where the decimation filter's transfer function is given by

$$|H(f)|^2 = \left[\frac{1}{K}\cdot\frac{\sin\left(K\pi\frac{f}{f_s}\right)}{\sin\left(\pi\frac{f}{f_s}\right)}\right]^{2(M+1)} \tag{7.80}$$

The mean-squared quantization noise is calculated by evaluating

$$V_{Qe,RMS}^2 = 2\cdot\overbrace{\frac{V_{LSB}^2}{12f_s}}^{|V_{Qe}(f)|^2}\cdot\int_0^{f_s/2}\overbrace{\left[2\sin\pi\frac{f}{f_s}\right]^{2M}}^{|NTF(f)|^2}\cdot\left[\frac{1}{K}\cdot\frac{\sin\left(K\pi\frac{f}{f_s}\right)}{\sin\left(\pi\frac{f}{f_s}\right)}\right]^{2(M+1)}\cdot df \tag{7.81}$$

or

$$V_{Qe,RMS}^2 = 2\cdot\frac{V_{LSB}^2}{12f_s}\cdot 2^{2M}\cdot\left[\frac{1}{K}\right]^{2(M+1)}\cdot\int_0^{f_s/2}\frac{\sin^{2(M+1)}\left(K\pi\frac{f}{f_s}\right)}{\sin^2\left(\pi\frac{f}{f_s}\right)}\cdot df \tag{7.82}$$

If we let $\theta = \pi\frac{f}{f_s}$, then we get

It should be $\frac{1}{2}\left(\frac{2}{K}\right)^{2(M+1)}$

$$V_{Qe,RMS}^2 = \frac{V_{LSB}^2}{12f_s}\cdot\left[\frac{2}{K}\right]^{2(M+1)}\cdot\frac{f_s}{\pi}\cdot\overbrace{\int_0^{\frac{\pi}{2}}\frac{\sin^{2(M+1)}(K\theta)}{\sin^2\theta}\cdot d\theta}^{=\frac{K}{2}\pi\cdot\prod_{m=1}^{M}\frac{2m-1}{2m}} \tag{7.83}$$

Finally, the RMS quantization noise associated with an M[th]-order modulator followed by an $M+1$ $(=L)$ Sinc averaging filter is

(7.83) should be

$$V_{Q_e,RMS}^2 = \frac{V_{LSB}^2}{12f_S}\frac{1}{2}\left(\frac{2}{K}\right)^{2(M+1)}\frac{f_S}{\pi}\int_0^{\frac{\pi}{2}}\frac{\sin^{2(M+1)}(K\theta)}{\sin^2(\theta)}d\theta$$

$$= \frac{V_{LSB}^2}{12f_S}\frac{1}{2}\left(\frac{2}{K}\right)^{2(M+1)}\frac{f_S}{\pi}\cdot\frac{K\pi}{2}\prod_{m=1}^{M}\frac{2m-1}{2m}$$

$$= \frac{V_{LSB}^2}{12}\frac{2^{2M}}{K^{2M+1}}\prod_{m=1}^{M}\frac{2m-1}{2m}$$

(7.84) should be

$$V_{Q_e,RMS} = \frac{V_{LSB}}{\sqrt{12}}\frac{2^M}{K^{M+\frac{1}{2}}}\sqrt{\prod_{m=1}^{M}\frac{2m-1}{2m}}$$

(7.85) should be

$$SNR\_increase = -20\lg\left(2^M\sqrt{\prod_{m=1}^{M}\frac{2m-1}{2m}}\cdot\frac{\sqrt{2M+1}}{\pi^M}\right)$$

For first-, second-, and third-order modulators, the improvement in the SNRs is 2.16dB, 5.1148dB, and 8.3677dB respectively.

Verification in matlab:

```
r(1)=2*sqrt(1/2)*sqrt(3)/pi;
r(2)=4*sqrt(1/2*3/4)*sqrt(5)/pi^2;
r(3)=8*sqrt(1/2*3/4*5/6)*sqrt(7)/pi^3;
20*log10(r)
```