ECE 615 CMOS Mixed-Signal Circuit Design
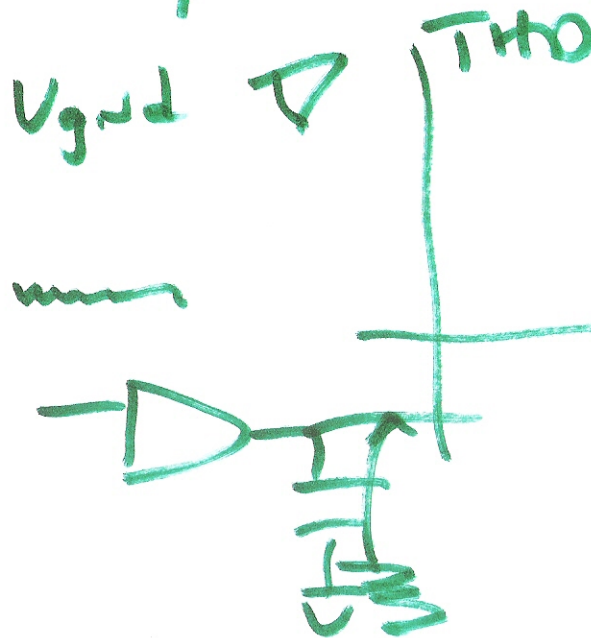
Lecture 13 OCT. 13, 2010

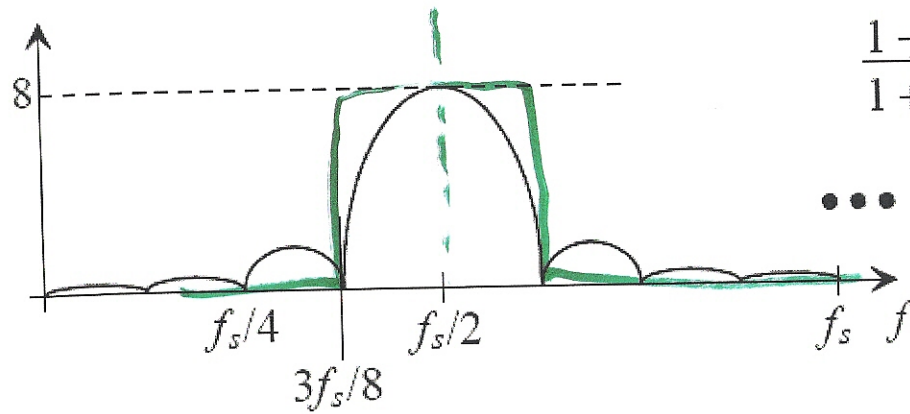$$THD = \frac{a_0^2}{a_1^2 + a_2^2 + \dots}$$

$$\frac{1}{4} = $$

$$\frac{1}{2} = $$

$$\frac{1}{12}$$

U_{gnd}
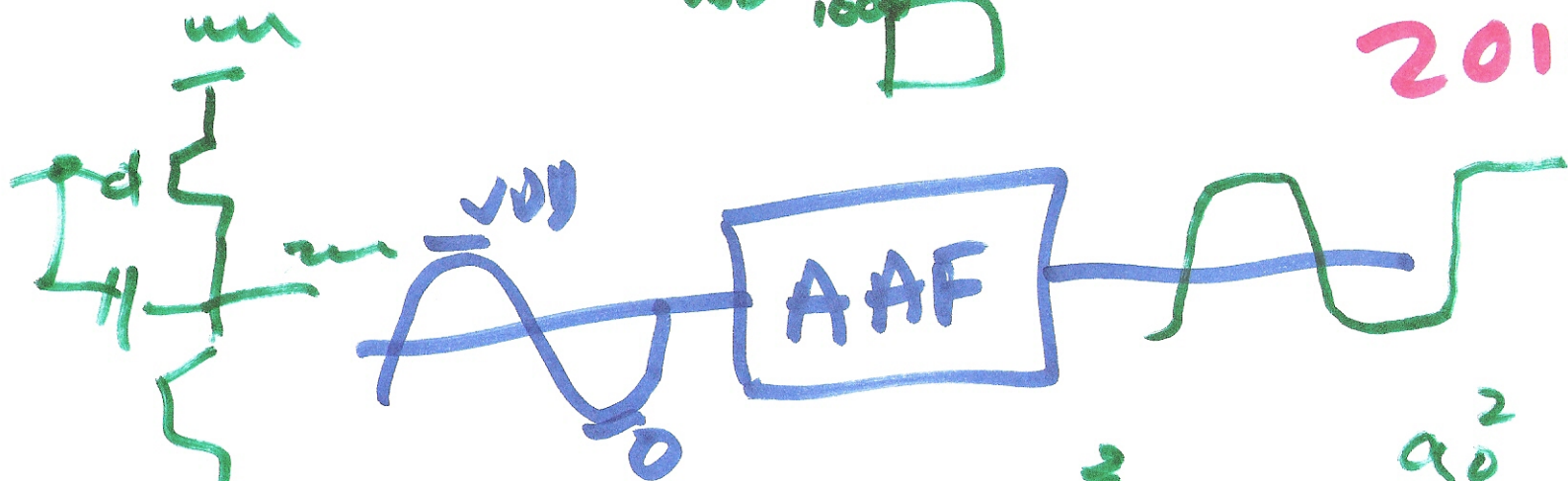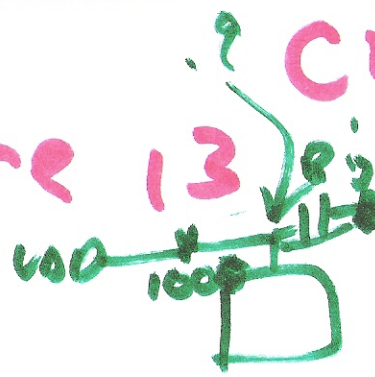
INPUT Amplitude

1)

.99532

$$\frac{1-z^{-8}}{1+z^{-1}} = \frac{z^8-1}{z^7(z+1)}$$

$f_s/2$

DC

$f_s/4$ $f_s/2$ f_s f

$3f_s/8$
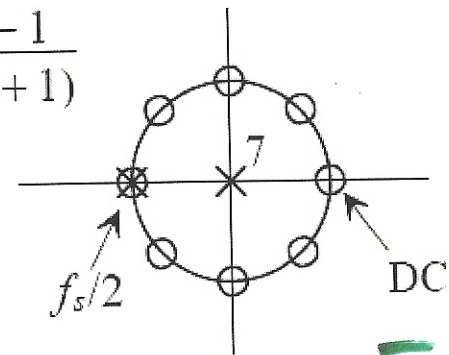
Integers!

Figure 4.19 A highpass filter implementation using a comb filter.

comb filter followed by a resonator

2)

$$K = 8$$

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z-plane

Comb filter	Digital resonator
$1 - z^{-K}$	$\dfrac{1}{1 - 2\cos\left[2\pi\dfrac{f}{f_s}\right]\cdot z^{-1} + z^{-2}}$

In ⟶ [Comb filter] ⟶ [Digital resonator] ⟶ Out

Figure 4.20 Implementing a sinc-shaped bandpass filter.

$$\frac{1}{1 + z^{-2}} = \frac{z^2}{z^2 + 1} = \frac{z^2}{(z+j)(z-j)}$$

$$\frac{1}{1 - z^{-1} + z^{-2}} = \frac{z^2}{z^2 + z + 1} \quad \text{change sign}$$

$$\frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{z^2}{\left(z + \frac{1}{2} - \frac{\sqrt{3}}{2}i\right)\left(z + \frac{1}{2} + \frac{\sqrt{3}}{2}i\right)}$$

$$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2}j \qquad K = 12$$
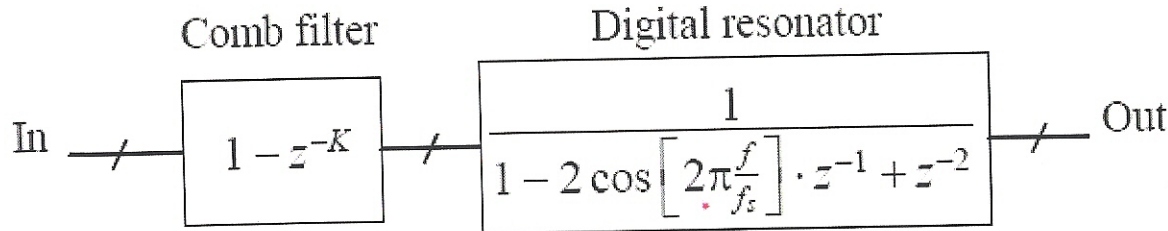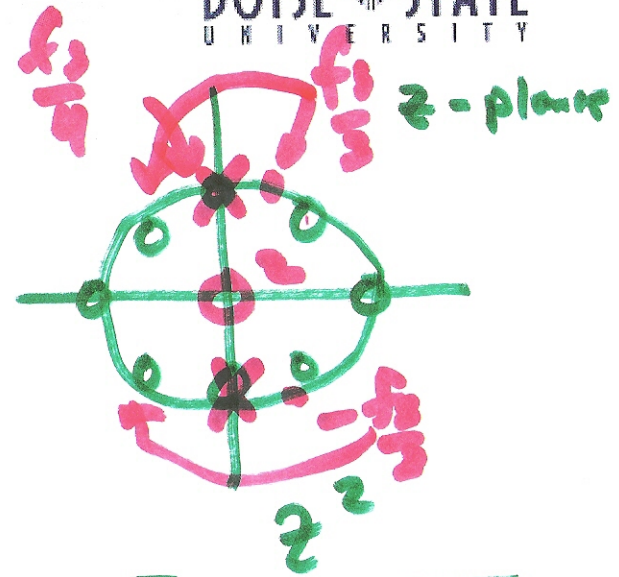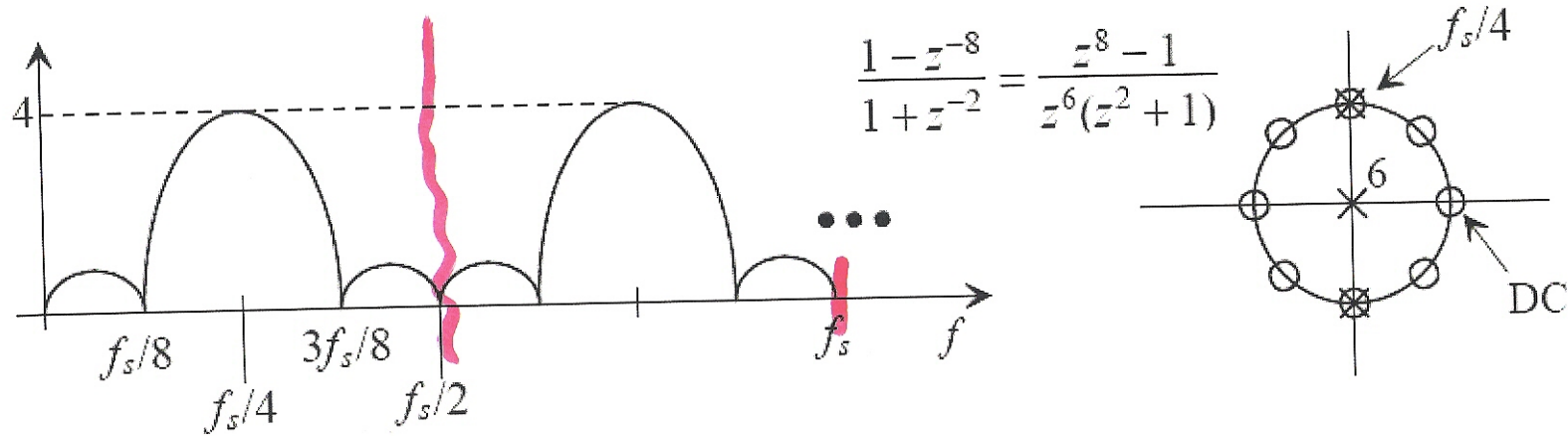
3)

$$\frac{1-z^{-8}}{1+z^{-2}} = \frac{z^8-1}{z^6(z^2+1)}$$

Figure 4.21 A bandpass filter implementation using a comb filter and digital resonator.

4)

$x[nT_s]$ → (+) → $y[nT_s]$

$\boxed{Q\ D\ clk}$ $\boxed{Q\ D\ clk}$

f_s

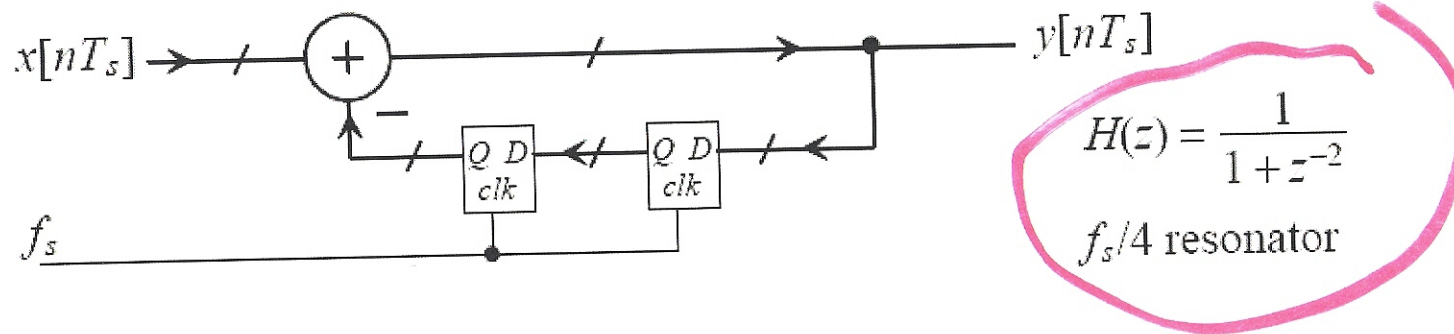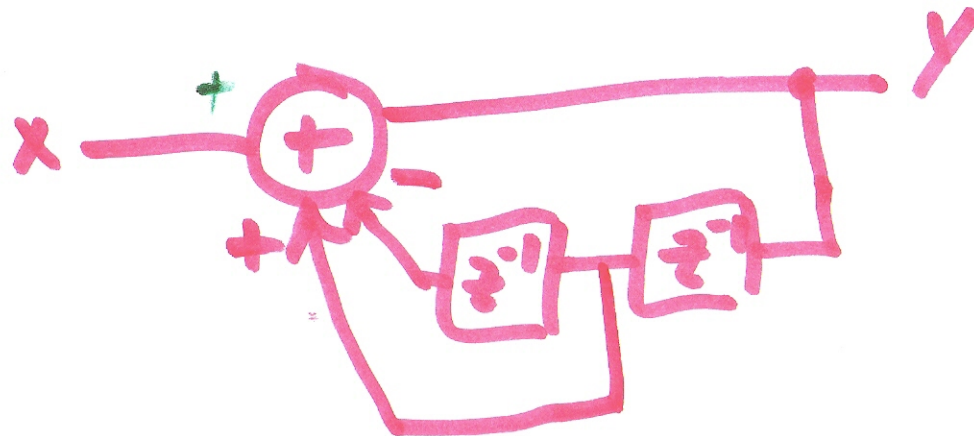$$H(z) = \frac{1}{1 + z^{-2}}$$

$f_s/4$ resonator

Figure 4.23 Implementation of a digital resonator.

$$\frac{1}{1 - z^{-1} + z^{-2}}$$

$$x + yz^{-1} - yz^{-2} = y$$

$$\frac{y}{x} = \frac{1}{1 - z^{-1} + z^{-2}}$$
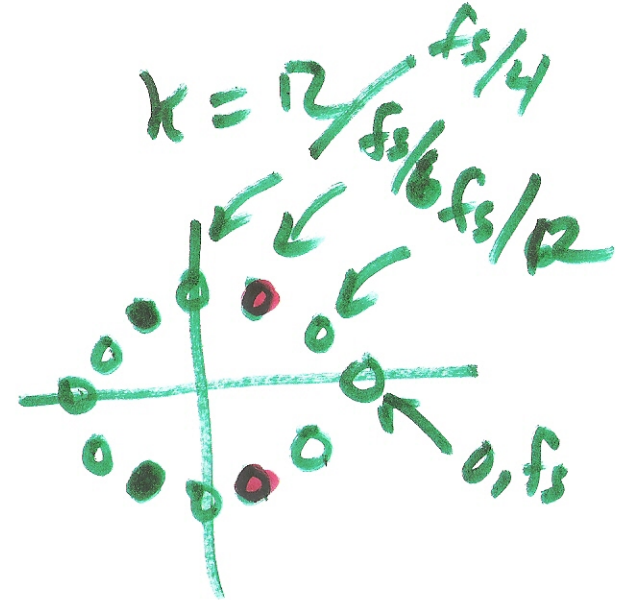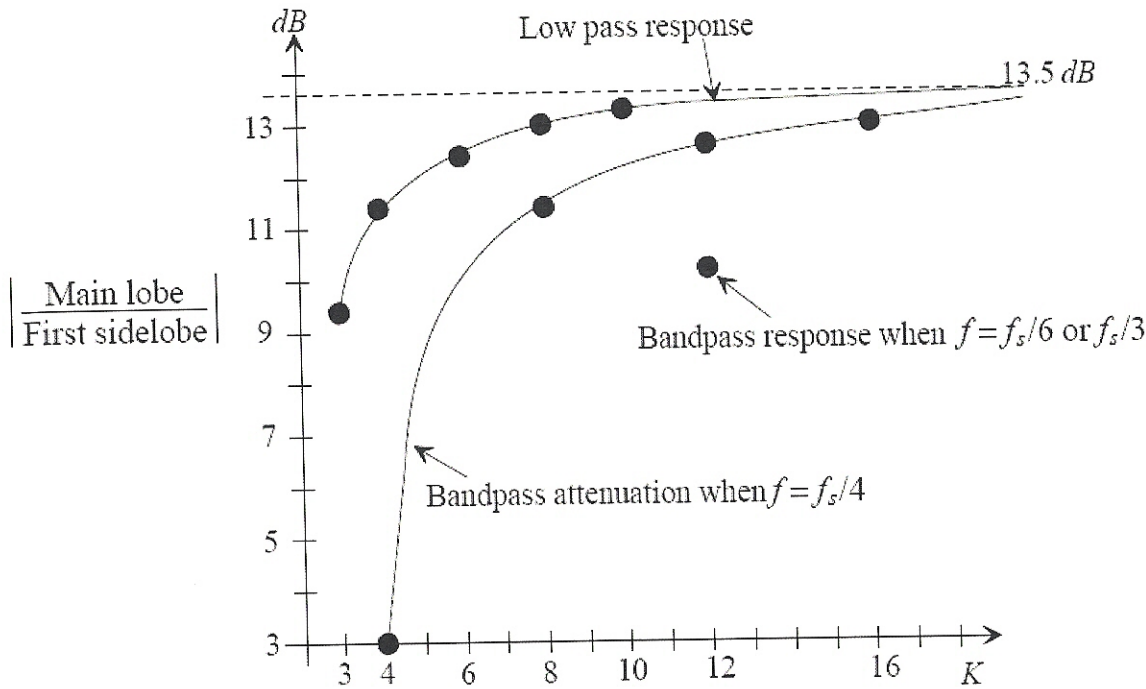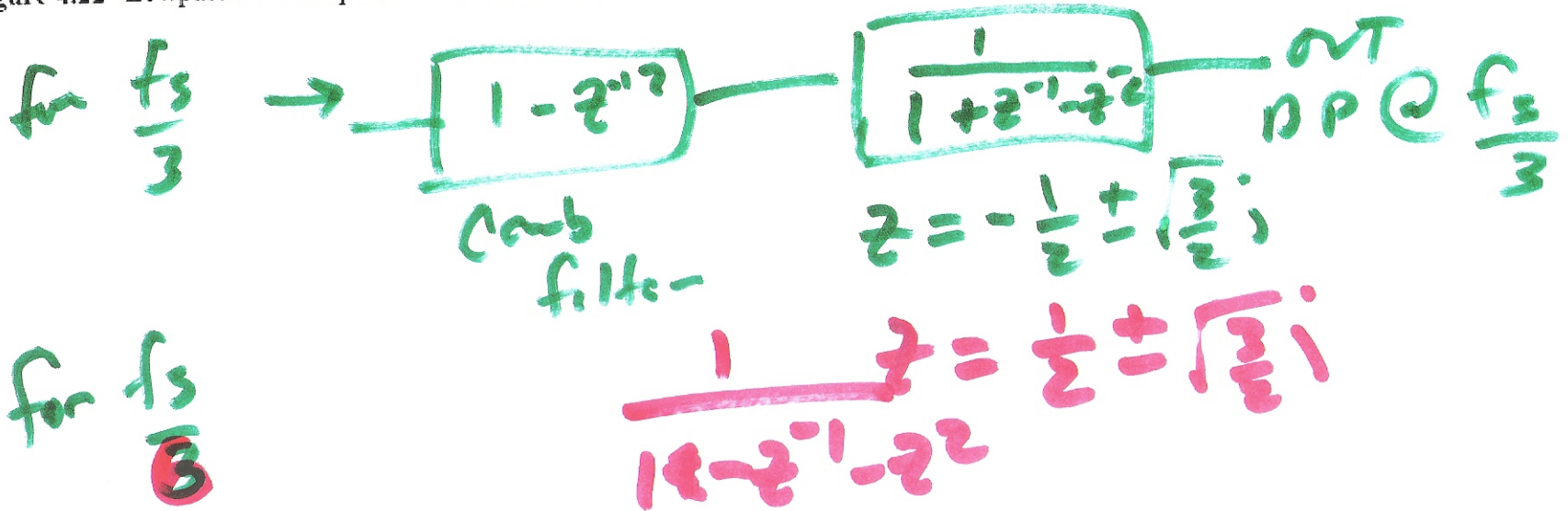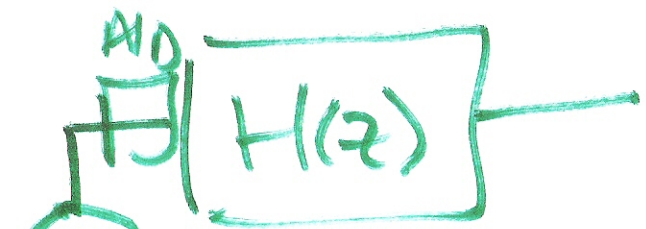
x → (+) → y

z^{-1} z^{-1}

5)

Figure 4.22 Lowpass and bandpass filter attenuation versus number of comb filter zeroes. K.

The figure shows axes labeled dB (vertical) and K (horizontal), with curves labeled:
- Low pass response
- 13.5 dB
- Bandpass response when $f = f_s/6$ or $f_s/3$
- Bandpass attenuation when $f = f_s/4$
- $\left|\dfrac{\text{Main lobe}}{\text{First sidelobe}}\right|$

Handwritten annotations (green):

$k = 12 \quad \begin{array}{l} f_s/4 \\ f_s/6, f_s/12 \end{array}$

$0, f_s$

$\text{for } \dfrac{f_s}{3} \rightarrow \boxed{1 - z^{-12}} - \boxed{\dfrac{1}{1 + z^{-1} - z^2}} \rightarrow n p @ \dfrac{f_s}{3}$

Comb filter

$z = -\dfrac{1}{2} \pm \sqrt{\dfrac{3}{2}} i$

Handwritten annotations (pink):

b)

$\text{for } \dfrac{f_s}{6}$

$\dfrac{1}{1 + z^{-1} - z^2} \qquad z = \dfrac{1}{2} \pm \sqrt{\dfrac{3}{2}} i$
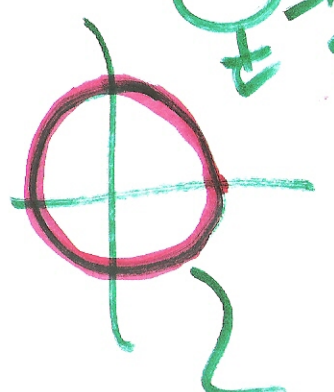
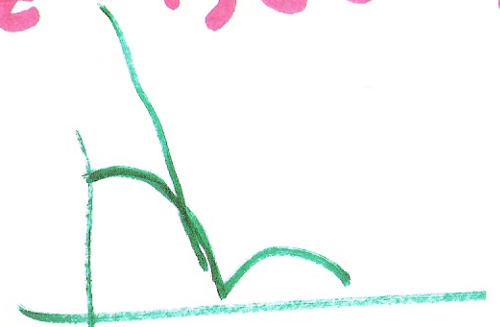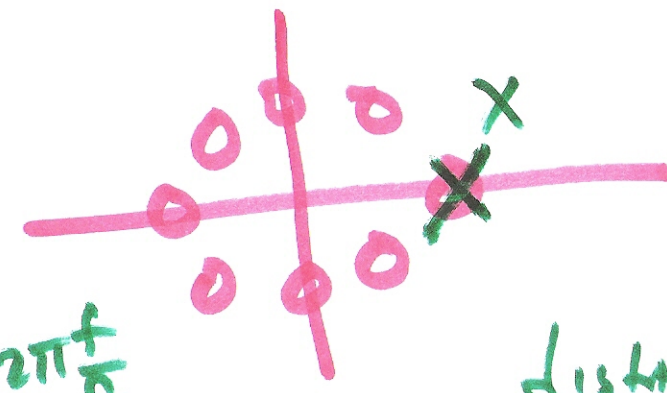$$\frac{1}{1 - 2z^{-1} + z^{-2}} = \frac{z^2}{z^2 - 2z + 1}$$

$$H(z) = \frac{z^2}{(z-1)(z-1)}$$

$$z = e^{j2\pi \frac{f}{f_s}}$$

$$\frac{\text{distance to zeroes}}{\text{distance to poles}}$$

$$\frac{1}{1 - \frac{1}{2}z^{-1} + z^{-2}} = \frac{z^2}{z^2 - \frac{1}{2}z + 1}$$

$$z = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 4}}{2}$$

$$= \frac{1}{4} \pm j \cdot \sqrt{\frac{1}{8} - 2}$$

$$= \frac{1}{4} \pm j \sqrt{\frac{15}{8}}$$

8)

$$\frac{1 - z^{-8}}{1 + z^{-2}}$$

Change into two's complement and extend sign bit by 1.

Extend the sign-bit

Change back to binary offset format.

In → 8-bit ADC →8 →7 →/2 →9 → 1 − z^{-8} →9 →9 →10 → $\frac{1}{1 + z^{-2}}$ → → 10-bit DAC → Out
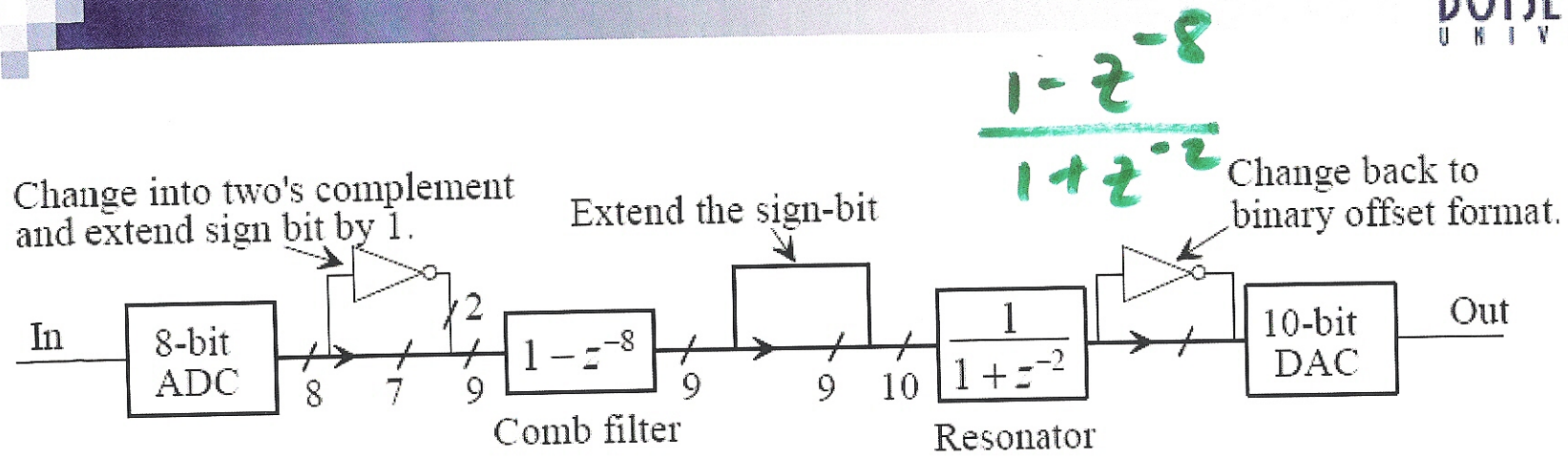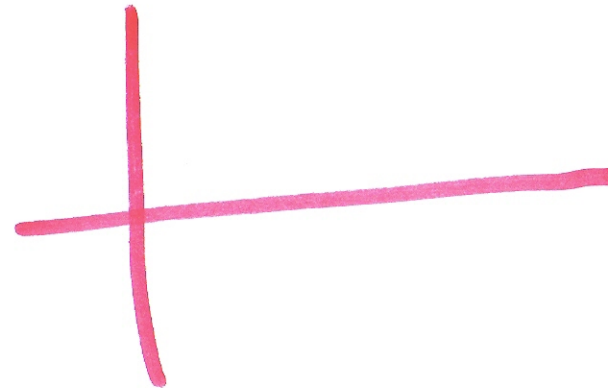
Comb filter

Resonator
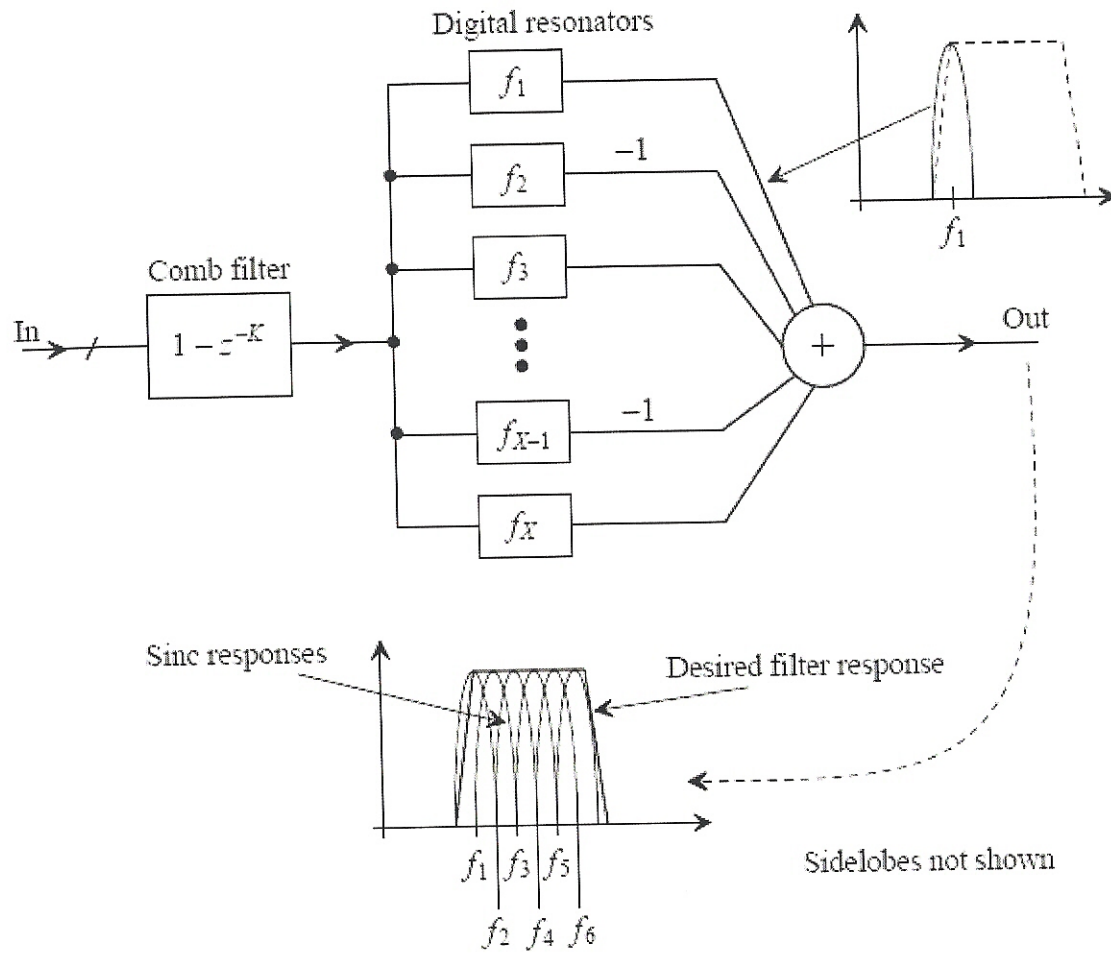
Figure 4.24 Digital filter sketch for Ex. 4.7.
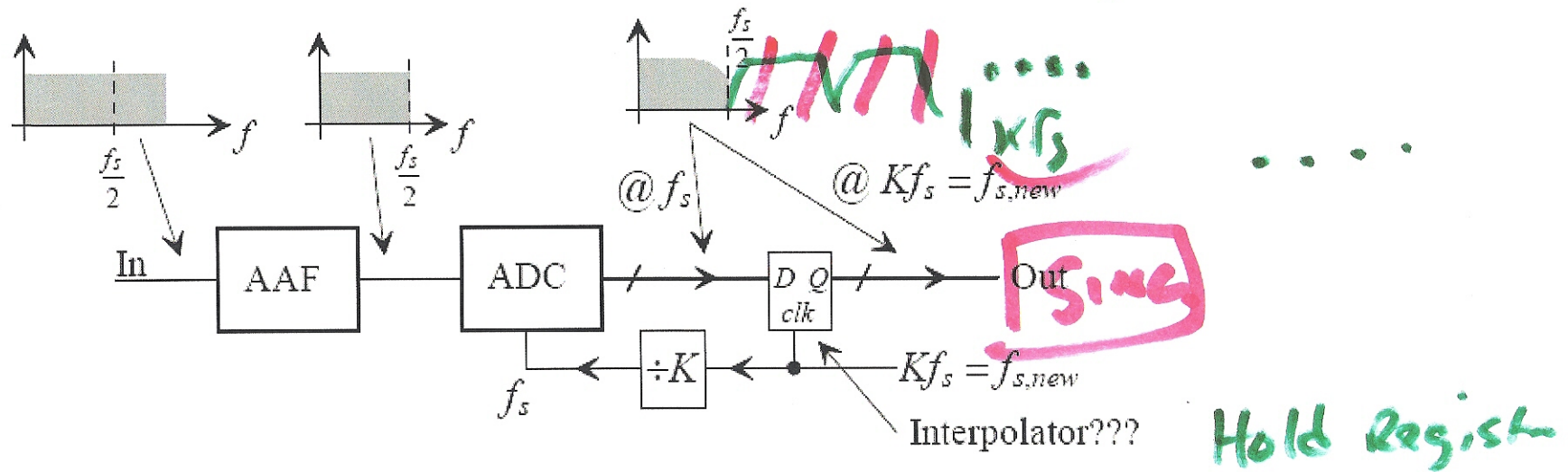
Figure 4.26 A frequency sampling filter.

Figure 4.27 Interpolation using a hold register (see Sec. 2.1.5).

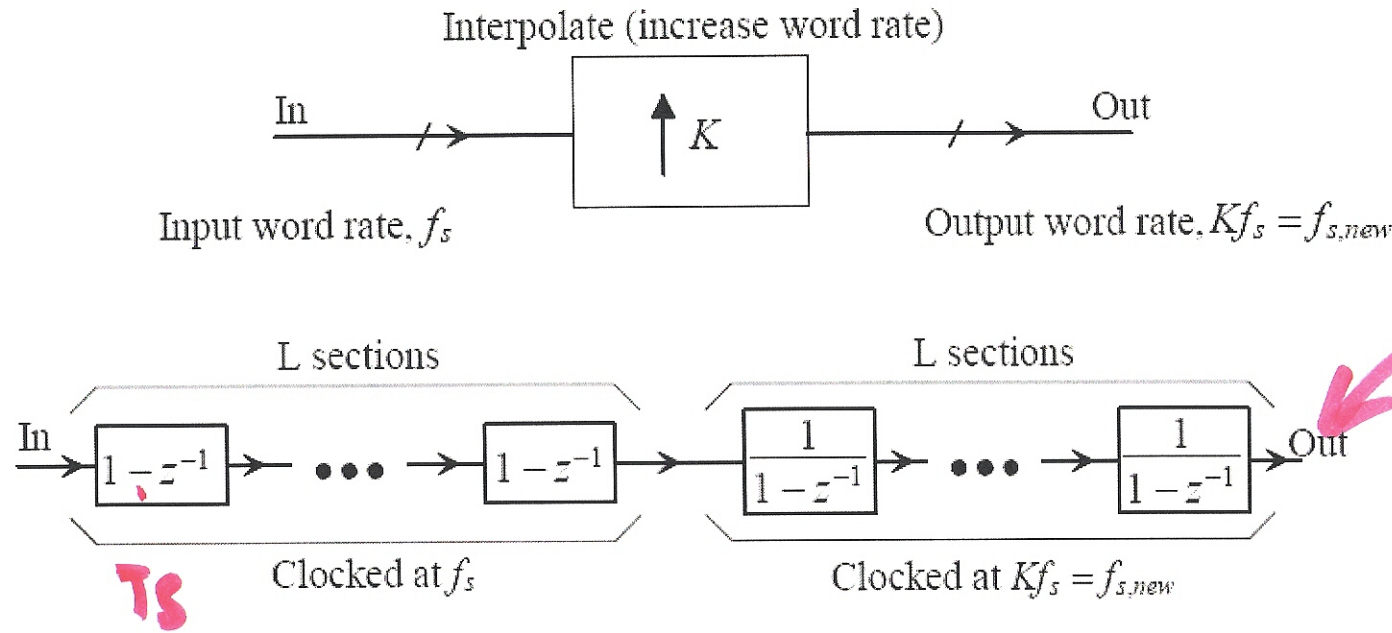**Figure 4.28** Interpolating using Sinc filters for image removal.

Handwritten annotations:

Kf_s

$\dfrac{TS}{K} \Rightarrow f_s \cdot K$

nf_s

TS

$\left(\dfrac{1 - z^{-K} - 1}{1 - z^{-2l} - 1} \right)^{2}$ @ f_s

2)

L sections   L sections

In $\rightarrow$ $\boxed{1 - z^{-M}}$ $\rightarrow$ $\bullet\bullet\bullet$ $\rightarrow$ $\boxed{1 - z^{-M}}$ $\rightarrow$ $\boxed{\dfrac{1}{1 - z^{-1}}}$ $\rightarrow$ $\bullet\bullet\bullet$ $\rightarrow$ $\boxed{\dfrac{1}{1 - z^{-1}}}$ $\rightarrow$ Out

Clocked at f_s Clocked at $Kf_s = f_{s,new}$

Figure 4.30 General interpolation using Sinc filters.

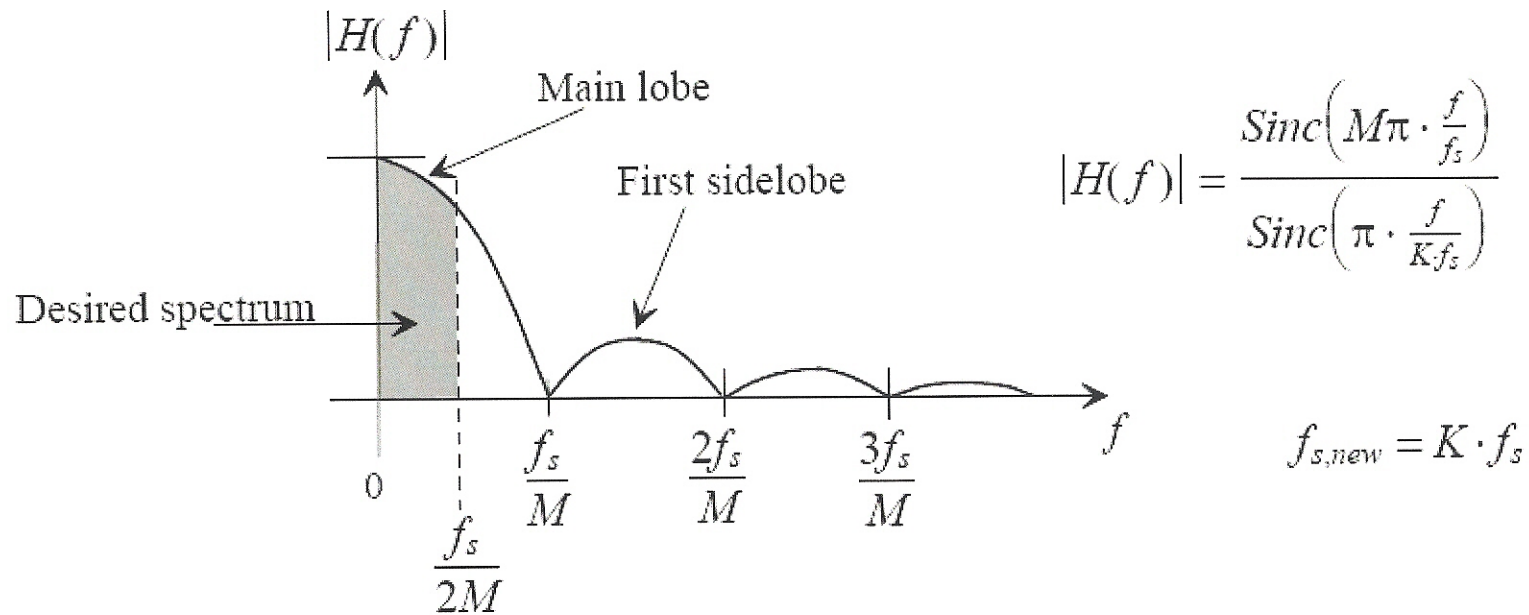$$\left(\frac{1 - z^{-KM}}{1 - z^{-1}} \right)$$

3)

Figure 4.31 Frequency response image removal filter using a Sinc interpolator, Fig. 4.30.

$$|H(f)| = \frac{Sinc\left(M\pi \cdot \frac{f}{f_s}\right)}{Sinc\left(\pi \cdot \frac{f}{K \cdot f_s}\right)}$$

$$f_{s,new} = K \cdot f_s$$

14)