Frequency Response of an Ideal Integrator

The frequency response of the integrator seen in Fig. 1.33 can be determined knowing the op-amp keeps the inverting input terminal at the same potential as the non-inverting input (here ground). The current through the resistor must equal the current through the capacitor so

\[
\frac{V_{in}}{R} + \frac{V_{out}}{1/j\omega C} = 0
\]

or

\[
\frac{V_{out}}{V_{in}} = -\frac{1}{j\omega RC} = \frac{-1 + j\cdot 0}{0 + j\omega RC}
\]

The magnitude of the integrator's transfer function is

\[
\left| \frac{V_{out}}{V_{in}} \right| = \frac{\sqrt{(1)^2 + (0)^2}}{\sqrt{(0)^2 + (2\pi fRC)^2}} = \frac{1}{2\pi fRC}
\]

while the phase shift through the integrator is

\[
\angle \frac{V_{out}}{V_{in}} = -\left( \tan^{-1} \frac{0}{1} - \tan^{-1} \frac{2\pi fRC}{0} \right) = +90^\circ
\]

Note that the gain of the integrator approaches infinity as the frequency decreases towards DC while the phase shift is constant.

Unity-Gain Frequency

It's of interest to determine the frequency where the magnitude of the transfer function is unity (called the unity-gain frequency, \(f_{u}\)). Using Eq. (1.15), we can write

\[
\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{2\pi fRC}
\]