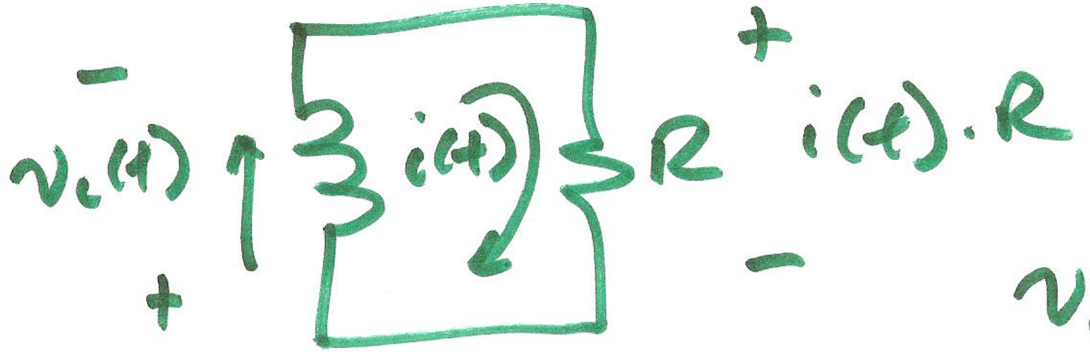
$\ln a - \ln b = \ln \frac{a}{b}$

Lecture 5

Sept. 3, 2010



$$v_L(t) + i(t)\cdot R = 0$$

$$L\frac{di(t)}{dt} = -R\cdot i(t)$$

$$\frac{L}{i(t)}\frac{di(t)}{dt} = -R$$

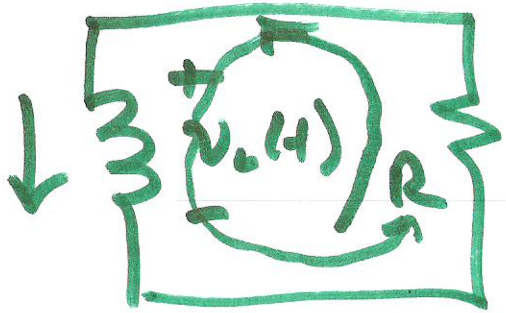
$$\int \frac{di}{i} = \int_0^t -\frac{R}{L}\cdot dt$$

$$\ln i(t) - \ln I(0^-) = -\frac{R}{L}\cdot t$$

$$\frac{i(t)}{I(0^-)} = e^{-t/(L/R)}$$

$$\boxed{i(t) = I(0^-)e^{-t/(L/R)}}$$

1)



$+$

$v_a(t) = -i(t) \cdot R$

$-$

$v_L(t) = u_L(t) = -i(t) \cdot R$

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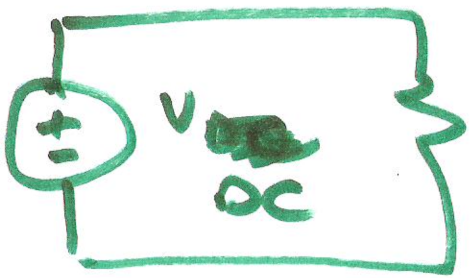
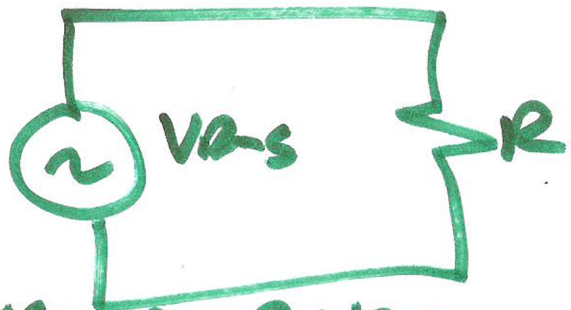


$R \to \frac{V_{DC}^2}{R}$



$\frac{V_{RMS}^2}{R}$

Average Power instantaneous power

$v(t) \cdot i(t)$

$$\frac{1}{T}\int_0^T p(t) \cdot dt$$

2)

$$P = \frac{V_{eff}^2}{R} = \frac{V_{rms}^2}{R}$$

$$= \frac{1}{T}\int_0^T \frac{v^2(t)}{R} \cdot dt$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t) \cdot dt}$$

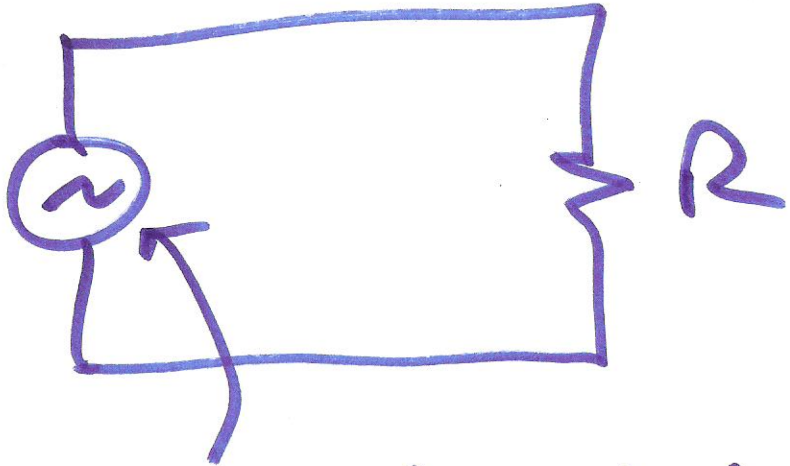
square ROOT of the MEAN

of the SQUARE

RMS

3)

$$V_{RMS} = \frac{V_p}{\sqrt{2}}$$



$V_p \sin(2\pi f t)$

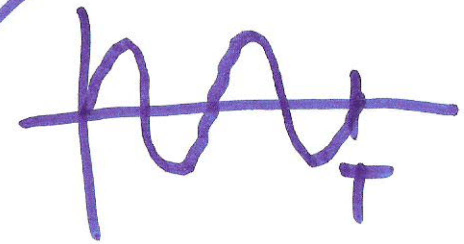
$$V_{RMS} = \sqrt{\frac{V_p^2}{T}\int_0^T \sin^2\left(2\pi \frac{t}{T}\right) dt}$$

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

4)

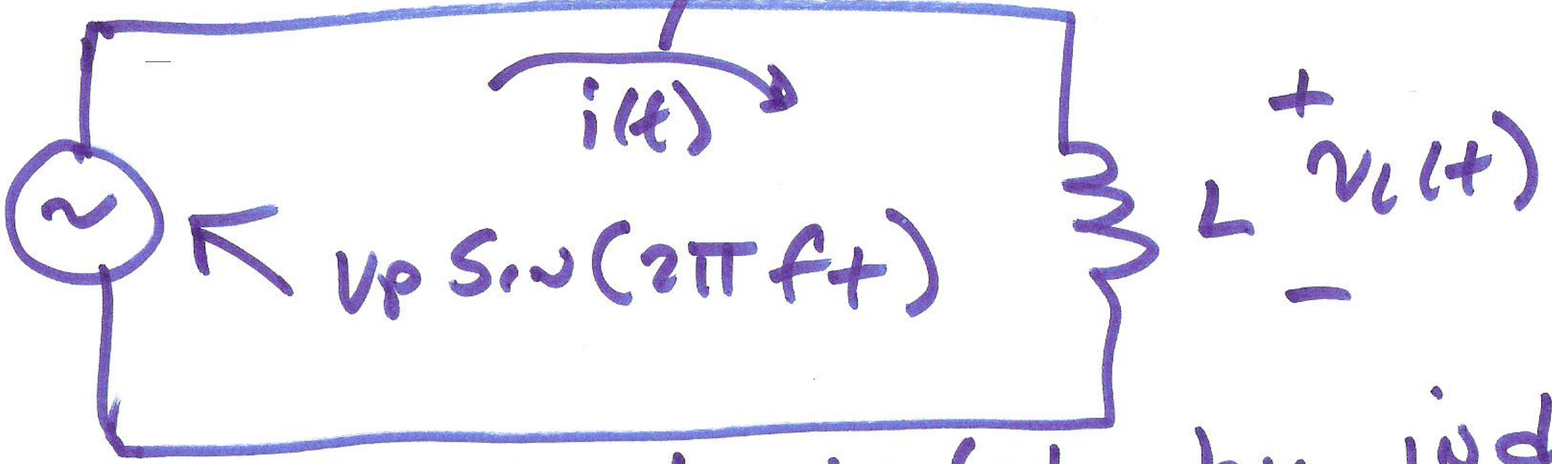
$$V_{RMS} = \underbrace{\frac{V_P}{\sqrt{2}} \cdot \sqrt{\frac{1}{T}\int_0^T \left(1 - \cos 4\pi f t\right) \cdot dt}}$$

$$\underbrace{\frac{1}{T}\int_0^T dT}_{1} - \underbrace{\frac{1}{T}\int_0^T \cos 4\pi f t \cdot dt}_{0}$$



5)

$= I_p \sin(2\pi f t + \theta)$



power dissipated by inductor

$$v_L(t) = V_p \sin(2\pi f \cdot t)$$

$$i(t) = \frac{1}{L}\int v_L(t)\cdot dt$$

$$P_L(t) = v_L(t)\cdot i(t)$$

$$i(t) = \frac{1}{L}\int_0^t V_p \sin(2\pi f \cdot t)\cdot dt$$

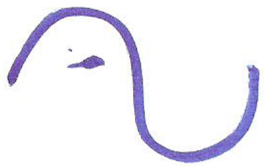
b)

$$i(t) = \frac{V_p}{L \cdot 2\pi f}\left(1 - \cos 2\pi f t\right)$$

$$P = \frac{1}{T}\int_0^T p(t) \cdot dt$$

Average power

$$= \frac{1}{T}\int_0^T V_p \sin(2\pi f t) \cdot$$



$$\frac{V_p}{L \cdot 2\pi f}\left(1 - \cos 2\pi f t\right) \cdot dt$$

$$\Rightarrow \quad P = \frac{V_p}{L \cdot 2\pi}\left[\overset{0}{\cancel{\int_0^T \sin(2\pi f t)\,dt}} - \int_0^T \sin(\ )\cos(\ )\,dt\right]$$

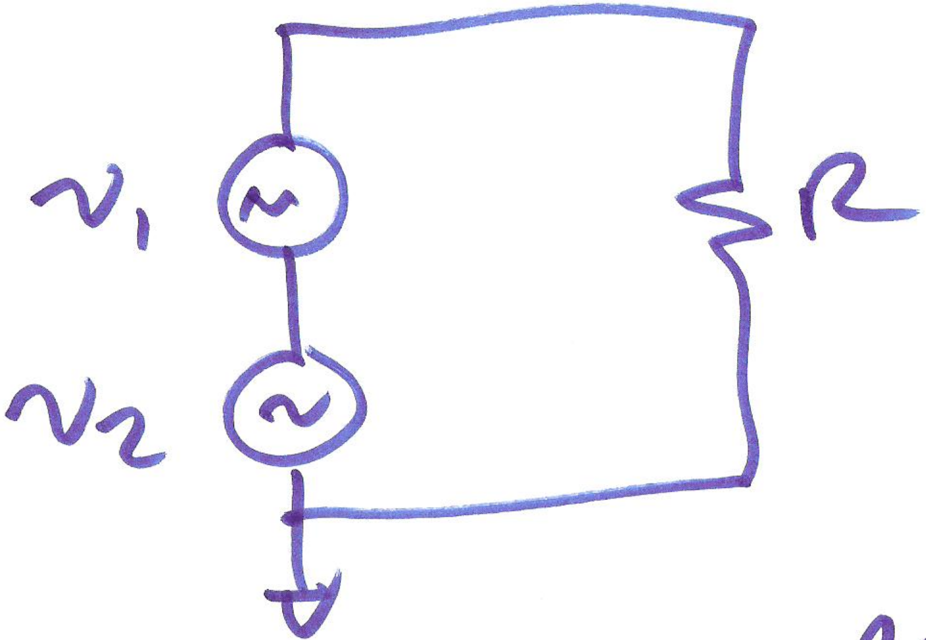
$$\int_0^T \sin(2\pi f t)\cdot\cos(2\pi f t)\cdot dt$$

$$2\sin x\cos x = \sin 2x$$

0

$$\int_0^T \frac{1}{2}\sin(4\pi f\cdot t)\cdot dt$$

8)



$$P = \frac{1}{T}\int_0^T p(t)\cdot dt$$

$$\cancel{p(t) = \frac{v_1^2}{R} + \frac{v_2^2}{R}}$$

$f_1 \neq f_2$

$$v_R = (v_1 + v_2)$$

$$P = \frac{1}{T}\int_0^T \frac{(v_1+v_2)^2}{R}\cdot dt \qquad p(t) = (v_1+v_2)\cdot\frac{v_1+v_2}{R} = \frac{(v_1+v_2)^2}{R}$$

$$\cancel{\frac{1}{T}\int_0^T v_1 v_2\, dT} \to 0$$

$$= \underbrace{\frac{1}{T}\int_0^T v_1^2\cdot dT}_{\to V_{1,rms}^2} + \underbrace{\frac{1}{T}\int_0^T v_2^2\cdot dT}_{\to V_{2,rms}^2}$$

9)

$$P = \frac{V_{1,RMS}^2}{R} + \frac{V_{2,RMS}^2}{R}$$

$$= \frac{V_{1,RMS}^2 + V_{2,RMS}^2}{R}$$

$f_1 = f_2$

$$v_{1p} \sin(2\pi f_1 t + \theta_1)$$

$$v_{2p} \sin(2\pi f_1 t + \theta_2)$$