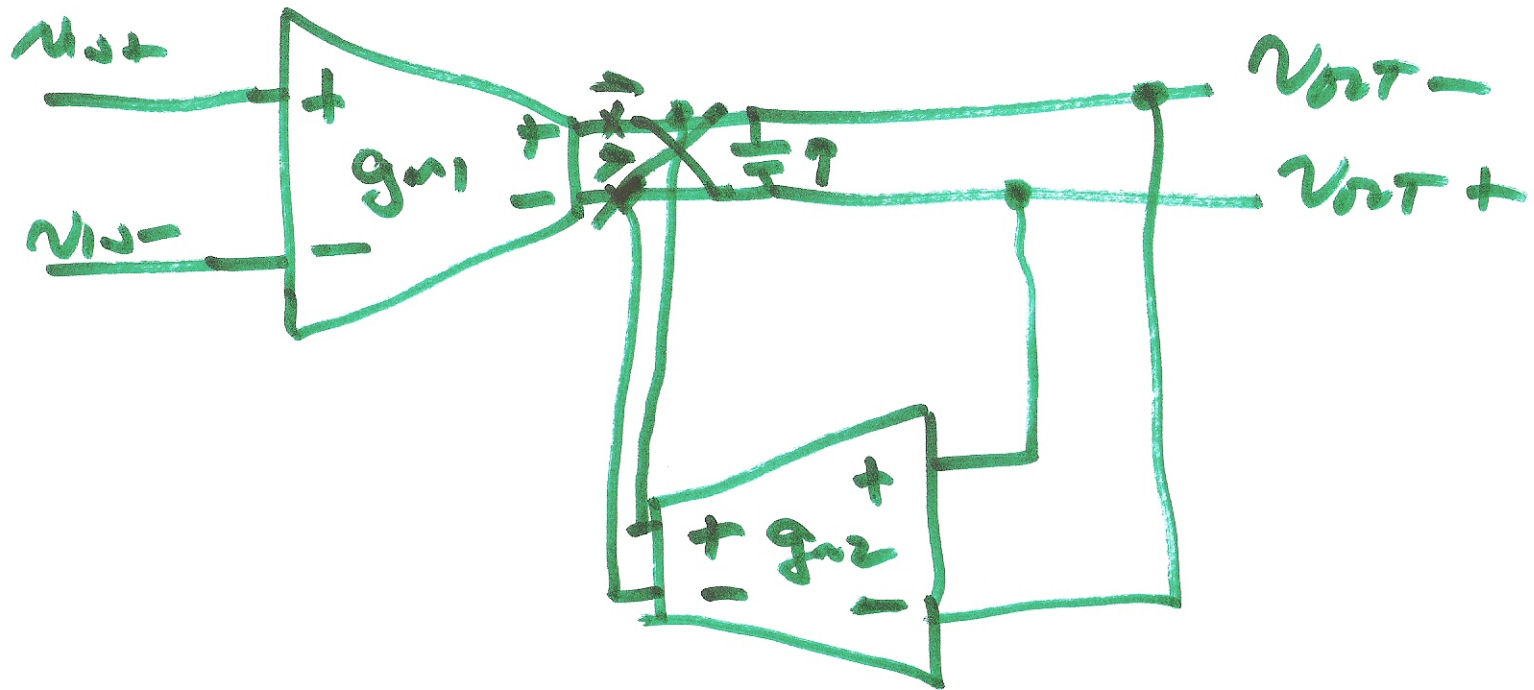


$$g_m (v_{out+} - v_{out-}) = (i_{out+} - i_{out-})$$

$$\frac{v_{out+} - v_{out-}}{i_{out+} - i_{out-}} = \frac{1}{g_m} \text{ (Res.!)}$$

10)

Fig. 3.16

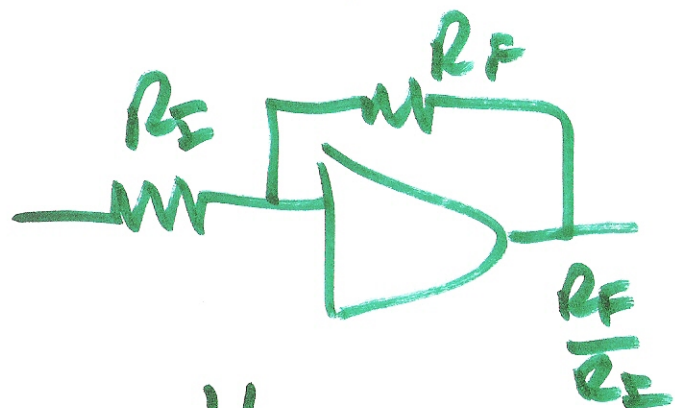


$$-g_{m1}(v_{in+} - v_{in-}) + g_{m2}(v_{out+} - v_{out-}) + (v_{out-} - v_{out+}) \cdot sC = 0$$

$$(v_{out+} - v_{out-}) \cdot (g_{m2} + sC) = g_{m1}(v_{in+} - v_{in-})$$

11)

$$\frac{V_{out+} - V_{out-}}{V_{in+} - V_{in-}} = \frac{g_{m1}}{g_{m2} + sC}$$



$$\frac{V_{out-}}{V_{in+}}$$

$$= \frac{\frac{g_{m1}}{g_{m2}}}{1 + \frac{sC}{g_{m2}}} = \frac{1}{G_2}$$

$$G_2 = \frac{g_{m2}}{g_{m1}} \left( 1 + \frac{s}{G_1 G_2} \right)$$

$$\frac{1}{G_1 G_2} = \frac{C}{g_{m2}} \rightarrow G_1 = \frac{g_{m1}}{g_{m2}} \cdot \frac{g_{m2}}{C} = \frac{g_{m1}}{C}$$


12)

$$\frac{v_{out}}{v_1(z) - v_2(z)} = \frac{C_I}{C_F} \cdot \frac{z^{-1/2}}{1 - z^{-1}}$$

$$\frac{1}{s}$$

$$z = e^{j2\pi f / f_s} \approx 1 + j2\pi \frac{f}{f_s}$$

$f \ll f_s$





$$\frac{C_I}{C_F} \cdot \frac{1}{z-1} \approx \frac{C_I}{C_F} \cdot \frac{-1}{j2\pi \frac{f}{f_s}}$$

$$\frac{v_1 \cdot z^{-1/2} - v_2 \cdot z^{-1/2}}{1 - z^{-1}}$$

$$\frac{C_I}{C_F}$$

$$\frac{1}{s} G$$

$$G = \frac{C_I}{C_F} \cdot f_s$$

$$f \ll f_s$$

13)