The small-signal circuit of the feedback circuit is seen in Fig. 31.14. Note that the forward path consists of the following nodes: 1, 2, 3. The feedback path consists of nodes 3 and 4, with the feedback variable, $v_f$, appearing across $R_1$. In the previous discussion, it was assumed that the $\beta$ network did not load the amplifier circuit. However, to accurately calculate the open-loop gain, $A_{OL}$, the loading of $R_1$ and $R_2$ on both the input and the output of the amplifier circuit needs to be considered. Note that the resistor $R_S$ is initially ignored, since it is essentially outside the feedback amplifier.

![Figure 31.14](image)

**Figure 31.14** Closed-loop small-signal model of Fig. 31.13.

Since we are analyzing a series-shunt amplifier, we may determine the loading caused by the $\beta$ network on the input, $R_{\beta i}$, and the output $R_{\beta o}$, in the following way (refer to Fig. 31.15). Looking into the $\beta$ network from the input, we observe the resistance seen with the output terminal shorted to ground. The equivalent resistance to ground seen is the loading of the $\beta$ network seen by the input of the amplifier. In this example, $R_2$ is seen. Therefore, in the open-loop model used to determine $A_{OL}$, we will include $R_2$ in parallel with $R_1$. The loading at the output is found similarly. Since the input mixing is series, we will remove M1 “out-of-socket” and look into the $\beta$ network from the output. The equivalent resistance seen is then attached to the output of the open-loop model. In this example, the equivalent resistance is $R_2 + R_1$ and is attached to the output of the

![Figure 31.15](image)

**Figure 31.15** Determining the loading due to the feedback network for a series-shunt amplifier.
The resulting open-loop model is seen in Fig. 31.16. We will initially assume that \( r_o \) for the MOSFETs is much larger than the discrete resistors and that the bulk and source are tied together \((v_{sb} = 0)\). As we progress through the chapter, more difficult circuits will include drain-to-source resistances in our small-signal analysis.

The open-loop model is now ready to be analyzed in order to calculate \( A_{OL} \). Since we are using a series (voltage)-shunt (voltage) feedback amplifier, the units of \( A_{OL} \) will be \( V/V \) and

\[
A_{OL} = \frac{v_2^*}{v_s^*} \tag{31.34}
\]

Solving for \( A_{OL} \) yields

\[
A_{OL} = \frac{v_2^*}{v_s^*} = \left( \frac{v_2^*}{v_{sg2}^*} \right) \left( \frac{v_{sg2}^*}{v_{sg1}^*} \right) = \left[ -g_{m2}R_L \right] \left[ (R_2 + R_1) \right] \left[ \frac{-g_{m1}R_3}{1 + g_{m2}R_4} \right] \left[ \frac{1}{1 + g_{m1}(R_1 || R_2)} \right]
\tag{31.35}
\]

Next, the value of \( \beta \) can also be calculated from the open-loop model. Remembering that \( \beta \) is defined as the gain from the output back to the input mixing variable, \( v_f \), we can write

\[
\beta = \frac{v_f^*}{v_2^*} = \frac{R_1}{R_1 + R_2} \tag{31.36}
\]

since the \( \beta \) network is simply a voltage divider relationship. Notice that the open-loop circuit now contains two values of \( R_2 \) and \( v_f^* \). In this example, since \( r_o \) was assumed to be infinite, the gain from \( v_2^* \) to \( v_f^* \) will be zero. If \( r_o \) had not been neglected, the gain from \( v_2^* \) to \( v_f^* \) would have been small but finite. Therefore, it can be said that a reverse path exists through the basic amplifier as well as through the feedback network. However, the gain from \( v_2^* \) to \( v_f^* \), though less than one, will be significantly larger than from \( v_2^* \) to \( v_f^* \).

Therefore, just as the forward path through the feedback network was neglected, the reverse path through the basic amplifier is assumed to be much smaller than the reverse path through the feedback path. Therefore, the value of \( \beta \) is calculated using the resistor, \( R_2 \), closest to the output.

Next, the value for \( R_i \) and \( R_o \) will be calculated. These values are determined using the open-loop model generated in Fig. 31.16. Since we are using MOS devices, it should
Figure 31.17  (a) Series-shunt circuit used in Ex. 31.1; (b) its closed-loop small-signal model; and (c) the resulting open-loop model.
Figure 31.20 (a) Closed-loop small-signal model of Fig. 31.19 and (b) method for determining the feedback network loading.

Figure 31.21 Open-loop small-signal model of Fig. 31.19.

Figure 31.22 (a) Solving a portion of Fig. 31.21, including the drain-to-source resistance, and (b) the equivalent transconductance model.
The value of $R_{Leq}$ can easily be found as

$$R_{Leq} = R_L || R_2 || R_{inD2} \quad (31.49)$$

where $R_{inD2}$ is the resistance seen looking into the drain of M2. From Ch. 20, we know that this resistance is

$$R_{inD2} = [(1 + g_{m2} R_4) v_{o2} + R_4] \quad (31.50)$$

The value of $G_M$ is the short-circuit transconductance and is defined as

$$G_M = \frac{i_o^*}{v_{g2}^*} (R_{Leq} = 0) \quad (31.51)$$

which means that the effective transconductance can be found by shorting the equivalent load resistance, in this case $R_L || R_2$, and finding the gain from the short-circuit current to the input voltage. As seen in Fig. 31.23, the equations used to find $G_M$ are

$$i_o^* = g_{m2} v_{sg2}^* + \frac{v_{g2}^*}{r_{o2}} \quad (31.52)$$
$$v_{s2}^* = -i_o^* R_4 \quad (31.53)$$
$$v_{s2}^* = v_{sg2}^* + v_{g2}^* \quad (31.54)$$

and solving Eqs. (31.52) - (31.54) yields

$$G_M = \frac{i_o^*}{v_{g2}^*} = \frac{-g_{m2}}{1 + g_{m2} R_4 + \frac{R_4}{r_{o2}}} \quad (31.55)$$

the gain, $\frac{v_2^*}{v_{g2}^*}$, becomes

$$\frac{v_2^*}{v_{g2}^*} = \frac{-g_{m2}(R_L || R_2 || [(1 + g_{m2} R_4) v_{o2} + R_4])}{1 + g_{m2} R_4 + \frac{R_4}{r_{o2}}} \quad (31.56)$$

Figure 31.23 Circuit used to determine the equivalent transconductance.

Referring back to Eq. (31.47), the second factor, $\frac{v_{g2}^*}{v_1^*}$, can be found by analyzing Fig. 31.21 as
Since the output and the feedback are connected to two separate terminals of the output device, the output variable is a current, sampling $i_o$. The small-signal model for this circuit is shown in Fig. 31.31 with the open-loop, small-signal model shown in Fig. 31.32. Since the output sampling is a current, loading of the $\beta$ network will be slightly different from that of the series-shunt example. The input utilizes series mixing; therefore the loading of the $\beta$ network on the output will be identical to the series-shunt example discussed previously ($R_{\beta o} = R_1 + R_2$). However, since the output sampling is series, the equivalent resistance, $R_{\beta i}$, will be the resistance seen looking into the $\beta$ network from the input, with the output device taken "out-of-socket" and $R_{\beta i} = R_2 + R_5$.

Once the open-loop model has been constructed, $A_{OL}$ can be calculated as

$$A_{OL} = \frac{i_o^*}{v_s} = \frac{i_o^*}{v_{g2}^*} \cdot \frac{v_{g2}^*}{v_s^*} \quad (31.80)$$

Figure 31.31 (a) Closed-loop small-signal model of Fig. 31.30 and (b) method for determining feedback loading.
where the term, \( \frac{i_o^*}{v_{g2}^*} \), can be determined by using straightforward circuit analysis to solve \( \frac{v_{g2}^*}{v_{s}} \) and then dividing the result by \( R_4 \),

\[
\frac{i_o^*}{v_{g2}^*} = \frac{g_{m2}}{1 + g_{m2}R_4 + \frac{R_4}{R_4 + R_5 + R_1}} \tag{31.81}
\]

The term, \( \frac{v_{g2}^*}{v_{s}} \), is found by using the \( G_m \) method presented in the previous section on shunt-shunt feedback and is

\[
\frac{v_{g2}^*}{v_{s}} = \frac{-g_{m1}(R_3 || [(1 + g_{m1}R_4)R_{o1} + R_4])}{1 + g_{m1}R_4 + \frac{R_4}{R_4 + R_1}} \text{ mhos} \tag{31.82}
\]

where \( R_4 = R_1 || (R_2 + R_3) \). The feedback factor, \( \beta \), is

\[
\beta = \frac{v_f^*}{i_o^*} \approx \frac{-R_5R_1}{R_5 + R_1 + R_2} \Omega \tag{31.83}
\]

And the closed-loop gain is simply

\[
A_{CL} = \frac{i_o}{v_s} = \frac{A_{OL}}{1 + A_{OL} \beta} \text{ mhos} \tag{31.84}
\]

The value of \( R_\beta \) is obviously infinite, resulting in an identical value of \( R_{inf} \). Therefore, \( R_\infty = R_{inf} || R_G = R_G \).

Calculating \( R_o \) for a series output requires some explanation. Examine Fig. 31.33.

The value of \( R_o \) is the value seen looking in series with the load resistor. In this case, the value of \( R_o \) becomes

\[
R_o = R_4 + \frac{R_B}{R_2 + R_1 + g_{m2}} \approx R_4 + \frac{1}{g_{m2}} \tag{31.85}
\]

where \( R_B = R_5 || (R_1 + R_2) \) and the closed-loop value becomes

\[
R_{oC} = R_o(1 + A_{OL} \beta) \tag{31.86}
\]
Notice, however, that $R_{of}$ is not the same as $R_{out}$, in this case. Typically, $R_{out}$ is designated as the resistance in parallel with the load. Taking the resistance in series with the load is not a practical specification. Therefore, the resistance $R_{out}$ can be described as seen in Fig. 31.34. In part (a), it can be seen that $R_{of} = R_o (1 + A_{OL}/\beta)$ and that $R'_{of} = R_{of} - R_4$. If we want to find a value for $R_{out}$, using Fig. 31.34b, $R_{out}$ is simply

$$R_{out} = R_4 || R'_{of} = R_4 || (R_{of} - R_4)$$  \hspace{1cm} (31.87)

31.7 The Current Amplifier (Shunt-Series Feedback)

The last feedback topology to be discussed is the shunt-series feedback amplifier, also known as a current amplifier. As can be expected, both $A_{OL}$ and $\beta$ have units of $\text{I}/\text{I}$, and we can expect the input impedance to be very low and the output impedance very high. Figure 31.35 illustrates the ideal shunt-series amplifier with open-loop values included. Based on past derivations, we can expect that

$$R_{inf} = \frac{R_i}{(1 + A_{OL}/\beta)}$$  \hspace{1cm} (31.88)

and $R_{of}$ to be

$$R_{of} = R_o (1 + A_{OL}/\beta)$$  \hspace{1cm} (31.89)

The derivations of this topology will be left to the reader in the Problems section.