Lecture 3

Aug. 30, 2010

$$x[0] + x[1] + x[2] + x[3] + x[4] + x[5]$$

$$y[3] \qquad y[4] \qquad y[5]$$

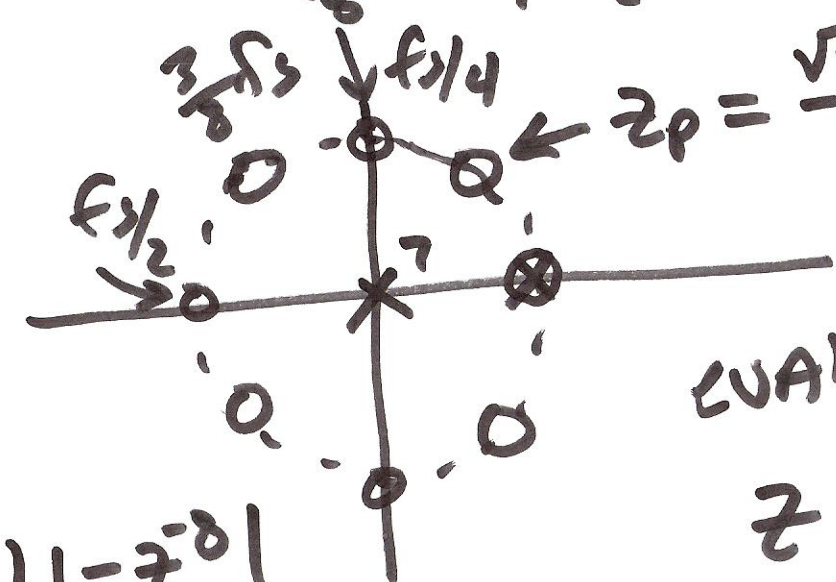
$$y[z] = x[z] + x[z]\cdot z^{-1} + x[z]z^{-2} + x[z]z^{-3}$$

$$H(z) = \frac{Y[z]}{X[z]} = \left(1 + z^{-1} + z^{-2} + z^{-3}\right)\frac{(1-z^{-1})}{1-z^{-1}}$$

$$H(z) = \frac{1-z^{-4}}{1-z^{-1}} = \frac{1 - z^{-1} + z^{-1} - z^{-2} + z^{-2} - z^{-3} \cdots}{1-z^{-1}}$$

$K = 8$

$$H(z) = \frac{1 - z^{-8}}{1 - z^{-1}} = \frac{z^8 - 1}{z^8 - z^7} = \frac{z^8 - 1}{z^7(z-1)}$$



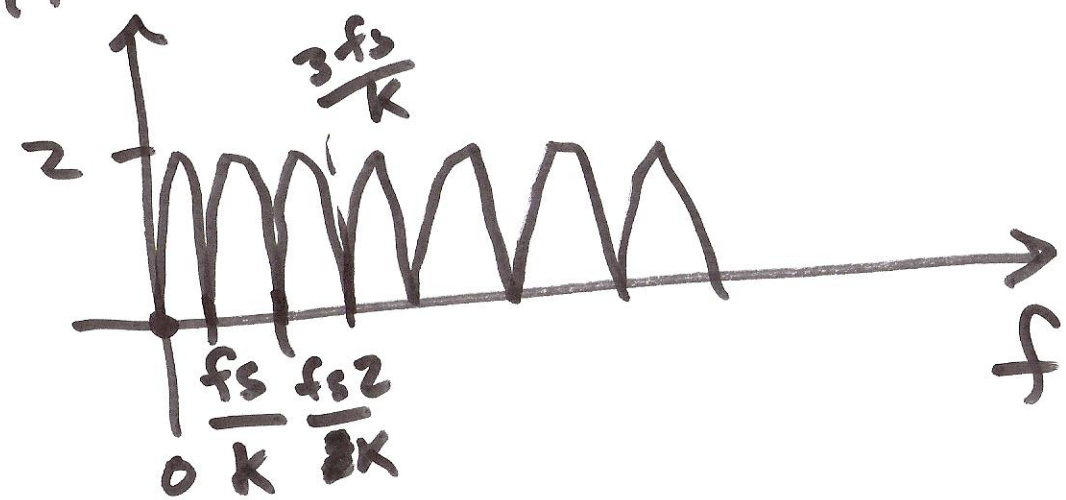
$$z_p = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$$

$$\left(z - \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right)$$

$$(z-1)$$
$$(z+1)$$
$$(z-j)$$
$$(z+j)$$

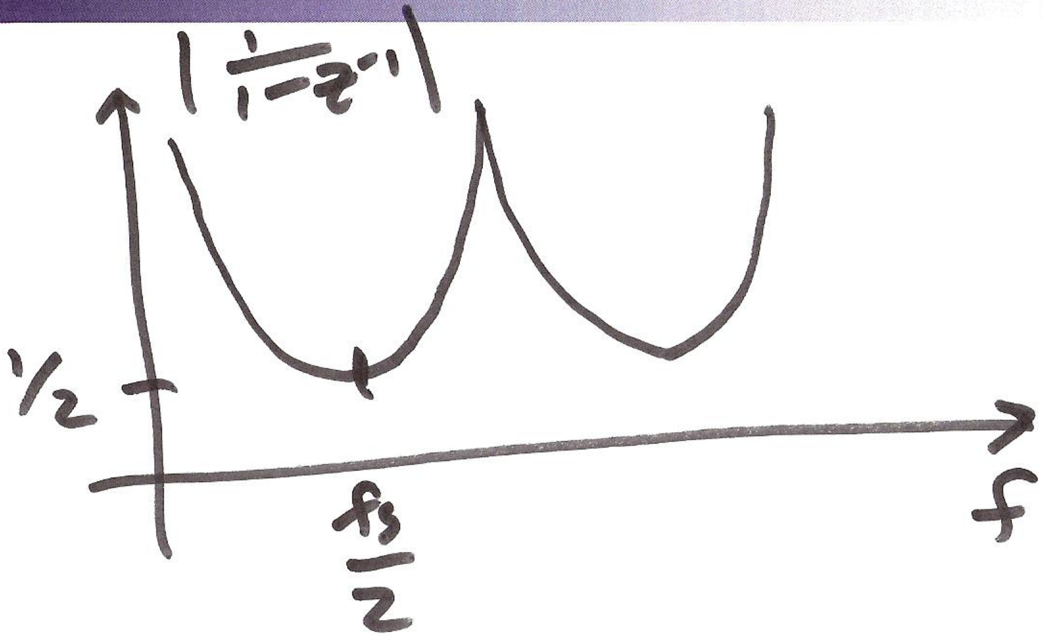
Evaluate $z = e^{j2\pi \frac{f}{f_s}}$

$|1 - z^{-8}|$

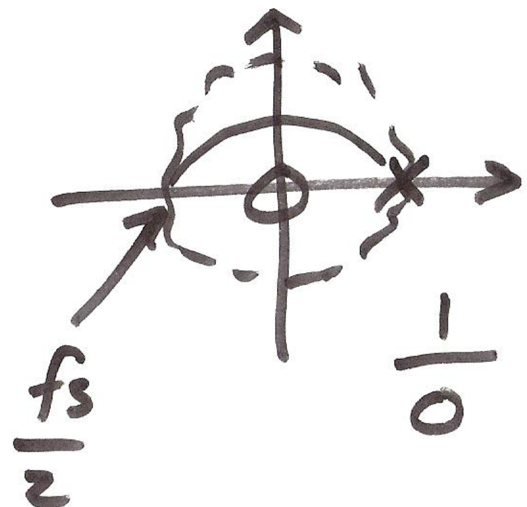


$$\frac{1}{1 - z^{-1}}$$

2)



$$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

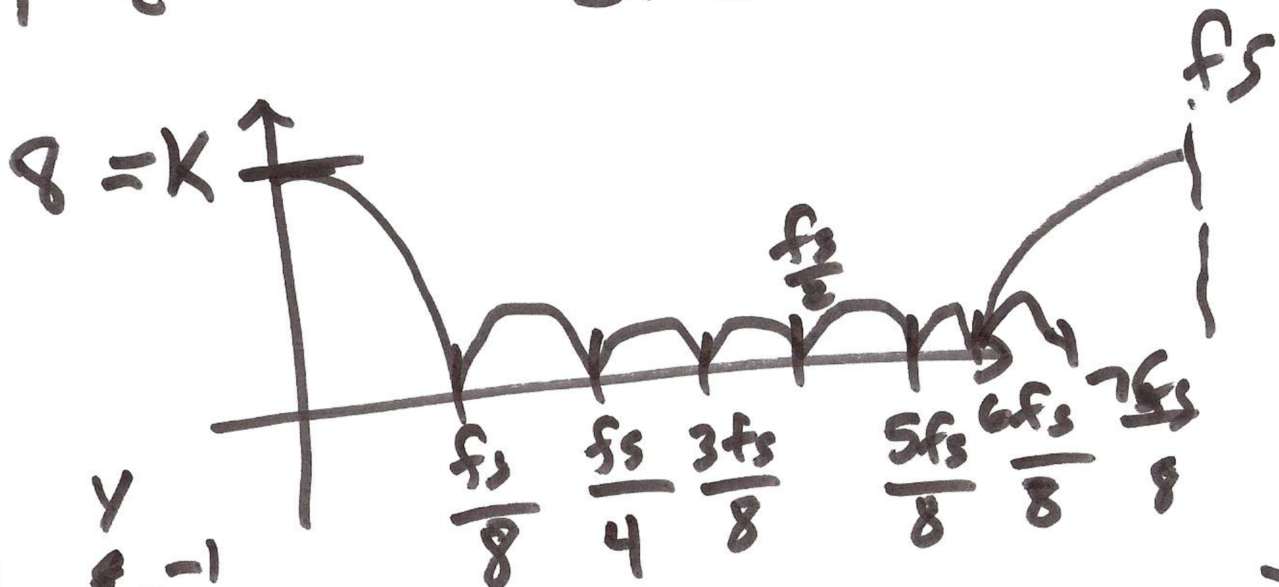


$$|\;| = \frac{1}{2}$$

3)

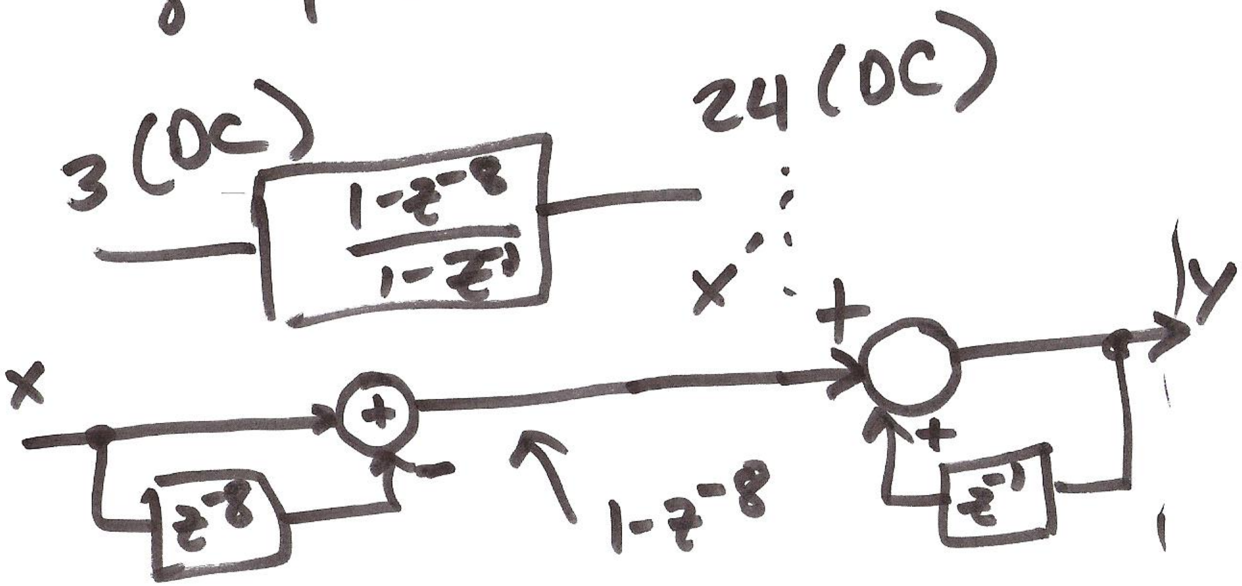
$$\frac{z^8 - 1}{z^7(z-1)} = \frac{1 - z^{-8}}{1 - z^{-1}} \cdot z^{-1}$$ moving Average filter

Sinc filter



$$y = x' + \overset{y}{\cancel{x}}\, z^{-1}$$

$$\frac{y}{x} = \frac{1}{1 - z^{-1}}$$

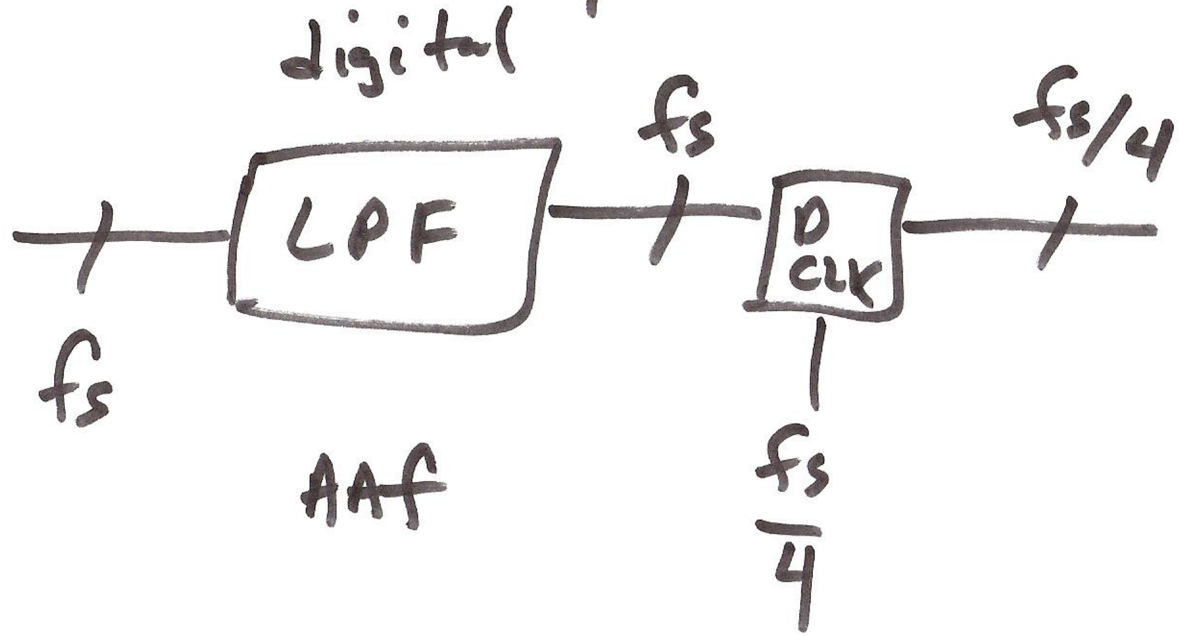


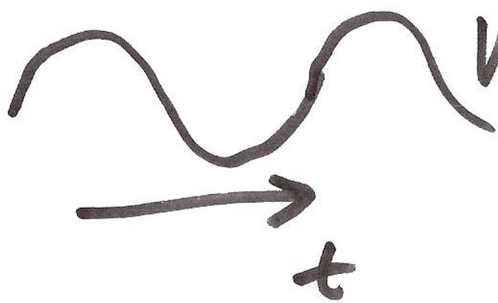
4)

Decimation

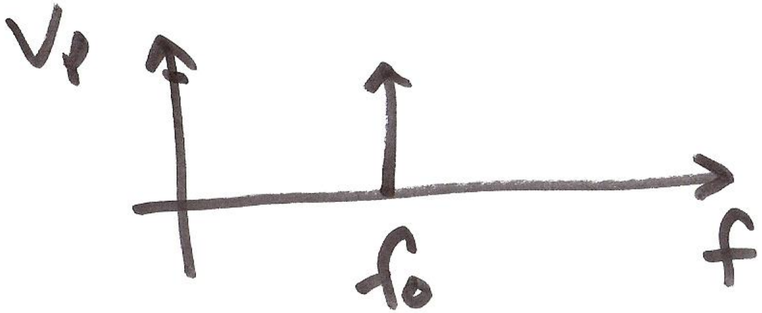
Reducing sample RATE

f_s $\qquad$ $\frac{f_s}{4}$





$V_p \cos(2\pi f_0 t)$



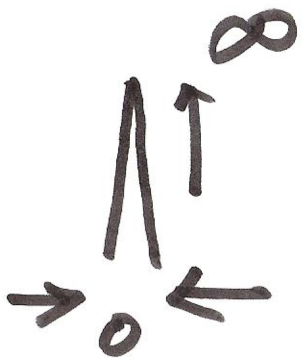
Fourier transform: $G(f) = \int_{-\infty}^{\infty} g(t) e^{-j2\pi ft} \cdot dt$

inverse Fourier transform:

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{+j2\pi ft} \cdot df$$

6)

Dirac delta function



$$\delta(t-t_0) = \infty \quad \text{when } t = t_0$$
$$= 0 \quad \text{when } t \neq t_0$$

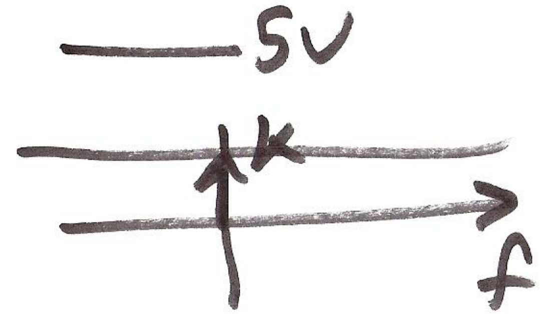
$$\int_{-\infty}^{\infty} \delta(t-t_0) \cdot dt = 1$$

Kronecker delta function

$$\delta(t-NT_s) = 1 \quad t = NT_s$$
$$0 \quad t \neq NT_s$$

$$\int_{-\infty}^{\infty} f(t)\cdot\delta(t-t_0)\,dt = f(t_0)$$

$$\int_{-\infty}^{\infty} K\cdot e^{-j2\pi ft}\cdot dt =$$



$$= K\cdot\delta(f)$$

$$\delta(100\,\text{MHz}) = 0$$

$$\delta(0) = \infty,$$

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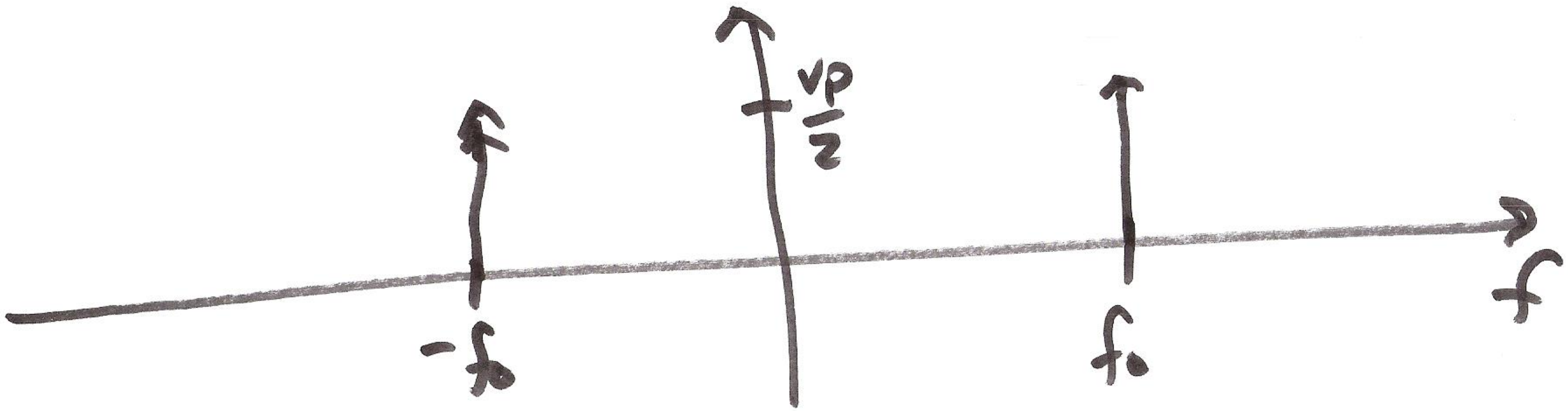
$$\int_{-\infty}^{\infty}\left[V_p \cos 2\pi f_0 \cdot t\right] \cdot e^{-j2\pi f t} \cdot dt$$

$$\cos 2\pi f_0 \cdot t = \frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}$$

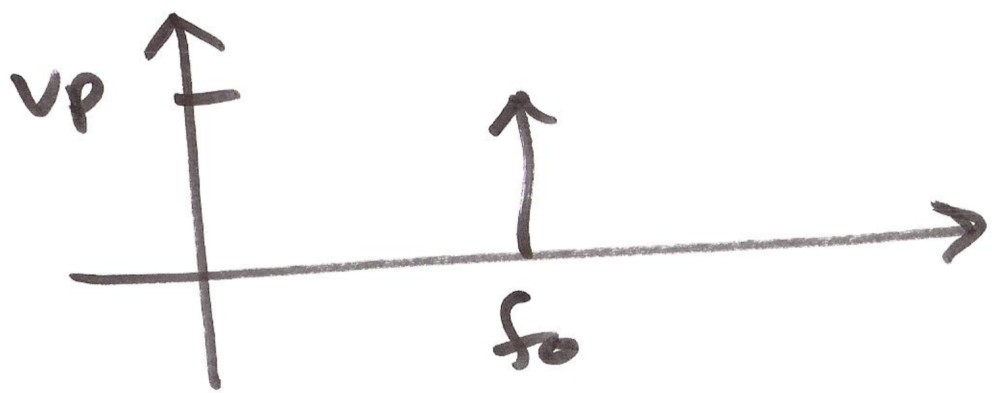
$$\frac{V_p}{2}\int_{-\infty}^{\infty}\left[e^{j2\pi t(f_0 - f)} + e^{-j2\pi t(f_0 + f)}\right]dt$$

$$\mathcal{F}\{\cos(\ )\} = \frac{V_p}{2}\left[\delta[f_0 - f] + \delta[f_0 + f]\right]$$

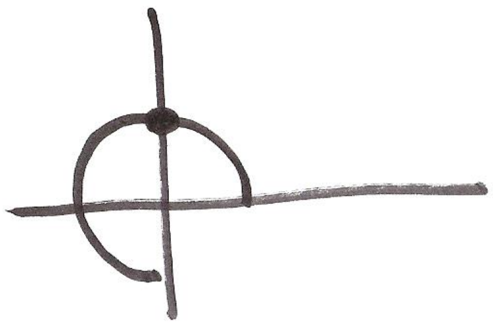
9)



$$\cos(2\pi(-f_0)t)$$
$$= \cos(2\pi f_0 \cdot t)$$



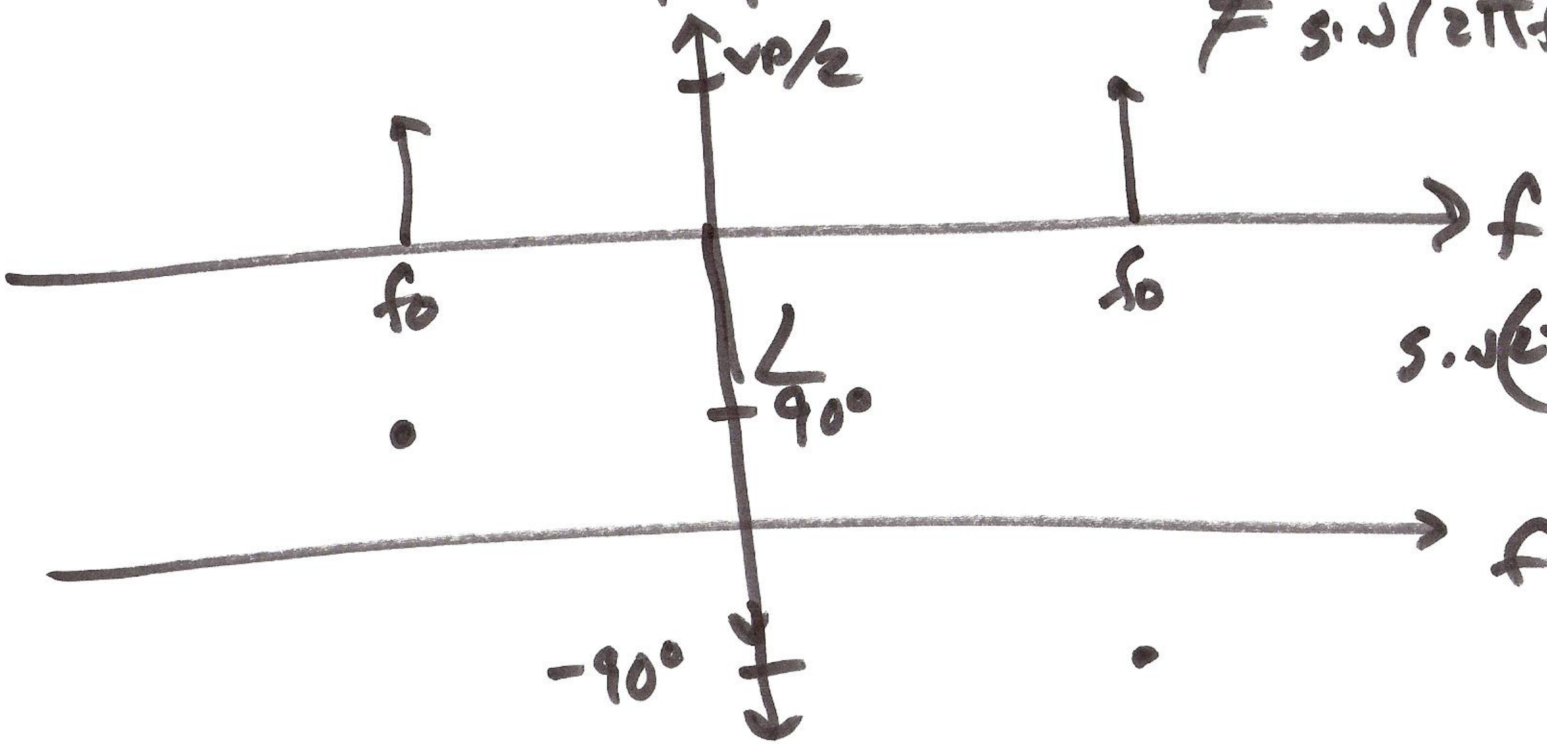
$$\mathcal{F}\{\sin(2\pi f_0 t)\} = \frac{V_P}{2j}\left[\delta(f_0 - f) - \delta(f_0 + f)\right]$$



$$= \frac{V_P}{2} \cdot j \left[\delta(f_0 + f) - \delta(f_0 - f)\right]$$

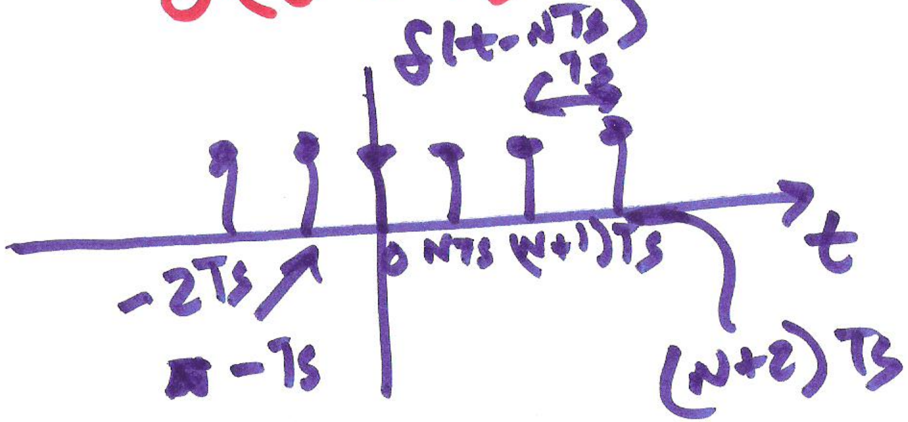
$\sin(2\pi(-f)t)$

$\neq \sin(2\pi f t)$



$\sin(2\pi f t + \pi)$

$$y(t) = V_p \sin 2\pi f_{in} \cdot t \cdot \delta(t - NT_s)$$



$$\sum_{N=-\infty}^{\infty} \int_{-\infty}^{\infty} V_p \sin 2\pi f_{in} t \cdot \delta(t - NT_s) e^{-j2\pi f t} \cdot dt$$

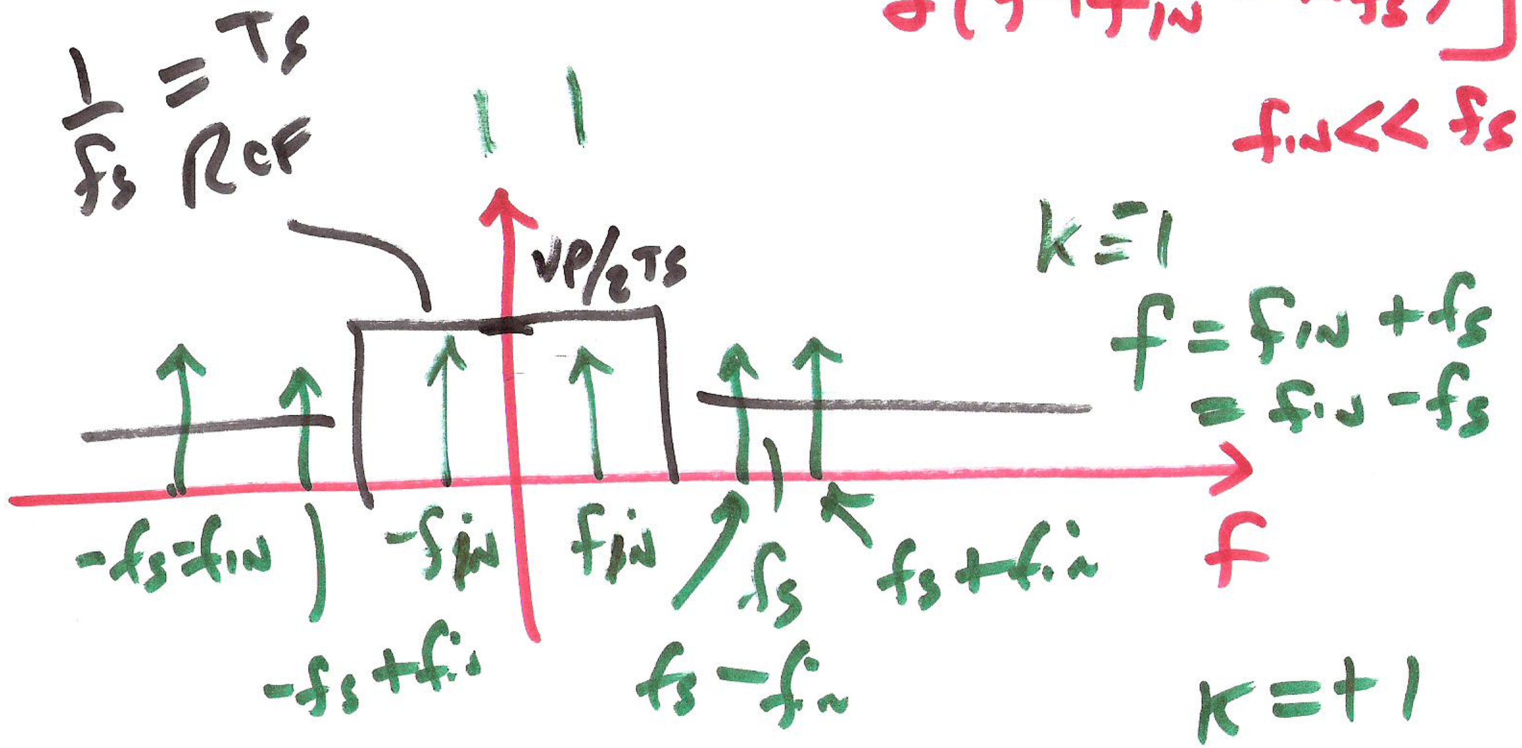
$$\sum_{N=-\infty}^{\infty} \delta(t - NT_s) = \frac{1}{T_s} \sum_{N=-\infty}^{\infty} e^{j2\pi N t / T_s}$$

$$= \frac{1}{T_s} \sum_{N=-\infty}^{\infty} e^{j2\pi N t \cdot f_s}$$

12)

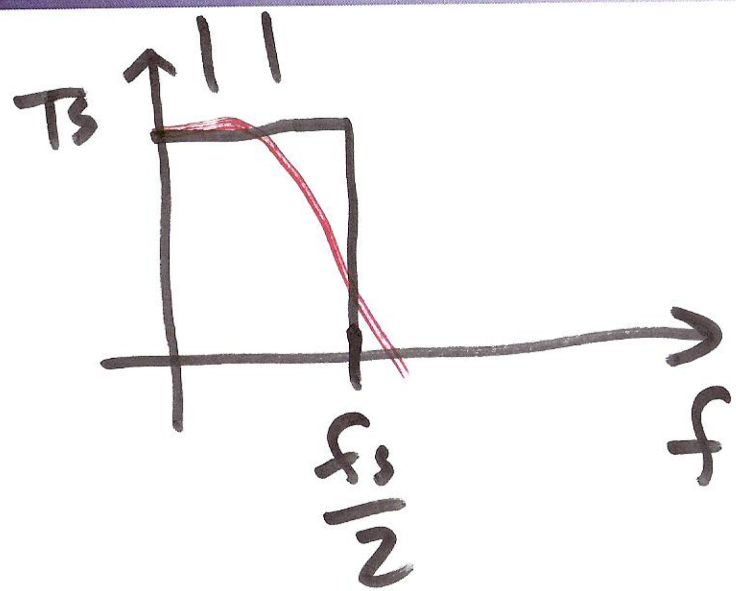
$$y(f) = \frac{V_P}{2jT_S} \sum_{k=-\infty}^{\infty} \left[\delta(f - f_{IN} - k f_S) - \delta(f + f_{IN} - k f_S) \right]$$

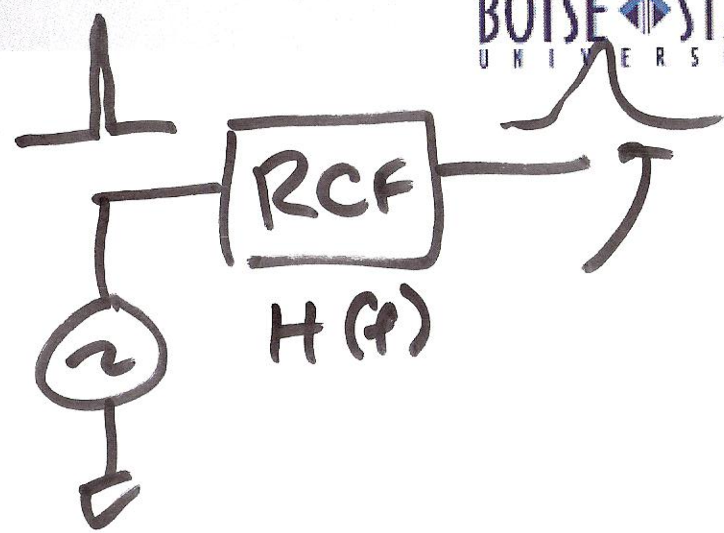
$f_{IN} << f_S$

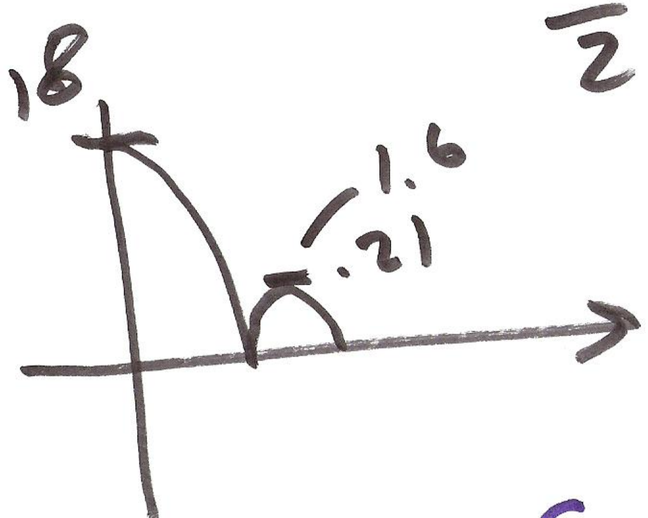


$$\frac{V_P}{2T_S}$$

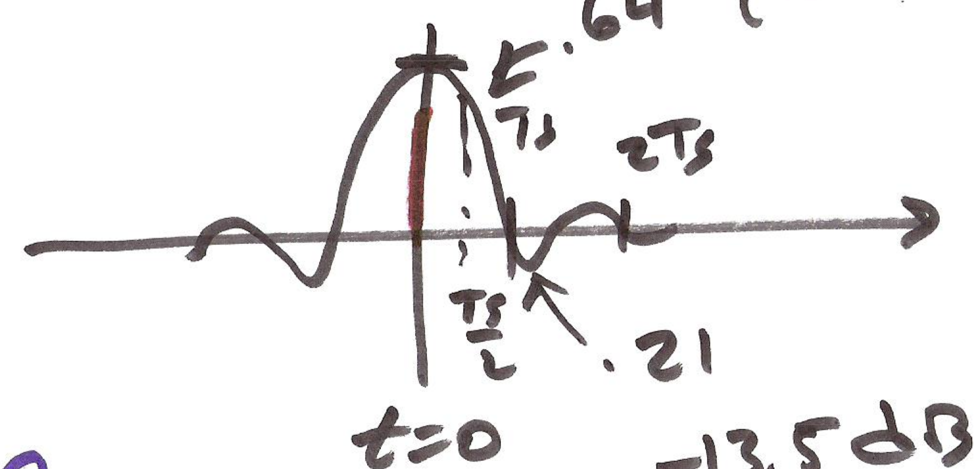
RCF



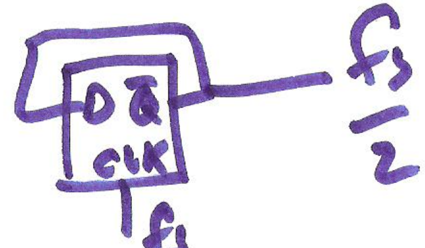




$$h(t) = \mathrm{Sinc}\ \pi f_s \cdot t$$



$f_s \rightarrow \frac{f_s}{2}$

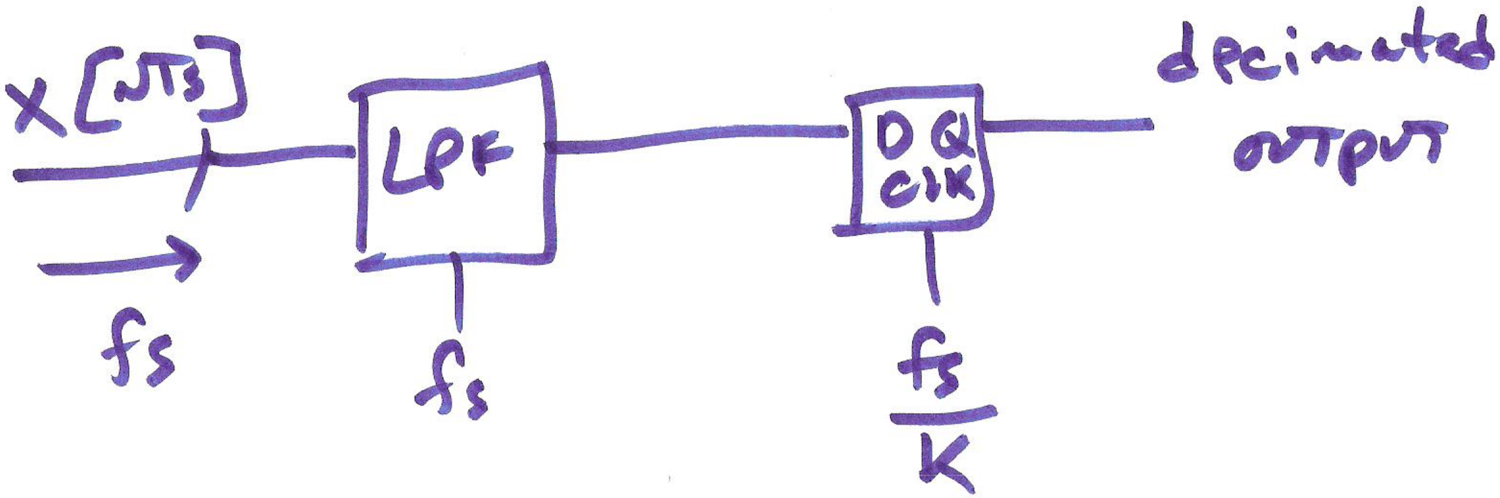


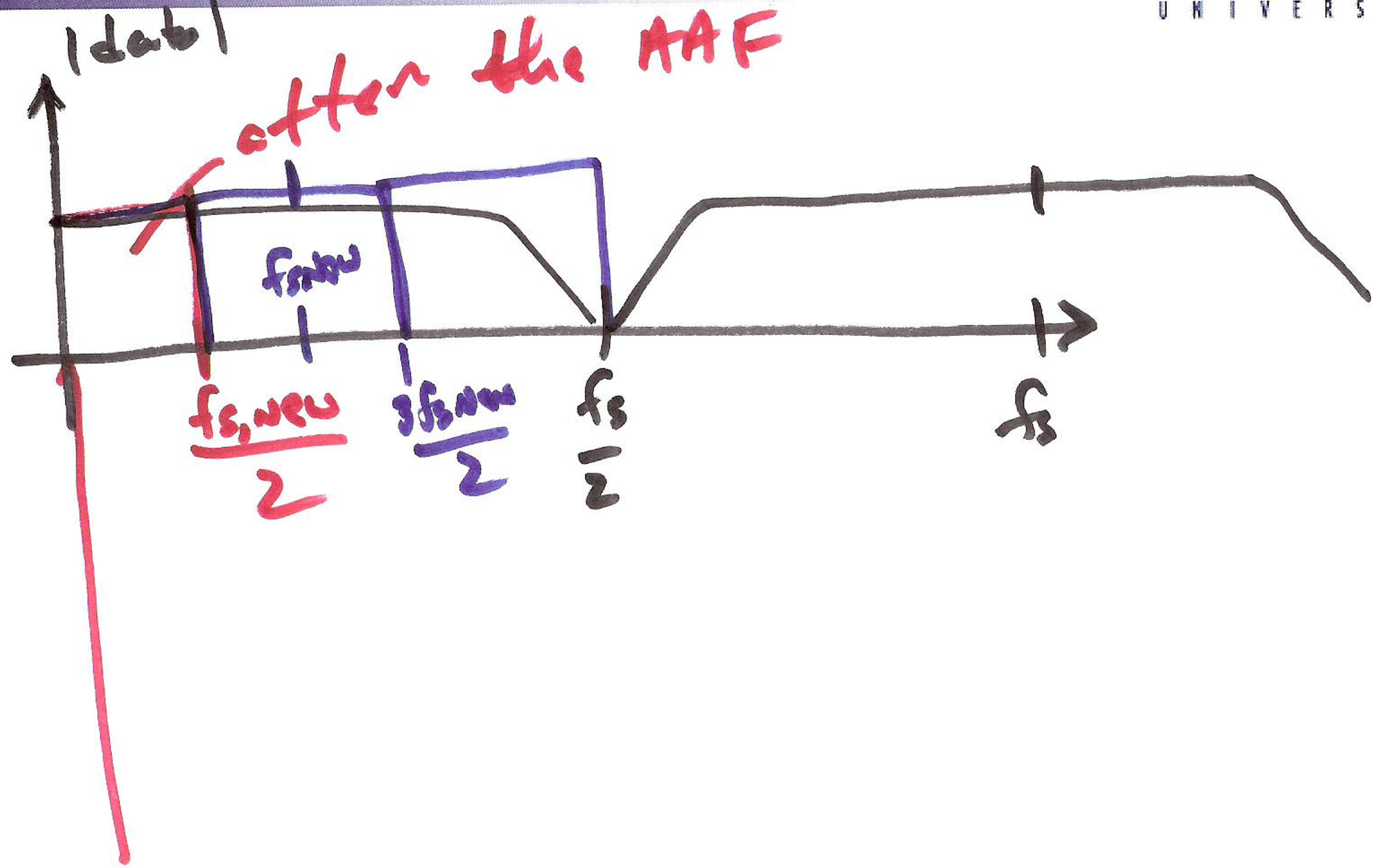
14)

Decimation

$$x[nT_s] \longrightarrow \boxed{\downarrow K} \longrightarrow y[ki \cdot T_s]$$

$$\underbrace{x_1 + x_2 + x_3 + x_4} + \underbrace{x_5 + x_6 + x_7 + x_8}$$




|data|
after the AAF
$f_{s,new}$
$\frac{f_{s,new}}{2}$
$\frac{3f_{s,new}}{2}$
$\frac{f_s}{2}$
f_s

16)