ECE 615 Sept. 8, 2010

BOISE STATE UNIVERSITY

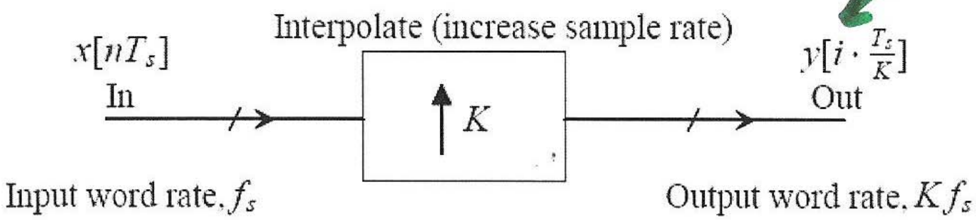


Figure 2.24 Block diagram of an interpolation block.

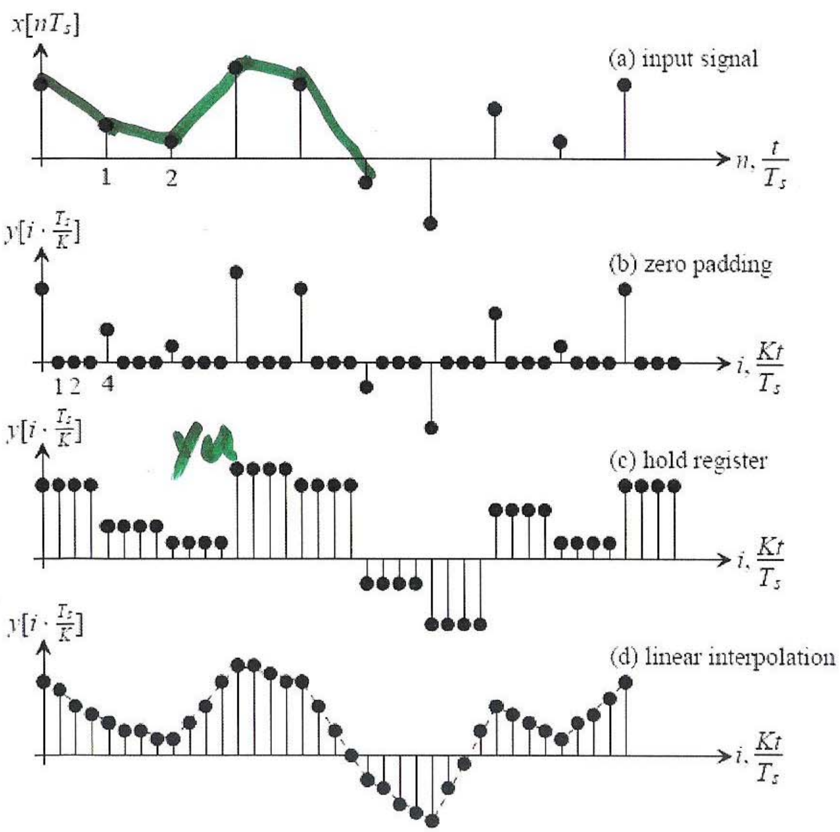
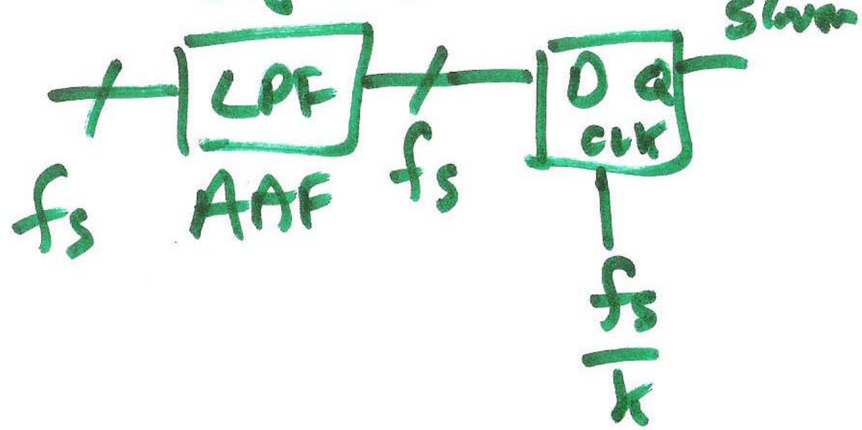


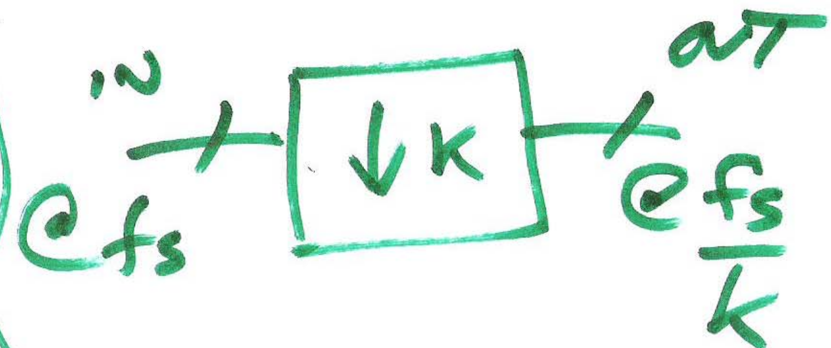
Figure 2.25 Types of interpolation.

Lecture 5

Decimation

Digital





1)

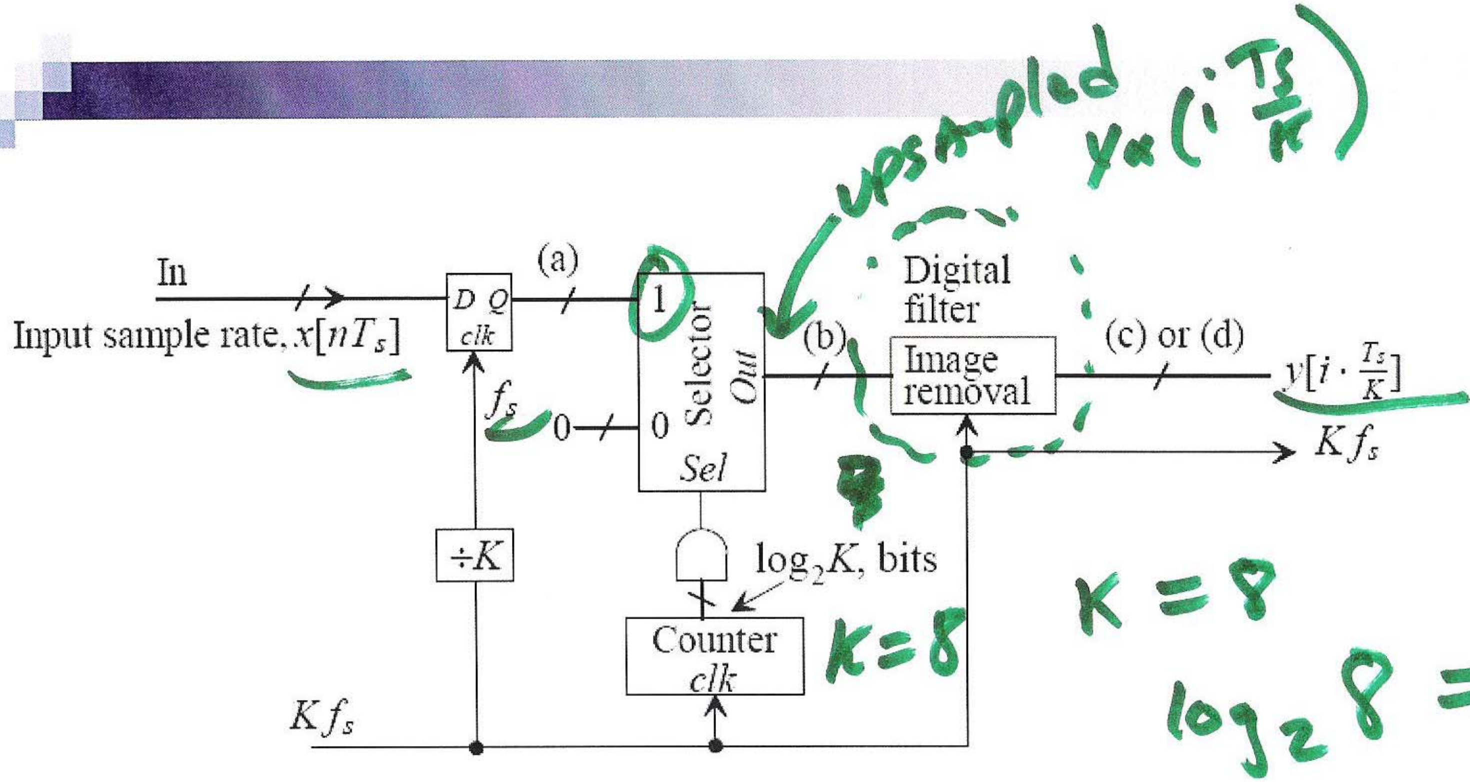
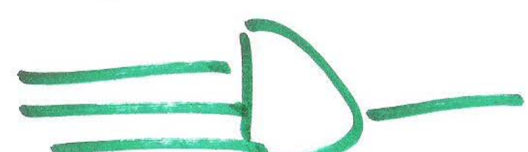


Figure 2.26 A zero-padding interpolation block, see spectrums in Fig. 2.27.

$K = 8$

$\log_2 8 = 3$

000 0
001
010
.
.
.
111 7



2)

$f_s/2 = f_N$

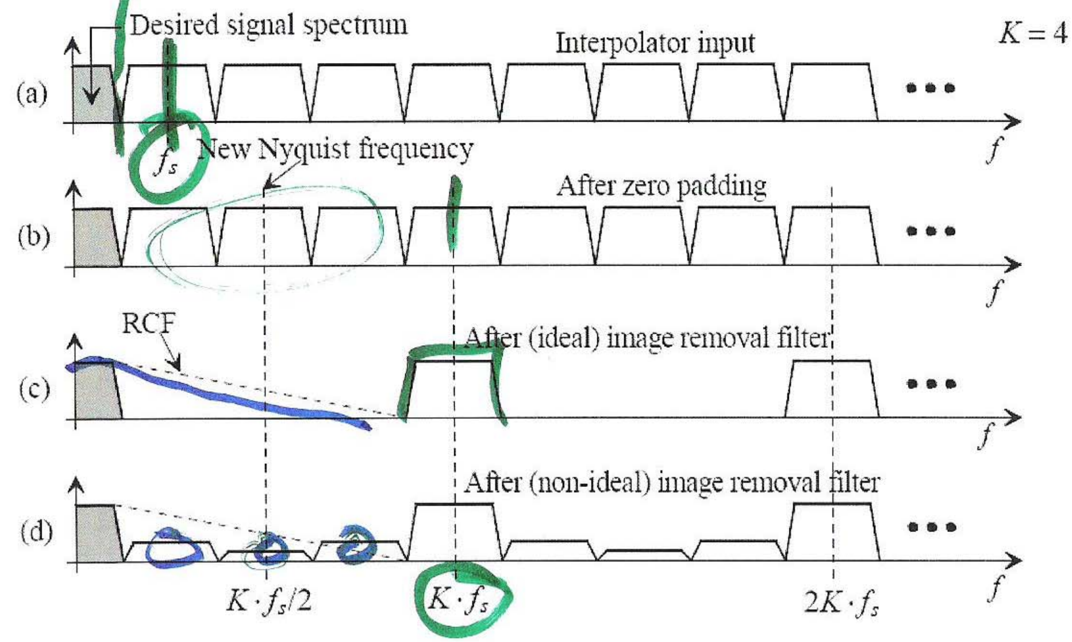
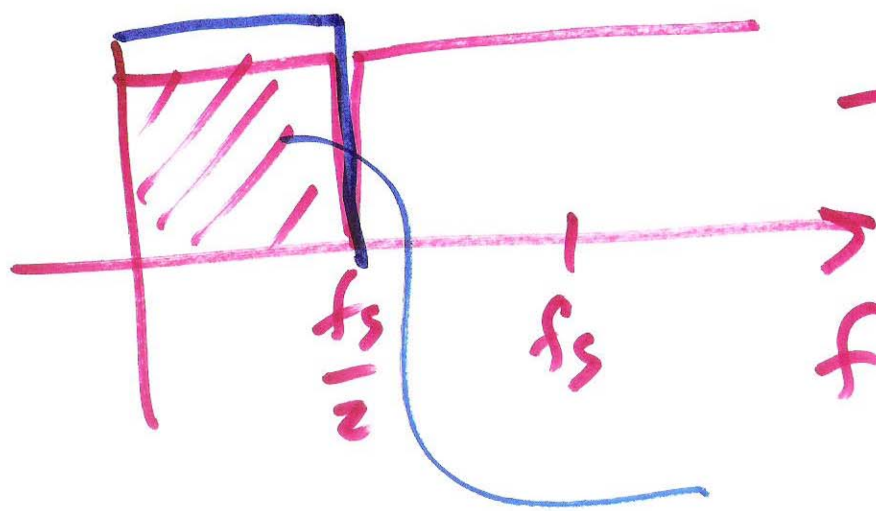


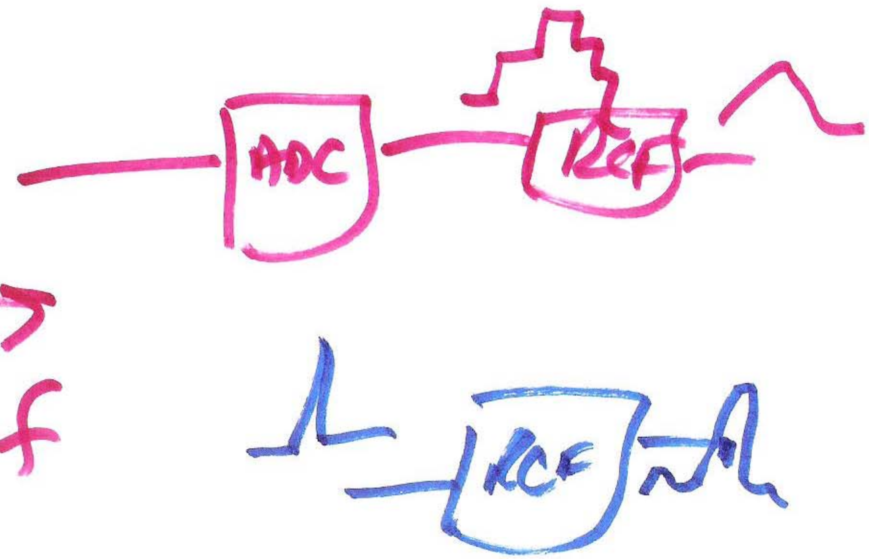
Figure 2.27 Example spectrums when zero padding interpolation is employed.

$f_N = \frac{f_s}{2}$

$f_s = 2f_N$

100MHZ





3)

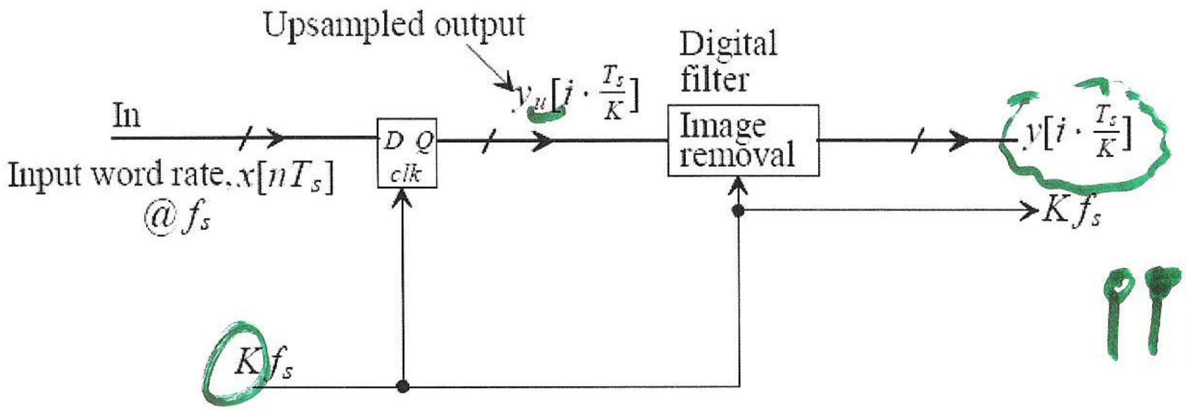
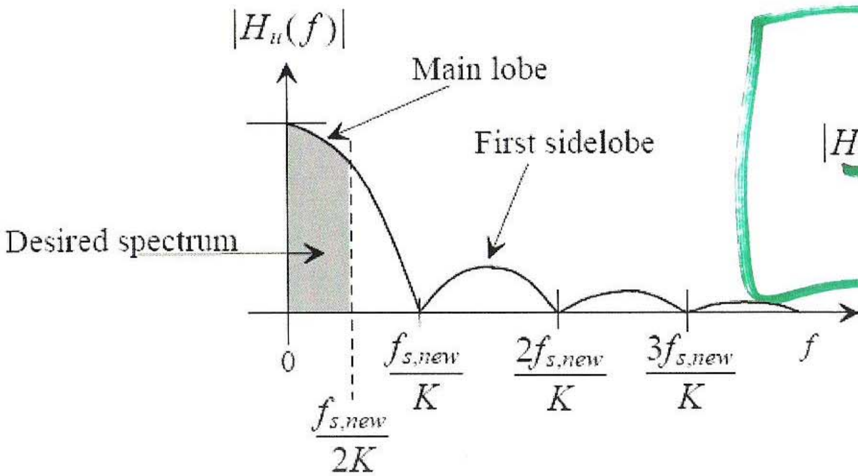


Figure 2.28 An interpolation block using a hold register, see spectrums in Fig. 2.32.



$$\boxed{|H_u(f)| = \frac{Sinc\left(K\pi \cdot \frac{f}{f_{s,new}}\right)}{Sinc\left(\pi \cdot \frac{f}{f_{s,new}}\right)}}$$

$$f_{s,new} = K \cdot f_s$$

Figure 2.29 Frequency response interpolated data sees using a hold register.

4)

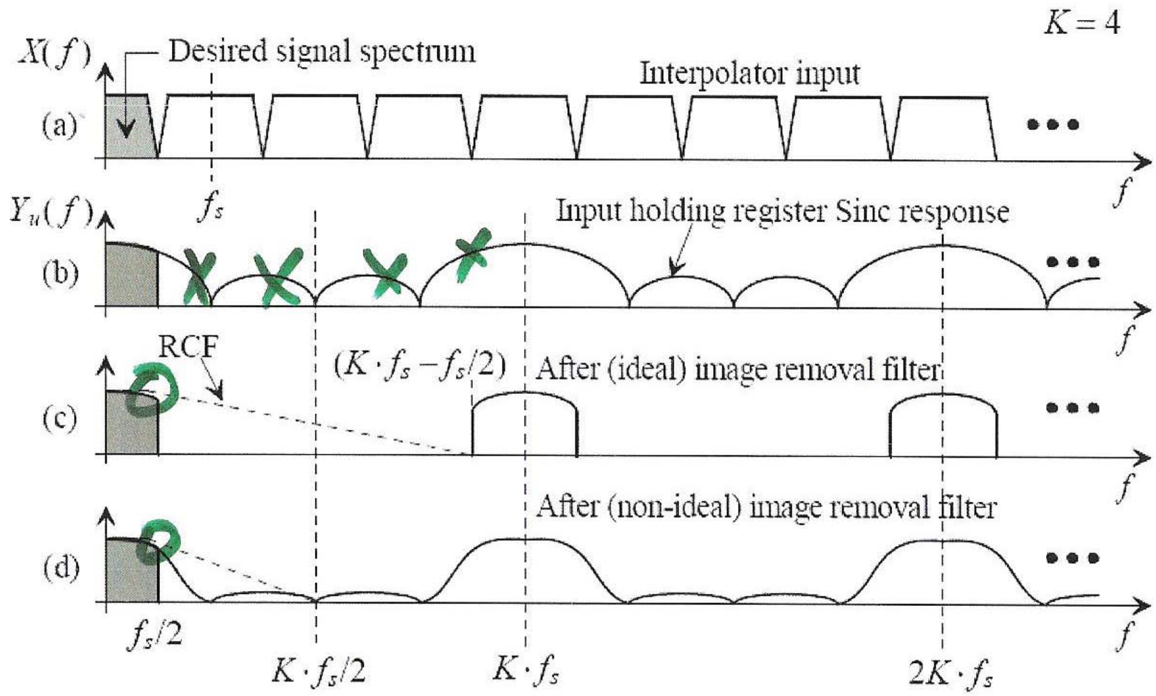


Figure 2.32 Example spectrums when interpolation using a hold register is employed.

5)

$$K \cdot (N+1) - 1$$

$$y_u\left(i \cdot \frac{T_s}{K}\right) = \sum_{i = K \cdot N} \cdot \frac{1}{K} x\left[i \cdot \frac{T_s}{K}\right] = x[NT_s]$$

$N = 0$, $i = 0 \rightarrow K - 1$

$N = 1$, $i = K \rightarrow 2K - 1$

$N = -1$, $i = -K \rightarrow -1$

$$x[0], \; x\left[\frac{T_s}{K}\right], \; x\left[\frac{2T_s}{K}\right], \; x\left[\frac{3T_s}{K}\right]$$

$$x\left[(K-1)\frac{T_s}{K}\right]$$

$$K \cdot N \le i \le K(N+1) - 1$$

6)

$$K \cdot y_a(NTs) = x\left[KN \cdot \frac{Ts}{K}\right] + x\left[(KN+1) \cdot \frac{Ts}{K}\right] + \cdots + x\left[(K(N+1)-1)\frac{Ts}{K}\right]$$

$$z^N \cdot K \cdot y_a(z) = z^N X(z)\left(1 + z^{1/K} + z^{2/K} + z^{\left(\frac{K-1}{K}\right)}\right)$$

$$\frac{y_a(z)}{X(z)} = \frac{1}{K}\left(1 + z^{1/K} + z^{2/K} + \cdots z^{\left(\frac{K-1}{K}\right)}\right)$$

$$= \frac{1}{K} \cdot \frac{1-z}{1-z^{1/K}}$$

7)

$$H_u(k) = \frac{1}{K} \cdot \frac{\operatorname{Sinc}\left(\pi \frac{K \cdot f}{f_{S\,new}}\right)}{\operatorname{Sinc}\left(\pi \cdot \frac{f}{f_{S,new}}\right)}$$

Linear Interpolation

$$y_u\left((i+1)\frac{T_S}{K}\right) = y_u\left(i \cdot \frac{T_S}{K}\right) + \frac{X[(N+1)T_S] - X[NT_S]}{K}$$

$$\frac{Y_u(z)}{X(z)} = \frac{1}{K} \cdot \frac{1-z}{1-z^{1/K}}$$

8)

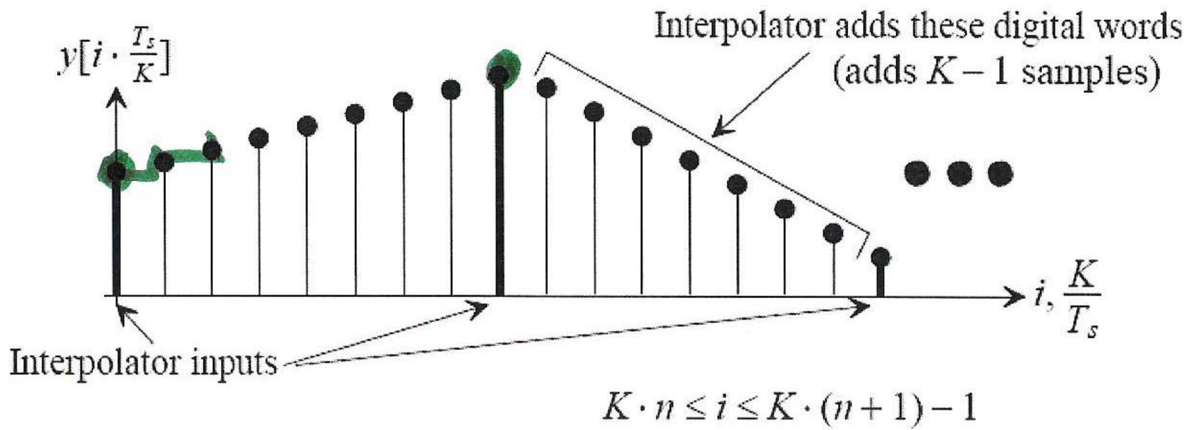


Figure 2.33 Using linear interpolation.

Liner Int.

$$|H_n(f)| = \frac{\text{Sinc}\left(\pi \frac{K \cdot f}{f_{s\,new}}\right)}{\text{Sinc}\left(\pi \frac{f}{f_{s\,new}}\right)}$$

9)

K-Path

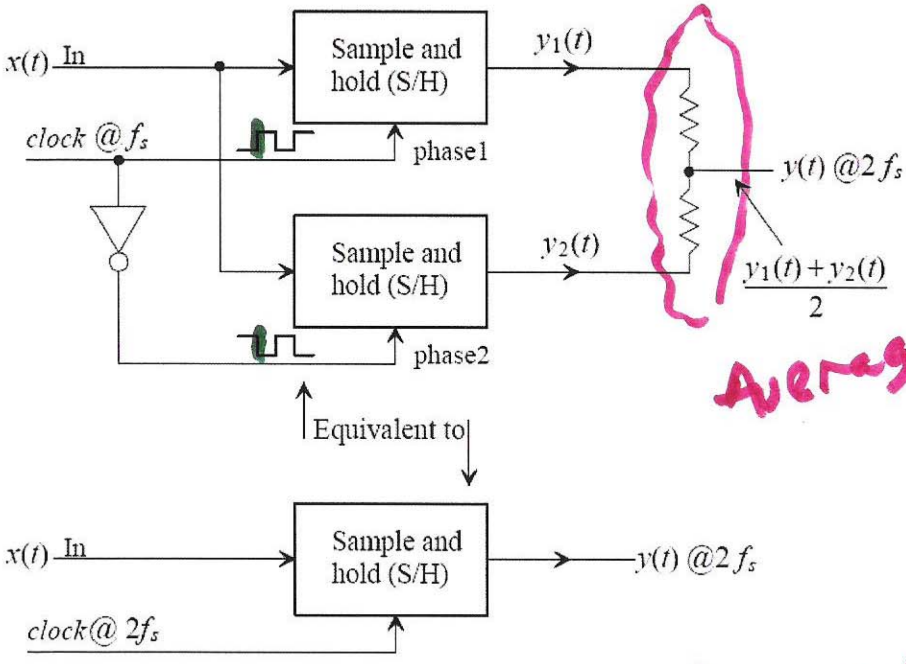
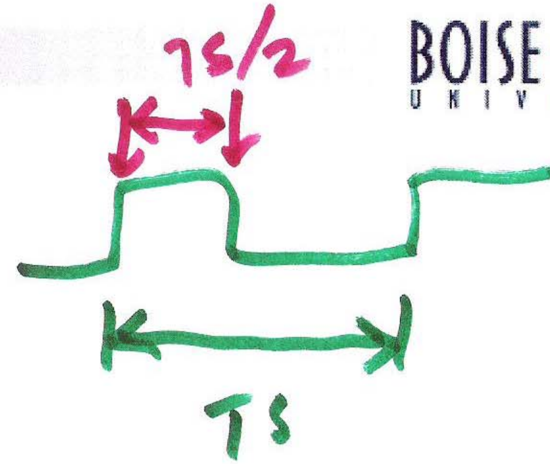


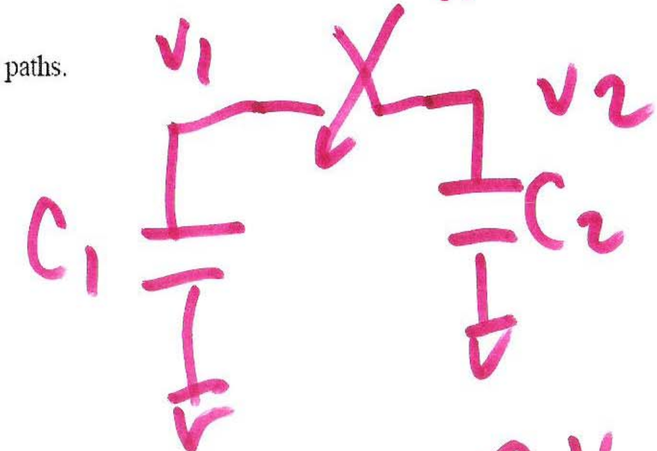
Figure 2.34 Using two S/H paths.

Averaging



$$f_{s,new} = \frac{2}{T_s} = 2f_s$$

$V_f = ?$



$$C_1V_1 + C_2V_2 = V_f(C_1 + C_2)$$

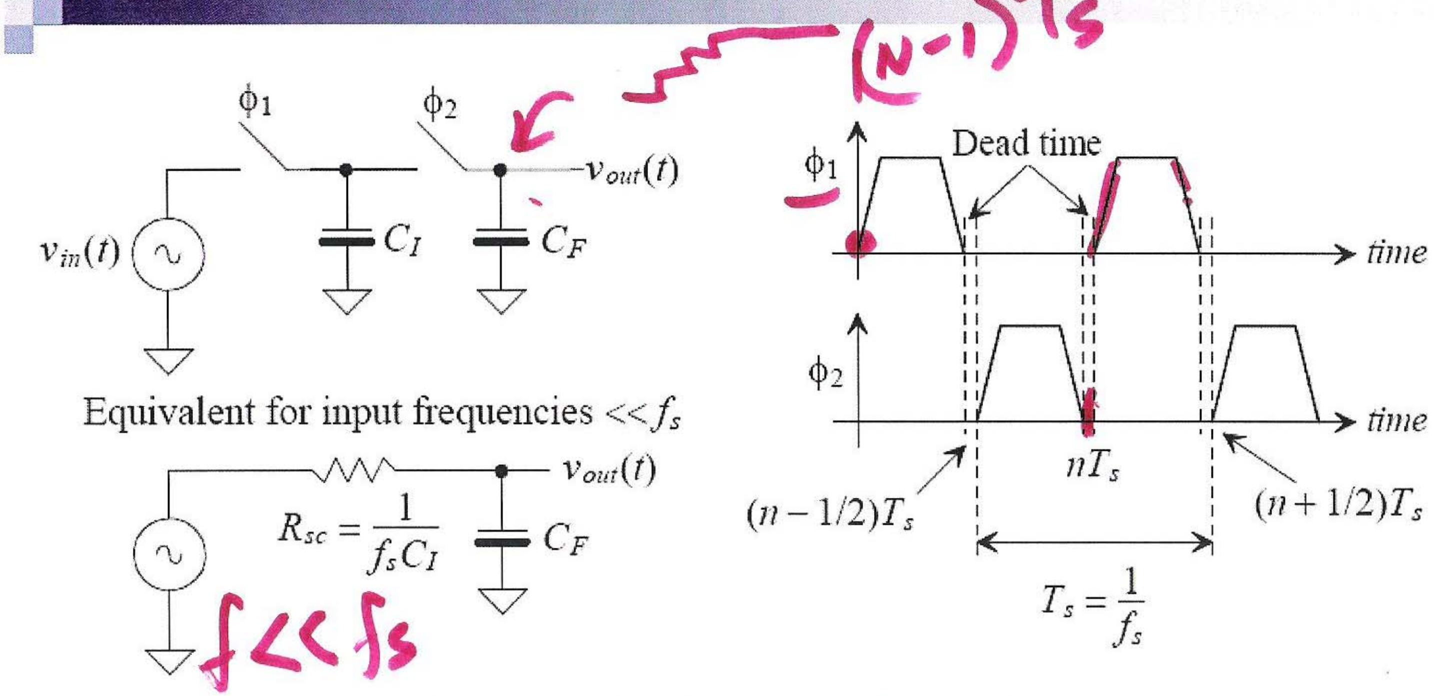


Figure 2.35 Switched-capacitor lowpass filter.

$$Q_1 = v_{IN}\left((N-\tfrac{1}{2})T_S\right)\cdot C_I$$

$$Q_{(N-1)} = v_{OUT}\left[(N-1)T_S\right]\cdot C_F$$

$$Q_2 = v_{OUT}\left[NT_S\right]\cdot(C_I + C_F) =$$

$$v_{IN}\left[(N-\tfrac{1}{2})T_S\right]C_I + C_F\cdot v_{OUT}\left[(N-1)T_S\right]$$

11)

$$V_{OUT}(z)\left(C_I + C_F\right) =$$

$$V_{IN}(z)\cdot z^{-1/2}\cdot C_I + V_{OUT}\, z^{-1}\cdot C_F$$

$$\frac{v_{OUT}}{v_{IN}} = \frac{C_I\cdot z^{-1/2}}{C_I + C_F - C_F\cdot z^{-1}}$$

$$z = e^{j2\pi\frac{f}{f_s}}$$

$f << f_s$

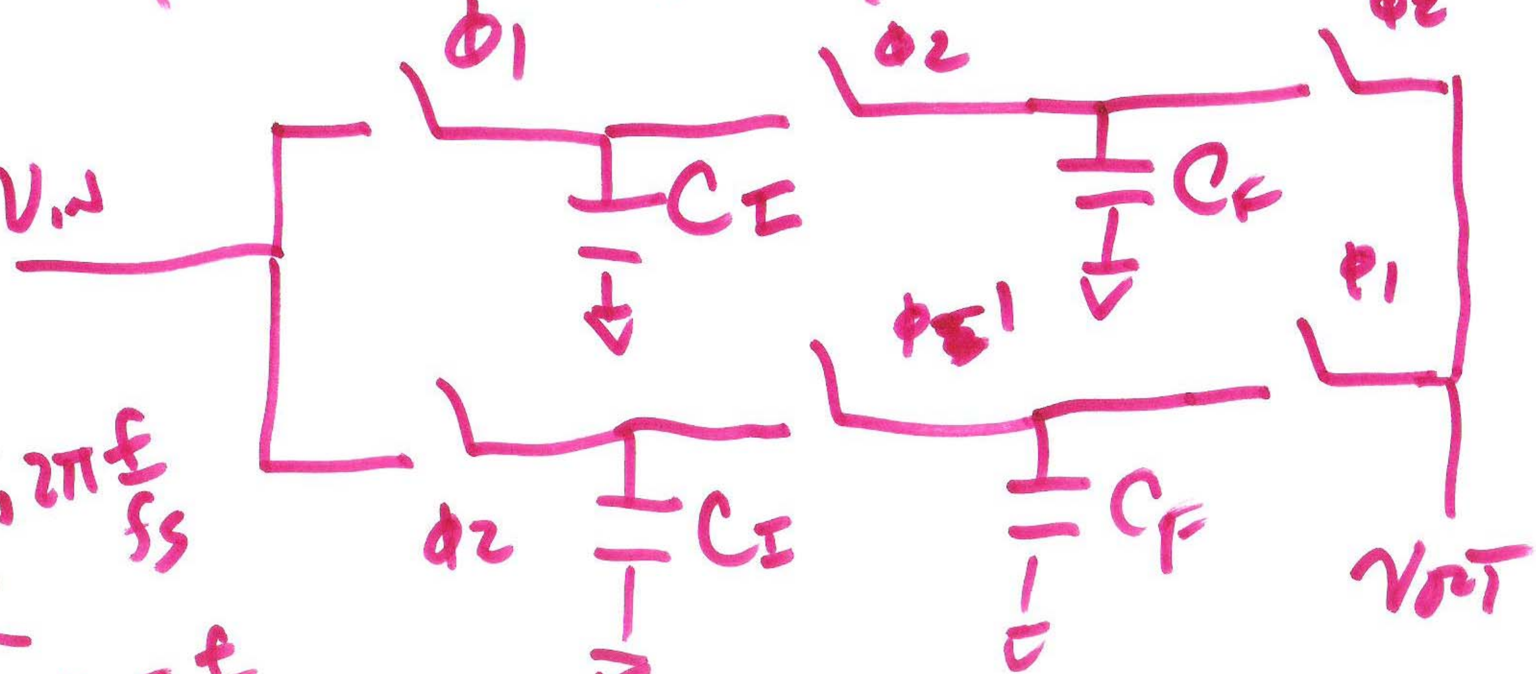
$$z = e^{j2\pi f/f_s}$$

$$\simeq 1 + j2\pi\frac{f}{f_s}$$

12)

$$R_{SC} = \frac{1}{f_s C_I}$$

$$\left|\frac{v_{out}}{v_{in}}\right| = \frac{1}{\sqrt{1 + (2\pi f R_{SC} \cdot C_F)^2}}$$

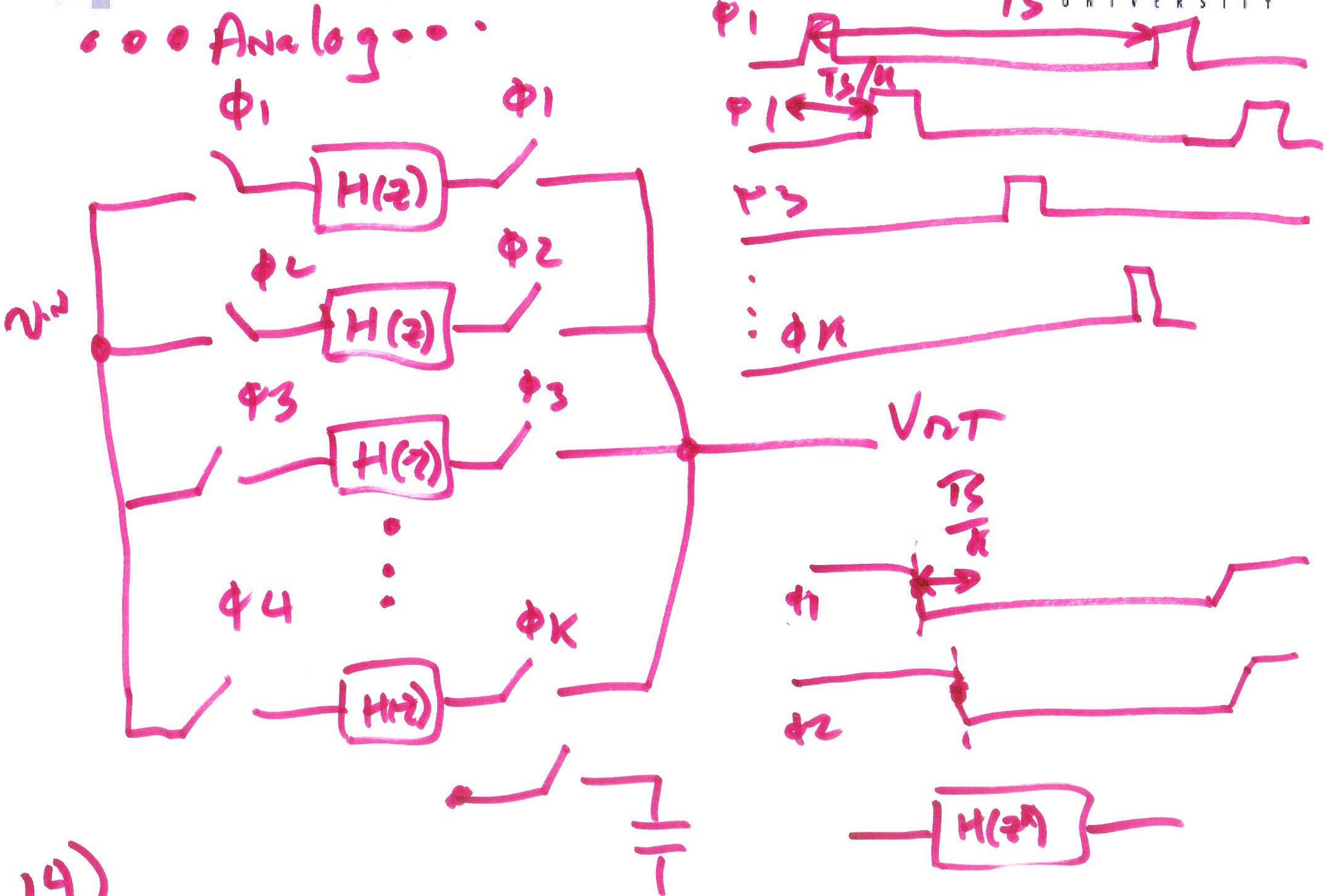


$$z = e^{j2\pi \frac{f}{f_s}}$$

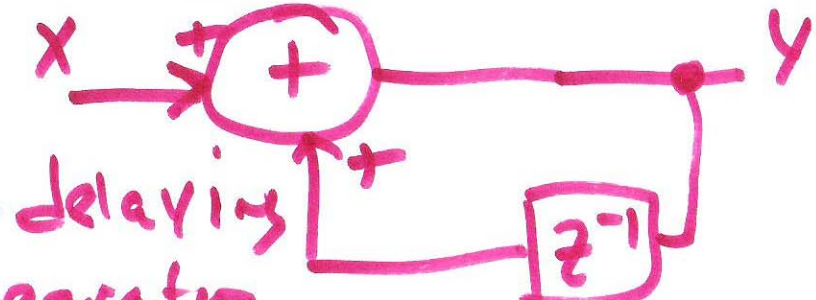
$$z^2 = \frac{e^{j2\pi \frac{f}{2f_s}}}{z} \quad z \to z^2$$

K - path $z \to z^K$

... Analog ...

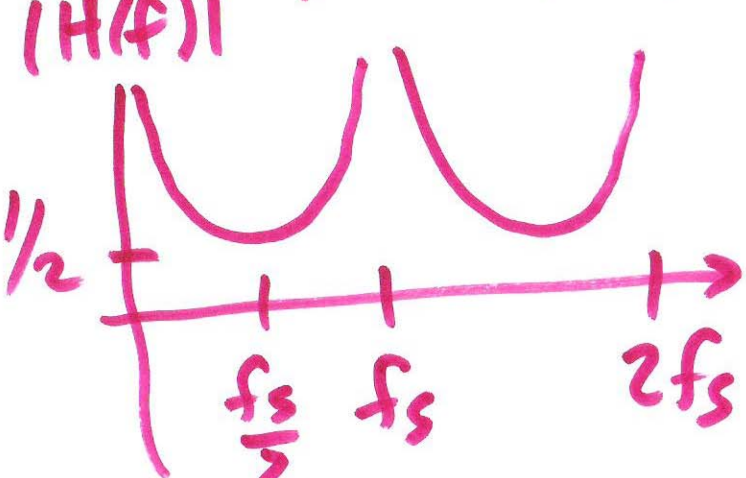


14)



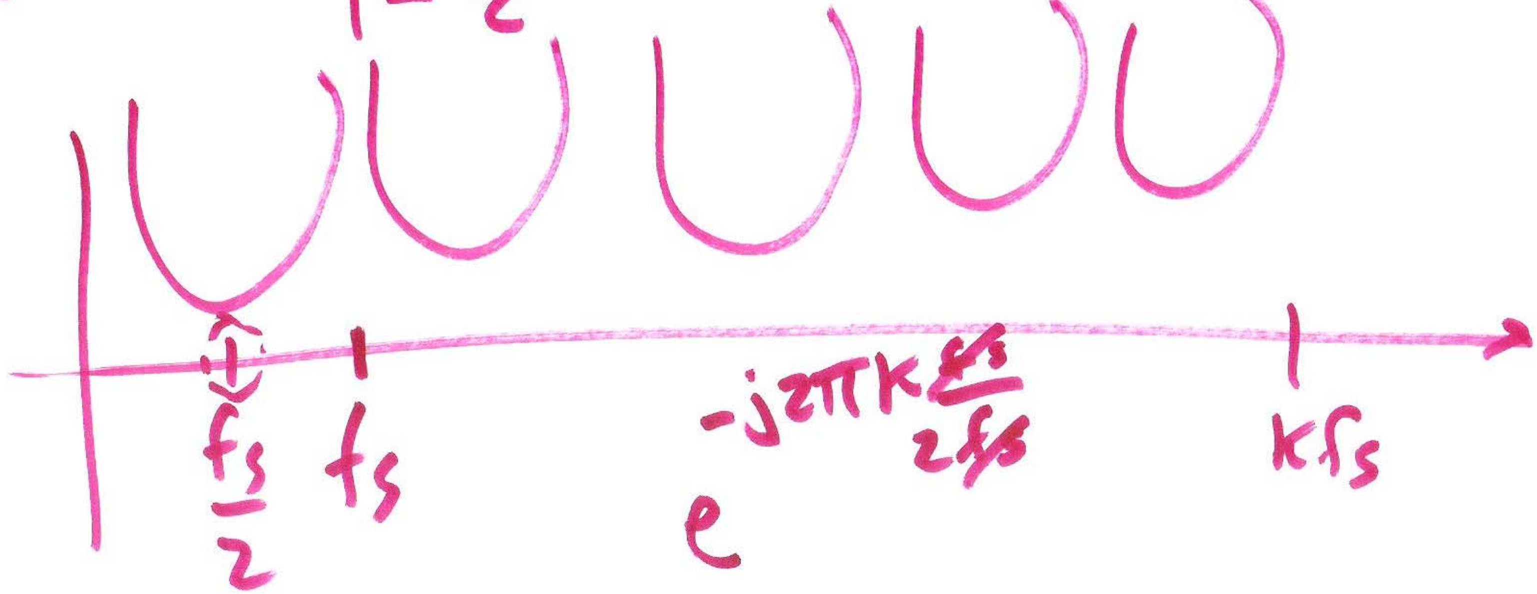
Non-delaying integrator

$$H(z) = \frac{Y}{X} = \frac{1}{1-z^{-1}}$$



$z \Rightarrow z^K$ K - in parallel

$$H(z^K) = \frac{1}{1-z^{-K}}$$



$$\frac{1}{1+z^{-1}} = H(z) \qquad \frac{1}{1+z^{-2}}$$

$$\frac{z}{z+1}$$

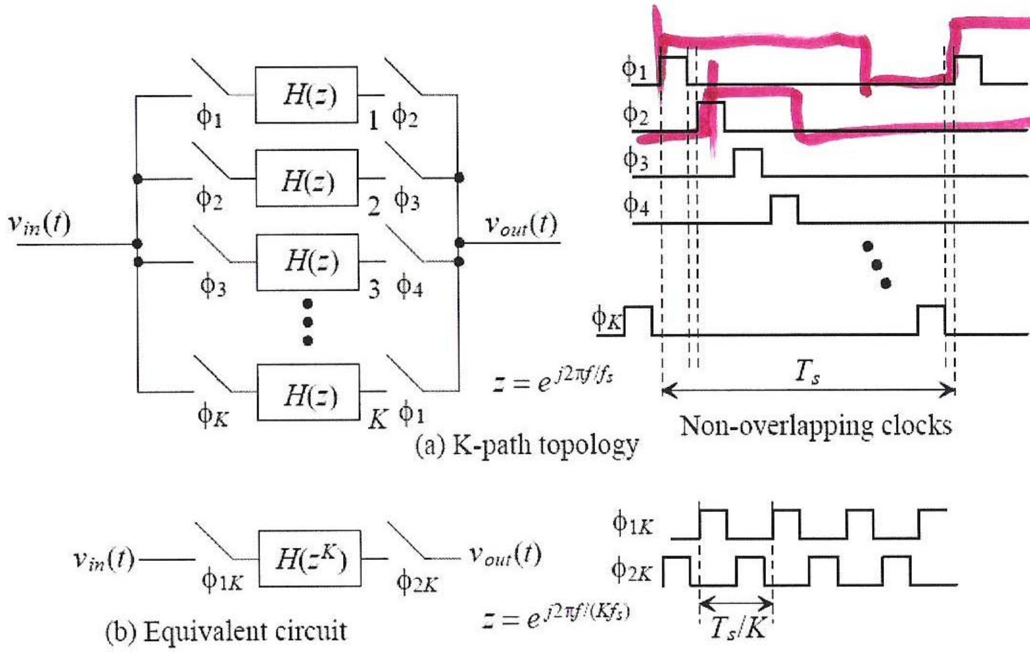
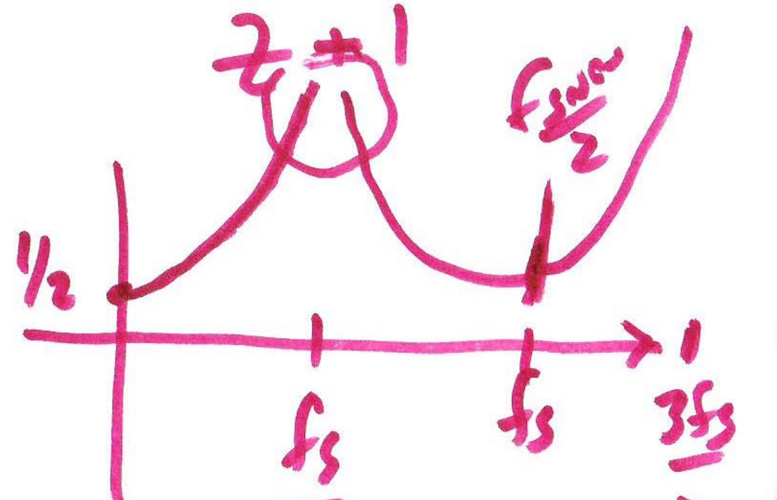
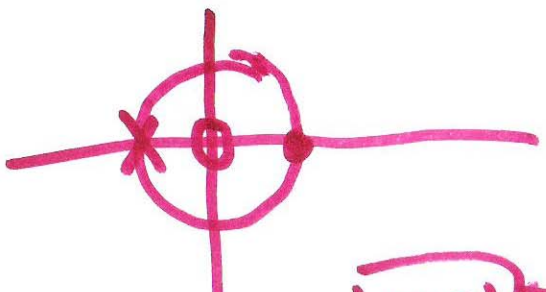
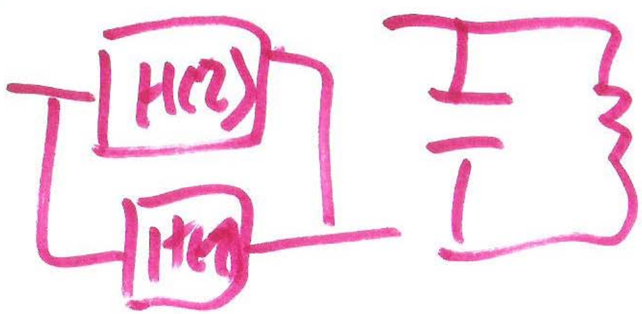


Figure 2.37 A K-path topology and its equivalent circuit.



$\frac{2f_s}{}$ $f_{s,new}$





1(a)