Lock-in time calculation - Wenlan Wu (http://cmosedu.com/jbaker/students/wenlan/wenlan.htm)

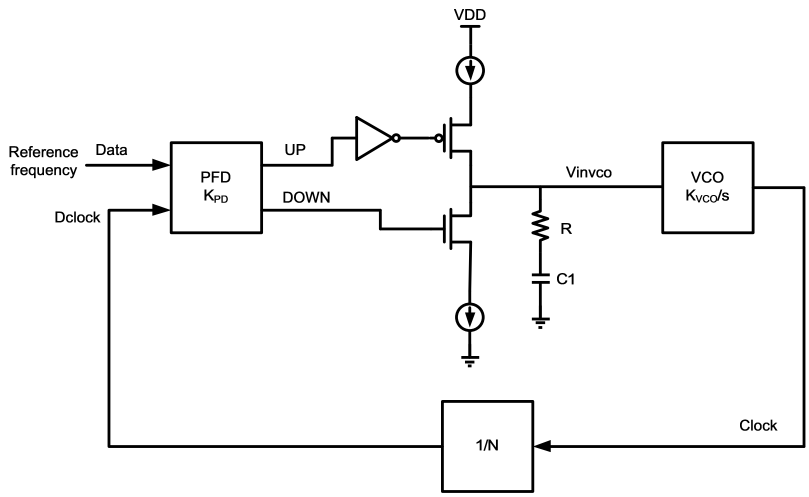

Figure 1 Charge-pump PLL block diagram

First, for the above feedback system, we can get the loop gain and transfer function of the close loop.

Loop Gain: $$T(s) = K_{FWD}(s) * K_{FB}(s) = K_{PD} * F(s) * \frac{K_{VCO}}{s} * \frac{1}{N} = \frac{K_{PD}K_{VCO}F(s)}{Ns}$$

Transfer function: $$H(s) = \frac{K_{FWD}(s)}{1+T(s)} = \frac{K_{PD}*F(s)*\frac{K_{VCO}}{s}}{1+T(s)}$$

$$= \frac{K_{PD} * F(s) * \frac{K_{VCO}}{s}}{1 + \frac{K_{PD}K_{VCO}F(s)}{Ns}}$$

Due to the loop filter s-function is $F(s) = R + \frac{1}{sC1} = \frac{1+sRC1}{sC1}$, so the transfer function can be revised as

$$H(s) = \frac{K_{PD} * \frac{1+sRC1}{sC1} * \frac{K_{VCO}}{s}}{1 + \frac{K_{PD}K_{VCO}\frac{1+sRC1}{sC1}}{Ns}} = \frac{K_{PD}K_{VCO} * \frac{1+sRC1}{C1}}{s^2 + \frac{K_{PD}K_{VCO}R}{N} * s + \frac{K_{PD}K_{VCO}}{NC1}}$$

From the denominator of the transfer function, it is a second-order system. We use the natural frequency and damping factor to redefine the transfer function, which are standard control system terminology. The denominator of the closed loop transfer function is revised into a “standard” form:

$$H(s) = \frac{K_{PD}K_{VCO} * \frac{1+sRC1}{C1}}{s^2 + \frac{K_{PD}K_{VCO}R}{N} * s + \frac{K_{PD}K_{VCO}}{NC1}} = \frac{K_{PD}K_{VCO} * \frac{1+sRC1}{C1}}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

So we can get

$$\omega_n = \sqrt{\frac{K_{PD}K_{VCO}}{NC1}}$$

$$\xi = \frac{\omega_n}{2}RC1$$

For the PLL system, we can calculate its step response. Assume the input signal is $F_{in}(s) = \frac{1}{s}$

$$F_{out}(s) = F_{in}(s) * H(s) = \frac{K_{PD}K_{VCO} * \frac{1 + sRC1}{C1}}{s(s^2 + 2\xi\omega_n s + \omega_n^2)} = \frac{N\omega_n^2 * (1 + sRC1)}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$
$$= \frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Based on the equation $A(s^2 + 2\xi\omega_n s + \omega_n^2) + Bs^2 + Cs = N\omega_n^2 * (1 + sRC1)$, we can get

$$A = N$$

$$Bs + C = N(\omega_n^2 RC1 - 2\xi\omega_n - s)$$

Use the equations of natural frequency and damping factor, we can get $RC1 = 2\xi/\omega_n$. And also we assume N=1 to simplify the calculations.

Hence, the closed loop response can be given as

$$F_{out}(s) = \frac{1}{s} + \frac{-s}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

<u>Sec-I:</u> assume the damping factor$0 < \xi < 1$, we can get the roots of denominator of the closed loop transfer function.

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1 - \xi^2}$$

Based on Laplace transform pairs, we have

$$\frac{\beta}{(s + \alpha)^2 + \beta^2} \xrightarrow{Inverse\ Laplace} e^{-\alpha t}\sin(\beta t)u(t)$$

$$\frac{s + \alpha}{(s + \alpha)^2 + \beta^2} \xrightarrow{Inverse\ Laplace} e^{-\alpha t}\cos(\beta t)u(t)$$

The output signal in time domain is expressed as

$$F_{out}(s) = \frac{1}{s} + \frac{-s}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{s} + \frac{-s - \xi\omega_n + \xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2} = \frac{1}{s} - \frac{s + \xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2} + \frac{\xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + \xi\omega_n}{s^2 + 2\xi\omega_n s + \omega_n^2} + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} * \frac{\omega_n\sqrt{1-\xi^2}}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$= \frac{1}{s} - \frac{s + \xi\omega_n}{(s + \xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2} + \frac{\xi\omega_n}{\omega_n\sqrt{1-\xi^2}} * \frac{\omega_n\sqrt{1-\xi^2}}{(s + \xi\omega_n)^2 + \left(\omega_n\sqrt{1-\xi^2}\right)^2}$$

$$\xrightarrow{Inverse\ Laplace} 1 - e^{-\xi\omega_n t}\cos\left(\omega_n\sqrt{1-\xi^2}t\right) + \frac{\xi}{\sqrt{1-\xi^2}} * e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}t\right) = f_{out}(t)$$

From the subtraction formula of the trigonometric identities, we have

$$cos\alpha t - sin\alpha t = \sqrt{2} * \left(\frac{\sqrt{2}}{2}cos\alpha t - \frac{\sqrt{2}}{2}sin\alpha t\right) = \sqrt{2} * \cos(\alpha t + 45°)$$

The output signal can be given as

$$f_{out}(t) = 1 - Ae^{-\xi\omega_n t}\cos\left(\omega_n\sqrt{1-\xi^2}t + \theta\right)$$

Where $A = \sqrt{1 + \left(\frac{\xi}{\sqrt{1-\xi^2}}\right)^2} = \frac{1}{\sqrt{1-\xi^2}}, \theta = \tan^{-1}\frac{\xi}{\sqrt{1-\xi^2}}$

The settling time or lock-in time is the time required for the oscillations to die down and stay within 2% of the final value.

Based on the output decaying signal, we get the lock-in time is when envelope decays to 2%.

$$Ae^{-\xi\omega_n t} = \frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t} = 0.02$$

$$T_L = -\frac{\ln\left(0.02\sqrt{1-\xi^2}\right)}{\xi\omega_n}, \quad 0 < \xi < 1$$

Damping factor $(0 < \xi < 1)$	Lock-in Time/Settling Time
$\xi = 0.2$	$T_L = 19.67/\omega_n$
$\xi = 0.5$	$T_L = 8.11/\omega_n$
$\xi = 0.707$	$T_L = 6.02/\omega_n$
$\xi = 0.9$	$T_L = 5.27/\omega_n$

Sec-II: assume the damping factor $\xi = 1$. We can get the roots of denominator of the closed loop transfer function.

$$s_{1,2} = -\xi\omega_n = -\omega_n$$

For N=1, the output signal in time domain is expressed as

$$F_{out}(s) = \frac{\omega_n^2(1+sRC1)}{s(s^2+2\omega_n s+\omega_n^2)} = \frac{1}{s} + \frac{-s}{s^2+2\omega_n s+\omega_n^2} = \frac{1}{s} + \frac{-s-\omega_n+\omega_n}{(s+\omega_n)^2} = \frac{1}{s} - \frac{s+\omega_n}{(s+\omega_n)^2} + \frac{\omega_n}{(s+\omega_n)^2}$$
$$= \frac{1}{s} - \frac{1}{s+\omega_n} + \frac{\omega_n}{(s+\omega_n)^2}$$

$$\xRightarrow{Inverse\ Laplace} \quad 1 - e^{-\omega_n t} + \omega_n t * e^{-\omega_n t} = 1 - e^{-\omega_n t}(1-\omega_n t) = f_{out}(t)$$

Based on the output decaying signal, we get the lock-in time is when the decay term becomes 2%.

$$e^{-\omega_n t}(1-\omega_n t) = 0.02$$

Define $K = \omega_n t$, the equation becomes

$$e^{-K}(1-K) = 0.02$$

$$T_L \approx \frac{0.95}{\omega_n}$$

For using the RC loop filter, the lock-in time is smaller than $\frac{2\pi}{\omega_n}$.

Let's calculate the lock-in time for PLL using only C in the loop filter.

Sec-III: Also assume damping factor $\xi = 1$. Because loop filter is only one capacitor, the closed loop transfer function is revised as

$$H(s) = \frac{K_{PD}K_{VCO} * \frac{1}{C1}}{s^2+2\omega_n s+\omega_n^2} = \frac{N\omega_n^2}{s^2+2\omega_n s+\omega_n^2}$$

Assume N=1, we can get the output signal in time domain can be given by

$$F_{out}(s) = \frac{\omega_n^2}{s(s^2+2\omega_n s+\omega_n^2)} = \frac{1}{s} - \frac{1}{s+\omega_n} - \frac{\omega_n}{(s+\omega_n)^2}$$
$$\xRightarrow{Inverse\ Laplace} 1 - e^{-\omega_n t}(\omega_n t + 1) = f_{out}(t)$$

Based on the output decaying signal, we get the lock-in time is when the decay term becomes 2%.

$$e^{-\omega_n t}(1+\omega_n t) = 0.02$$

Define $K = \omega_n t$, the equation becomes

$$e^{-K}(1+K) = 0.02$$

$$T_L \approx \frac{5.83}{\omega_n}$$

The lock-in time is very close to $\frac{2\pi}{\omega_n}$.

Conclusion:

As we know, adding a resistor series with the capacitor of loop filter can get a better stability for the charge-pump PLL. In addition, from the above Sec-II and Sec-III, we can see the resistor can reduce the lock-in time when the damping factor equals to 1.

Section-2: Lock-in range calculation [1]

Initially, the PLL is out of lock. The input frequency or reference frequency is ω_1 and the output is working at center frequencyω_0. In the lock-in process, the output frequency is a function of time defined as$\Delta\omega_{vco}$. So the initial frequency offset is also a function of time $\Delta\omega = \omega_1 - \omega_0$.

The input signal of the PLL is a square wave and given by

$$x(t) = A_{in} rect(\omega_0 t + \Delta\omega t)$$

The output signal is given by

$$y(t) = A_o rect(\omega_0 t)$$

So the phase difference can be given by $\theta_e = \Delta\omega t$ which is a ramp function.

The average output of the phase detector is a triangular signal when phase error ramps up linearly with time. Its peak amplitude is $K_{PD} * 2\pi$ for using the PFD in a charge-pump PLL. Therefore, the average output of the PFD is written by

$$\overline{y_{PD}} = K_{PD} * 2\pi * tri(\Delta\omega t)$$

The average output of the loop filter is written by

$$\overline{y_{LP}} = F(\Delta\omega) K_{PD} * 2\pi * tri(\Delta\omega t)$$

The peak deviation for the VCO output becomes

$$\Delta\omega_{vco,max} = K_{VCO} F(\Delta\omega) K_{PD} * 2\pi * tri(\Delta\omega t)$$

For the lock-in process, the PLL acquires lock within one beat note when $\frac{\Delta\omega_{vco,max}}{N}$ equals to the frequency offset $\Delta\omega$. Under this condition, the $\Delta\omega$ is identical with the lock range $\Delta\omega_L$.

$$\Delta\omega_L = K_{VCO} F(\Delta\omega_L) K_{PD} * 2\pi$$

For practical considerations, the lock range is always much greater than corner frequency of loop filter,$\frac{\Delta\omega_L}{\tau} \gg 1$. For the filter gain $|F(\Delta\omega_L)|$, we can make the approximation.

$$|F(\Delta\omega_L)| = \left|\frac{1 + sRC}{sC}\right| = \frac{\sqrt{1 + (\Delta\omega_L RC)^2}}{\Delta\omega_L C} \approx \frac{\sqrt{(\Delta\omega_L RC)^2}}{\Delta\omega_L C} = R$$

So the lock range is equal to

$$\Delta\omega_L = K_{VCO}K_{PD} * 2\pi R$$

From the loop gain of second order system of the CPPLL, we get $2\xi\omega_n = \frac{K_{VCO}K_{PD}R}{N}$

Hence, the lock range is

$$\Delta\omega_L = 4\pi\xi\omega_n$$

Which is the lock range for the Type-II charge-pump PLL.

References:

1. Roland E. Best, “Phase-Locked Loops: Design, Simulation and Applications”, McGraw-Hill, 6th ed., 2007. ISBN 978-0071493758.